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Concept for Numerical Calculation of 3D MHD Equilibria with Flow and FLR Effects

流れと有効ラーモア半径効果を取り入れた 3次元MHD平衡の数値解析の進展

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One of the simplest models for describing plasma equilibrium is the single-fluid magnetohydrydynanimc (MHD) model under axisymmetry with no fluid flow. However, both non-axisymmetric effects and fluid flow can significantly alter plasma equilibrium. A code which can rapidly calculate 3D MHD equilibrium with fluid flow is important for future modeling of fusion plasmas. Presently, there are many codes which can look at a subset of the physics of interest. Based on those codes, a new code for calculating 3D MHD equilibrium with fluid flow is being developed.

1. Overview of MHD Equilibrium

Single-fluid MHD equilibrium is governed by conservation of mass, Ohm's law, force-balance, and Maxwell's equations with quasi-neutrality. Closure requires an equation of state, often taken to be an adiabatic law. Consider a cylindrical coordinate system (R, ϕ, Z), where R is the major radius, ϕ is the toroidal angle, and Z is the distance above the mid-plane. For the case of an axisymmetric plasma ($\partial/\partial\phi = 0$) with no bulk fluid flow, equilibrium is governed by the well-known Grad-Shafranov equation [1]:

$$\nabla\cdot\left(\frac{1}{R^2}\nabla\psi\right) = \frac{1}{2R^2}\frac{dI^2}{d\psi}+\mu_0\frac{dp}{d\psi} \qquad (1)$$

where ψ is the poloidal magnetic flux, I is the poloidal current, and p is the pressure. Equilibrium depends on the boundary conditions for ψ, as well as the two free functions $p(\psi)$ and $I(\psi)$.

With the inclusion of fluid flow, equilibrium is governed by a *generalized* Grad-Shafranov equation and a Bernoulli equation [2]. Equilibrium in this case depends on five free functions of the poloidal flux, which can be taken to be the flux-surface averages of the following quantities: (1) the poloidal current I; (2) the pressure p; (3) the density n; (4) the electrostatic scalar potential Φ; and, (5) the stream function Ψ, which measures the poloidal flow [3].

However, the inclusion of fluid flow can produce a sharp profile pedestal [2-6]. In the single-fluid MHD model, the pedestal occurs where the poloidal flow transitions from sub- to super-poloidal sonic. However, because of the small scale-length of the pedestal, the inclusion of finite Larmor radius (FLR) effects are generally necessary to accurately model the plasma. Ito and Nakajima have developed a formulation for MHD equilibrium with flow and FLR effects using an expansion in the inverse aspect-ratio (ε), which includes FLR effects by the ordering $\rho/a \sim \varepsilon$, where ρ is the ion Larmor radius and a is the plasma minor radius [6]. In this formulation, equilibrium depends on five free profiles of the lowest-order poloidal flux, which are related to the lowest non-constant order of the following quantities: (1) the poloidal current I; (2) the electron pressure p_e; (3) the ion pressure p_i; (4) the density n; and, (5) the electrostatic scalar potential Φ [7]. However, because the effects of flow are included as a linear perturbation, the pedestal becomes a singularity, and the formulation cannot be directly applied to trans-poloidal-sonic flows.

In 3D cases, even with no flow, equilibrium is complicated by the absence of a guarantee of nested flux-surfaces: islands and stochastic regions are possible. Even in nominally axisymmetric devices, magnetic islands may form through tearing modes. 3D MHD equilibrium without flow depends on two free functions, which must be constant on each magnetic field line: (1) the field-line average of the parallel electrical current density divided by the magnetic field strength $\langle J_\parallel/B\rangle_B$; and, (2) the pressure p [8].

2. Overview of Existing Codes

There presently exist many codes for calculating MHD equilibrium in a variety of cases. Here, we briefly review three codes: (1) the FLOW code; (2) a code for calculations under the formulation by Ito and Nakajima; and, (3) the PIES code.

The FLOW code is capable of calculating axisymmetric single-fluid MHD equilibrium with flow [2]. FLOW handles the Bernoulli equation explicitly, identifying the location of the trans-poloidal-sonic surface, and separates the computational domain in to two regions, with the outer region assumed to have super-poloidal-sonic flow.

We have written a solver for reduced two-fluid MHD equilibrium in the large aspect-ratio axisymmetric limit, as per the formulation by Ito and Nakajima [7]. Because of the limitations of the formulation, the code cannot handle trans-poloidal-sonic flows.

The PIES code is capable of calculating 3D single-fluid MHD equilibrium without flow [8]. The algorithm is based on that originally suggested by Spitzer [9] and by Grad and Rubin [10], and involves iterated direct integration of the equilibrium equations. However, there are substantial complexities involved with determining the topology of the magnetic field lines, and PIES uses a system of magnetic coordinates to effectively treat magnetic islands and stochastic field lines. As a consequence of the complications, PIES is prohibitively slow for some applications.

Recently, an external wrapper for the PIES code has been developed to improve the speed of convergence [11]. The external wrapper makes use of a custom Jacobian-free Newton-Krylov (JFNK) numerical solver with adaptive preconditioning and a novel subspace-restricted Levenberg-Marquardt backtracking algorithm. The details of the solver are beyond the scope of this document, but for information on JFNK, see Ref. [12], and, for information on adaptive preconditioning, see Ref. [13].

3. Concept for a New Code

We have begun work on a code for calculating 3D MHD equilibrium with fluid flow and FLR effects. We refer to the code as KITES – Kyoto ITerative Equilibrium Solver. Based on an analysis of the governing equations, we believe that equilibrium is governed by the magnetic field-line averages of six quantities, which can taken to be the following: (1) the parallel current density $J_{\parallel}$; (2) the electron temperature T_e; (3) the ion temperature T_i; (4) the density n; (5) the electrostatic scalar potential Φ; and, (6) the parallel fluid velocity $v_{\parallel}$. For a numerical solver, we plan to use the custom JFNK solver which was used with PIES. Unlike PIES, we hope to work entirely in physical coordinates without the use of magnetic coordinates: this is expected to reduce the speed of the solver but increase the robustness and maintainability.

There appear to be three major hurdles to the development of the code. The first is to have a way to accurately and rapidly determine the topology of the magnetic field lines. We have already developed an algorithm for field line tracing in physical space, and the results appear to be competitive with PIES. The second major hurdle is accurately solving the differential equation along each field line; these determine the variation of several quantities along each field line. We hope to be able to do this by brute-force by following the field lines for a sufficient length. The third hurdle is handling the trans-poloidal-sonic surface. The presence of FLR terms may eliminate the pedestal, in which case this hurdle would be moot. Otherwise, we would attempt a multi-region approach similar to that used in the FLOW code.

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