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2D extended MHD simulation of R-T and K-H instabilities

レイリー・テイラー．ケルビン・ヘルムホルツ不安定性の二次元拡張MHDシミュレーション

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Hall and gyroviscosity effects to the Rayleigh-Taylor(R-T) instability are studied by 2D numerical simulations of extended magnetohydrodynamic(MHD) equations. Numerical simulations over wide range of parameters such as the Hall parameter, gyroviscosity, beta value, and wave number of unstable modes are carried out. Effects on the growth of unstable modes and nonlinear saturation process are reported.

1. General

Dynamical aspects of magnetohydrodynamic (MHD) instability such as ballooning instability in a torus device have been extensively studied. Though numerical simulations using the single-fluid MHD model for high wave number ballooning modes have been carried out[1], not all the mechanism of the ballooning instability are clarified. The single-fluid model ignores some effects such as Hall effect and Finite Larmor Radius (FLR) effect. These effects come under higher order terms than the pressure gradient, the $\mathbf{J}\times\mathbf{B}$ force and so on in the MHD ordering, and have short distinctive length. Since the single-fluid model can be insufficient for high wave number ballooning modes, we consider studying these effects on the instability by using an extended MHD model. Furthermore, toroidal and poloidal flows exist in a realistic plasma. Since the flow is not uniform, the shear of flow (velocity difference) can affect the growth of the ballooning instability through the Kelvin-Helmholtz(K-H) instability.

Since the ballooning instability is Rayleigh-Taylor(R-T) type instability, here we concentrate on a simple R-T instability. The R-T instability is instability of an interface between two fluids of different densities, and occurs when the lighter fluid is pushed by the heavier fluid. Here we assume that heavy fluid is sustained both by the magnetic and thermal pressures, as it is assumed in Ref.[2]. The schematic view is shown in Fig. 1.

In this presentation, we study growth of unstable R-T modes by the use of the extended MHD model with the Hall term and the FLR effect as gyro-viscosity[2]. For simplicity, 2D slab geometry is assumed. A change of the growth rate and saturation level due to the two effects, depending on the wave number of the unstable modes is concerned. Effect of shear flow on the growth is also going to be studied.

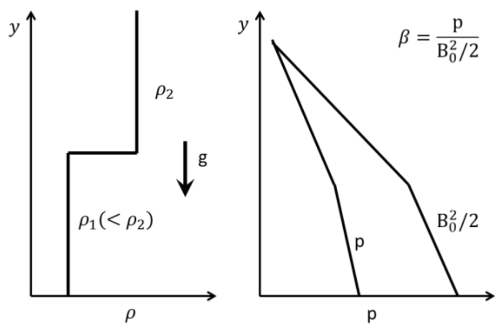

Fig.1. Schematic view of the R-T instability

2. Numerical Simulation

Extended MHD equations in 2D slab geometry are expressed as follows.

$$\frac{\partial\rho}{\partial t}+u_x\frac{\partial\rho}{\partial x}+u_y\frac{\partial\rho}{\partial y}=-\rho\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right), \quad (1)$$

$$\rho\left(\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}\right)=-\frac{\partial p}{\partial x}+(J_yB_z-J_zB_y)-\frac{\partial(\pi_i)_{xx}}{\partial x}-\frac{\partial(\pi_i)_{xy}}{\partial y}, \quad (2)$$

$$\rho\left(\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}\right)=-\frac{\partial p}{\partial y}-\rho g+(J_zB_x-J_xB_z)-\frac{\partial(\pi_i)_{yx}}{\partial x}-\frac{\partial(\pi_i)_{yy}}{\partial y}, \quad (3)$$

$$\frac{\partial p}{\partial t} + u_x\frac{\partial p}{\partial x} + u_y\frac{\partial p}{\partial y} = -\gamma p\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right), \qquad (4)$$

$$\frac{\partial b_x}{\partial t} = -\frac{\partial E_z}{\partial y}, \qquad (5)$$

$$\frac{\partial b_y}{\partial t} = \frac{\partial E_z}{\partial x}, \qquad (6)$$

$$\frac{\partial b_z}{\partial t} = -\frac{\partial E_y}{\partial x} + \frac{\partial E_x}{\partial y}. \qquad (7)$$

Here we have set $u_z = 0, \partial/\partial z \to 0$ in eqs.(1)-(7), and the current density is $J_x = \frac{\partial B_z}{\partial y}, J_y = \frac{\partial B_z}{\partial x}, J_z = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}$.

The electric field and gyro-viscosity are given as

$$E_x = -u_y B_z + \frac{\epsilon}{\rho}\left(J_y B_z - J_z B_y - \frac{\partial p_e}{\partial x}\right), \qquad (8)$$

$$E_y = u_x B_z + \frac{\epsilon}{\rho}\left(J_z B_x - J_x B_z - \frac{\partial p_e}{\partial y}\right), \qquad (9)$$

$$E_z = -u_x B_y + u_y B_x + \frac{\epsilon}{\rho}\left(J_x B_y - J_y B_x\right), \qquad (10)$$

$$(\pi_i)_{xx} = -(\pi_i)_{yy} = -\delta p_i\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right), \qquad (11)$$

$$(\pi_i)_{xy} = (\pi_i)_{yx} = \delta p_i\left(\frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y}\right). \qquad (12)$$

Equations (1)-(12) have been already normalized by some representative quantities. The symbol g is the gravitational acceleration, and the symbols ϵ, δ, γ are the Hall parameter, gyroviscosity, the ratio of the specific heats, respectively. We also denote the total pressure as $p = p_i + p_e = (\alpha + 1)p_e$, $\alpha = p_i/p_e$. Where p_i and p_e are the ion and the electron pressure, respectively.

In our numerical simulation, the Kawamura-Kuwahara scheme[3] for advection terms, 4th order central difference method for the other terms, and 4th order Runge-Kutta-Gill method for time evolution are used. The system size in the x and y directions are set 2π and 8π, respectively. The system is periodic in the x-direction, and the boundary condition $\partial/\partial y = 0$ is imposed at the upper and lower boundaries $y = \pm 4\pi$.

Figure 2 shows typical evolutions of R-T instability. Figs.2(a)-(b) show the R-T instability in the single-fluid MHD model and (c)-(d) show the R-T instability in the extended MHD model. The control parameters are m=2 (the wave number of perturbation in the x-direction), $\alpha = 0.5$ and $\beta = 0.1$ in the former, and $m = 2, \beta = 0.1, \alpha = 0.5, \delta = \epsilon = 0.01$ in the latter. In Figs.2(a)-(b), we can observe that typical mushroom-like structures grow due to the R-T instability. In Figs.2(c)-(d), the growth of the unstable mode is under the Hall and the gyro-viscosity effects. Because of these effects, the structures of Fig.2(c) is slightly different from those in Fig.2(a), and the difference becomes obvious in a later stage, between Fig.2(b) and Fig.2(d).

Numerical simulations are carried out wide range of β, δ, ϵ and m to characterize the influence of the Hall term and the gyroviscosity effects. We will also consider the flow shear, which can cause the K-H instability (Fig.3) in the presentation.

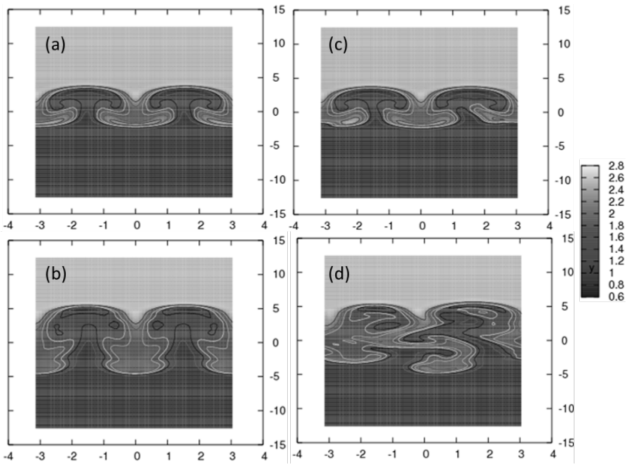

Fig.2. Time evolution of the R-T instability. Panels (a)-(b) are for the single-fluid MHD model, and (c)-(d) are for the extended MHD model.

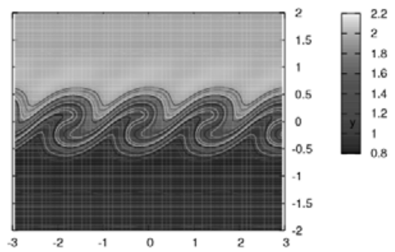

Fig.3. An example of the K-H instability

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