23P076-P

Effect of Resistivity on Mode Structure of Interchange Instability

交換型不安定性のモード構造に対する抵抗率の影響

Ryosuke Ueda, Yutaka Matsumoto, Masafumi Itagaki, Shun-ichi Oikawa, Kiyomasa Watanabe and Masahiko Sato

上田亮介，松本　裕，板垣正文，及川俊一，渡邊清政，佐藤雅彦

Graduate School of Engineering, Hokkaido University, Sapporo 060-8628, Japan
National Institute for Fusion Science, Oroshi-cho 322-6, Toki, 509-5292, Japan

The interchange instability mode of the straight heliotron plasma is analyzed by use of the reduced MHD equations transformed to eigenvalue equation. The differences of the instabilities between the first eigenmode and the second eigenmode are investigated. It is found that the instabilities due to first eigenmode are larger than second eigenmode. It is also found that the differences of the mode structure appear substantially when the magnetic Reynolds number or β become large.

1. Introduction

It is one of the important subjects for a realization of the fusion reactor to investigate the Magnetohydrodynamics (MHD) instabilities. The interchange mode which is one of the pressure-driven instabilities is considered to play an important role in the plasma confinement property. In LHD, the interchange instability modes are observed in various magnetic Reynolds numbers.

In this study, the interchange mode instabilities of the straight heliotron plasma with various magnetic Reynolds numbers and various beta values are analyzed by eigenvalue analysis. Especially the difference of the effect on mode structure due to the first and second eigenmodes is studied.

2. Numerical Method

In this study, the interchange mode with the various magnetic Reynolds number S ($10^4 \leqq S \leqq 10^{10}$) are analyzed. For such analyses, the following normalized reduced MHD equations [1] are useful. These equations with the cylindrical coordinates (r, θ, z) are written as

$$\frac{\partial}{\partial t}\nabla_\perp^2\varphi = [\varphi, \nabla_\perp^2\varphi] + [\nabla_\perp^2 A, \psi] + \frac{\partial}{\partial z}\nabla_\perp^2 A + [\Omega, p], \tag{1}$$

$$\frac{\partial\psi}{\partial t} = [\varphi,\psi] + \frac{\partial\varphi}{\partial z} + \eta\nabla_\perp^2 A, \tag{2}$$

$$\frac{\partial p}{\partial t} = [\varphi, p], \tag{3}$$

where

$$\nabla_\perp^2 = \nabla^2 - \frac{\partial^2}{\partial z^2}, \tag{4}$$

$$[f,g] = (\nabla f \times \nabla g)\cdot e_z. \tag{5}$$

Here, φ is the stream function, p is the pressure. The poloidal flux is $\psi = A + \psi_h$, where A and ψ_h are due to the plasma current and the helical coils, respectively. In Eq. (1), the $\nabla\Omega$ term denotes the contribution of the averaged magnetic curvature. The η is resistivity expressed by $\eta = S^{-1}$.

We assume that $f(r,\theta,z)$ are written as $f = f_0(r) + \hat{f}(r)\exp(i(m\theta - nz) + \gamma t)$ and linearize Eqs. (1)-(3). The number m and n indicates the poloidal mode number and toroidal mode number. Then the following equations are obtained:

$$i\gamma\hat{\psi} = -k_\parallel\hat{\varphi} + i\eta\nabla_\perp^2\hat{\psi}, \tag{6}$$

$$\gamma^2\nabla_\perp^2\hat{\varphi} = k_\parallel\gamma\nabla_\perp^2\hat{\psi} + k_\perp^2\Omega' p_0'\hat{\varphi}, \tag{7}$$

where $k_\parallel = m\iota - n$, $k_\perp = m/r$ and ι is the rotational transform. Ω' is expressed by

$$\frac{d\Omega}{dr} = \frac{N\varepsilon}{\ell}\frac{1}{r^2}\frac{d}{dr}(r^4\iota). \tag{8}$$

N is the pitch number and ℓ is the pole number of heliotron device [3].

We solve the above equations in $0 \leqq r \leqq 1$. The boundary conditions are set as followings:

$$\begin{aligned} &\psi = 0, \quad \varphi = 0 \quad (r = 0) \\ &\frac{d\psi}{dr} = 1, \quad \varphi = 0 \quad (r = 1) \end{aligned} \tag{9}$$

The equilibrium profile of the pressure and the poloidal flux are given by

$$p_0 = (1 - p_a)(1 - r^2)^2 + p_a, \tag{10}$$

$$\iota = -\frac{1}{r}\frac{d\psi_0}{dr} = \iota_a + \iota_b r^2. \tag{11}$$

Here, $p_a = 10^{-3}$, $\iota_a = 0.461$, $\iota_b = 1.1$.

In this way, the growth rate γ and the mode structure of instabilities can be obtained. We focus on the instabilities with (m, n) = (1, 1) mode.

3. Saturation level

We are unable to obtain the saturation level of

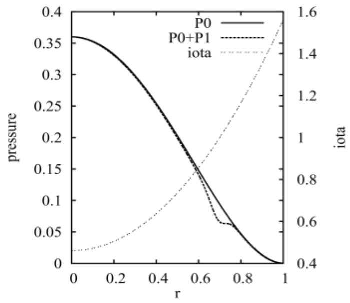

Fig. 1 Flattened pressure profile $p=p_0+p_1$, around the rational surface.

the instability by only use of the linear eigenvalue analysis. We assumed pressure perturbation p_1 so that the pressure profile ($p=p_0+p_1$) around $\iota=1$ rational surface is flattened. Under such an assumption, the saturation level can be defined. .

4. Differences between the first and second eigenmodes.

Equations (6) and (7) have multiple eigenvalues. We evaluate the differences between the first and second eigenmodes. Specifically we investigate the mode structure, the growth rate and the mode width with various magnetic Reynolds numbers.

4.1 Mode Structures of instabilities

Figure 2 shows the mode structure of ψ and φ ($S = 10^4$ and β = 2%).The left figure of Fig. 2 is the mode structure of first eigenmode and the right is that of the second eigenmode. It can be seen the first eigenmode structure of φ is even around the rational surface. On the other hand, the second eigenmode structure of φ is odd. The peak value and the mode width of φ of the first eigenmode are larger than those of the second eigenmode. It is noted that the mode width is evaluated by FWHM (Full width at half maximum).

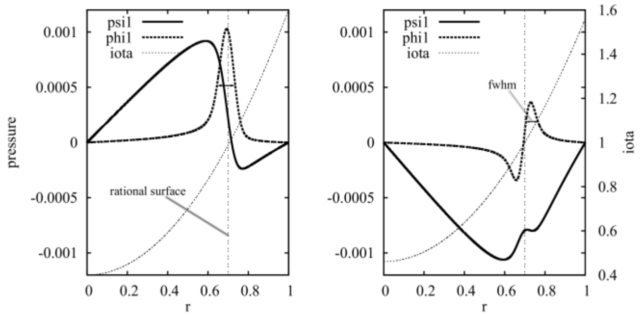

Fig. 2 Mode structure of ψ (solid) and φ (dashed). Left figure is the case of first eigenvalue and right is the case of second eigenvalue.

4.2 Growth rate

Figure 3 shows the growth rate of the first eigenmode (left) and the second eigenmode (right). It can be seen that the growth rate of the first eigenmode is larger than the second eigenmode. Moreover, when β > 3% and $S > 10^8$, the growth rate is different substantially.

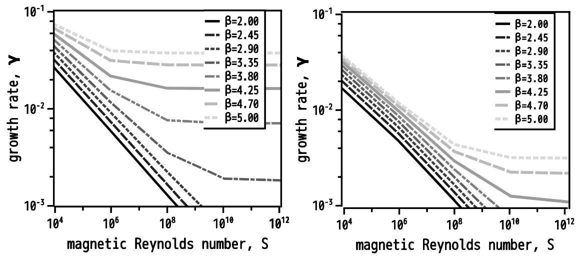

Fig. 3 Growth rate as a function of the magnetic Reynolds number in the case of first eigenmode (left) and in the case of second eigenmode (right).

4.3 FWHM of displacement

We evaluate the FWHM of the displacement $|\xi_r|$ as a function of the magnetic Reynolds number. Here, the displacement is written as

$$\xi_r = -p_1 \frac{dp_0}{dr}. \tag{12}$$

Here, p_1 is the perturbation of pressure. Figure 4 shows the FWHM of $|\xi_r|$ as a function of S. It can be seen that the FWHM of first eigenmode is larger than those of the second eigenmode. The dependence of the FWHM on S is different in the high β cases. Moreover, we can find that the dependence of FWHM on S is close to theory ($\propto S^{-1/3}$) in low β cases.

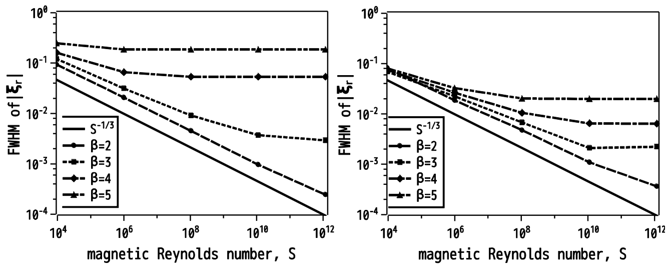

Fig. 4 FWHM of displacement ξ_r in the case of first eigenmode (left) and second eigenmode (right).

6. Summary

We investigate the interchange instabilities of the heliotron plasma. The differences of the instabilities between the first eigenmode and the second eigenmode are shown. As a result of analyses, we found the instabilities due to first eigenmode are larger than second eigenmode. Additionally, when the magnetic Reynolds number or β become large, the differences of the mode structure appear substantially. For this reason, it may be expected that the second eigenmode also affects on the interchange instabilities.

References

[1] H. R. Strauss, Plasma Phys., **22**, 733 (1980).
[2] K. Miyamoto, Nucl. Fusion **18**, 243 (1978).