

Neoclassical Toroidal Viscosity Calculations in Tokamaks using a delta-f Simulation and Their Verifications

delta-f法を用いたトカマクにおける新古典トロイダル粘性計算法の検証

Shinsuke Satake Hideo Sugama, Ryutaro Kanno, Jong-Kyu Park¹
佐竹真介, 洲鎌英雄, 菅野龍太郎, J.-K. Park

National Institute for Fusion Science
322-6 Oroshi, Toki, Gifu 509-5292, Japan
核融合科学研究所 〒509-5292 岐阜県土岐市下石町322-6

¹ Princeton Plasma Physics Laboratory
Princeton, New Jersey 08543, USA

Neoclassical toroidal viscosity, which arises in tokamaks when non-axisymmetric magnetic perturbation is applied, is known to affect the toroidal rotation of tokamak plasma even if the perturbation amplitude is quite weak. FORTEC-3D, a neoclassical transport code using the δf Monte Carlo method, is utilized to evaluate the neoclassical toroidal viscosity caused by weak magnetic perturbations. The calculation method is applied to investigate the dependence of neoclassical toroidal viscosity on the radial electric field strength. It is shown that the neoclassical toroidal viscosity is sensitive to the radial electric field of which amplitude is about the same order as that is expected from the force balance relation and vanishes when the force balance relation is satisfied.

1. Introduction

The control of the toroidal rotation is an important issue in tokamaks in order to improve the stability of the contained plasmas. Recent studies have shown that the non-axisymmetry as small as $\delta B/B \sim 10^{-4}$ can induce significant damping in toroidal rotation which otherwise would be much better maintained. Such small perturbation can be applied for the purpose of stabilizing or destabilizing edge localized modes[1], or is inherent in tokamaks because of the error in coil systems.

One of the main physics which affect the toroidal rotation of the plasmas is the neoclassical toroidal viscosity (NTV), which arises when the toroidal symmetry of the magnetic field is broken. The NTV torque has been observed and studied in many tokamak experiments[2–4] and with a numbers of theories[5–6]. However, theoretical prediction of NTV is nontrivial because it involves complicated diffusion process in the perturbed magnetic field, which is also affected by collisions and radial electric field. Conventional theories have been therefore developed in each limited regime to simplify or ignore other processes; the effects by passing particles are ignored, effects by trapped particles are studied in many different collisionality regimes, the finite drift width of particle orbits are neglected, and so on. In order to give a more general and quantitatively reliable method to estimate the NTVs in weakly perturbed tokamaks, we have adapted FORTEC-3D code[7] for direct numerical simulation of NTV. Since it is a first

application of the δf transport code to neoclassical viscosity calculation, we have been carefully verifying its applicability step by step. Previous benchmarks[8, 9] have demonstrated that it can be applied to the wide range of collision frequency and that the calculation results agree very well with the combined NTV formula by Park[10] in the limit of zero radial electric field.

In this paper, as the second step of the verification of the code, the NTV dependence on the radial electric field (E_r) is investigated. It is shown that the NTV in low collisionality plasma is very sensitive to E_r of which amplitude is about the same order as that is expected from the force balance relation. When E_r satisfies the force balance, the NTV is found to vanish completely, as expected from the neoclassical theory.

2. Numerical method

FORTEC-3D solves the drift-kinetic equation for the perturbed distribution function δf in the 5-D phase space [7]:

$$\left[\frac{\partial}{\partial t} + (\mathbf{v}_d + \mathbf{v}_\parallel) \cdot \nabla + \dot{K} \frac{\partial}{\partial K} \right] \delta f - C_T(\delta f) = - \left(\mathbf{v}_d \cdot \nabla + \dot{K} \frac{\partial}{\partial K} \right) f_M + P f_M, \quad (1)$$

where f_M is the local Maxwellian, and C_T and $P f_M$ are the test-particle and field-particle part of the linearized collision terms respectively. The magnetic field is expressed by Fourier spectrum in Boozer coordinates as $B(r, \theta, \zeta) = B_0(1 - \epsilon_t \cos \theta + \sum_{m,n} \delta_{m,n} \cos[m\theta - n\zeta])$. Utilizing the field expression, the NTV in perturbed tokamak can

be expressed as [8]

$$\langle \mathbf{e}_\zeta \cdot \nabla \cdot \mathbf{P} \rangle = B_0 \sum_{m,n \neq 0} \delta_{m,n} \left\langle \frac{\delta P}{B} \sin[m\theta - n\zeta] \right\rangle, \quad (2)$$

where $\delta P \equiv \int d^3v \delta f M (v_\perp^2/2 + v_\parallel^2)$ and $\langle \dots \rangle$ means the flux-surface average. Eq. (2) is numerically evaluated in the code from the quasi-steady state solution of δf .

3. Simulation condition

We used the same simple tokamak geometry as is used in Ref. 8. Single (7, 3)-mode perturbation $\delta_{7,3} = 0.01(r/a)^2$ is applied, which has a resonant rational flux surface $q = 7/3$ at $r/a = 0.487$. Concerning E_r , we utilize the force balance relation (for ions) in tokamaks[10],

$$\mathbf{u} \cdot \nabla \zeta = -\frac{T}{e} \left[\frac{d \ln p}{d\chi} + \frac{e}{T} \frac{d\Phi}{d\chi} - k \frac{d \ln T}{d\chi} \right], \quad (3)$$

where χ is the poloidal flux and Φ is the electrostatic potential, $E_r = -\frac{d\Phi}{d\chi} \frac{d\chi}{dr}$. The factor k depends on the plasma collisionality ν_* and ϵ_t , which is ≈ 1.17 in the ν_* , $\epsilon_t \rightarrow 0$ limit. To avoid the complexity and for future benchmark with analytic theories which sometimes uses an approximation in k , we used a constant T_i profile. The collisionality is banana regime, $\nu_* \approx 0.06$. Since the simulation starts from zero toroidal flow and the flow seldom develops during a simulation run, the E_r profile which satisfies the force balance can be obtained from Eq. (3) by setting $\mathbf{u} = d \ln T/dr = 0$. Each simulation is carried out by multiplying the E_r between ± 2.0 times of its force-balance value.

4. Simulation results

Figure 1 shows the radial profile of NTV density for several amplitude of E_r . It has a peak at the resonant surface. One can see that NTV completely vanishes when the E_r profile satisfies the force balance ($E_r \times 1.0$ case). Though Eq. (3) is derived assuming the toroidal symmetry, the relation still holds true when a weak non-axisymmetric perturbation is applied.

The NTV dependence on the E_r amplitude is shown in Fig. 2. NTV has the maximum at $E_r \approx 0$ and becomes almost constant for large $|E_r|$. It is expected that the E_r and $\mathbf{u} \cdot \nabla \zeta$ profiles of a steady-state in perturbed tokamak plasmas are close to the force balance solution if there is not strong torque input (NBI) or non-ambipolar flux. The simulation result anticipates that the NTV is almost zero if the force balance is completely satisfied, while it is also sensitive to the deviation from the balanced state. It has nearly a linear dependence, $\langle \mathbf{e}_\zeta \cdot \nabla \cdot \mathbf{P} \rangle \propto |E_r - E_{r(\text{force balance})}|$, when the deviation is small.

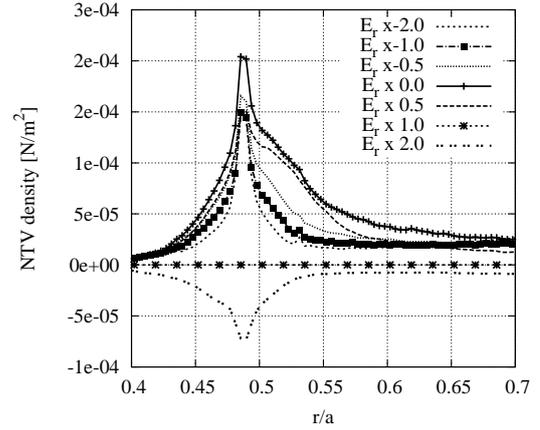


Fig.1. NTV profile for each amplitude of E_r .

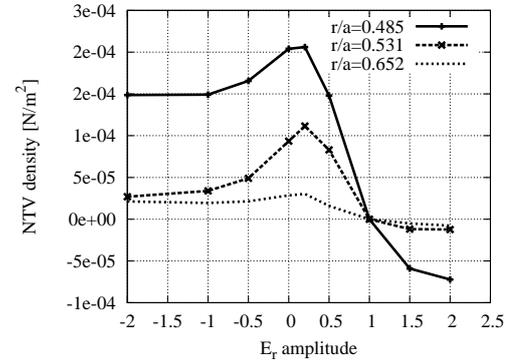


Fig.2. Dependence of NTV on E_r . The horizontal axis represents the magnification factor of the E_r amplitude.

Acknowledgments

This study is supported by JSPS Grant-in-Aid for Young Scientists (B), No. 23760810 and NIFS collaborative Research Programs 11KNST014.

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