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2D kinetic full wave analysis in tokamak plasmas using FEM
有限要素法によるトカマクプラズマ中の運動論的２次元波動伝播解析

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For describing electron cyclotron (EC) waves in spherical tokamaks with high density, full wave analysis is necessary owing to the existence of evanescent layers and mode conversion to the Bernstein waves. Full wave analysis requires a lot of computational resources and finite element method (FEM) is expected to be suitable for parallel computing since it requires less computational resources compared with full or partial spectral method. We have already developed 3D full wave code using FEM, TASK/WF. In order to obtain better spatial resolution for the analysis of EC wave in tokamak plasmas, we have developed a 2D version with mixed basis functions. The kinetic response of plasmas will be implemented as an integral representation of the dielectric tensor.

1. Introduction

Because there is no central Ohmic solenoid, the structure of the spherical tokamak (ST) is much simpler than other reactor designs. This makes the ST attractive. For plasma initiation and current start-up without central Ohmic solenoid, the use of electron cyclotron heating and current drive (ECH/ECCD) is planned. For high density STs, a cutoff layer exists and we cannot heat the center of plasma by EC waves directly. Owing to the kinetic effects of plasma (finite Larmor radius effects), EC waves are converted into Electron Bernstein waves (EBW) near the evanescent layer. EBW can propagate into high-density region and heat the central part of the plasma[1]. Therefore, it is necessary to analyze the wave propagation including kinetic effects, such as mode conversion of EC waves and electron cyclotron harmonic damping. This requires a numerical code implementing the following two essential features:

(i) Full wave analysis

EC wave propagation has been mostly analyzed by the ray tracing technique. For STs with high density, however, full wave analysis is necessary owing to the existence of evanescent layers and mode conversion to the Bernstein waves. Full wave analysis requires a lot of computational resources and finite element method (FEM) is expected to be suitable for parallel computing since it requires less computational resources compared with full or partial spectral method.

(ii) Integral formulation of dielectric tensor

For $k_\perp \rho_s > 1$ where $k_\perp$ is the wave number perpendicular to the static magnetic field and ρ_s is the electron gyro radius, differential formulation of dielectric tensor becomes invalid and we have to use the integral formulation to describe non-local kinetic wave-particle interaction.

We have already developed 3D full wave code using FEM, TASK/WF. In order to analyze the mode conversion of EC wave to waves with shorter wave length, however, we need higher spatial resolution, therefore we have developed a 2D version of the full wave code with mixed basis functions.

2. Formulation

We have formulated a 2D version FEM with mixed basis functions, because a vector interpolation function is used for the components on the 2D plane and a scalar interpolation function for the component perpendicular to the 2D plane.

2.1 *Fourier decomposition*

We assume that the tokamak plasma is axisymmetric and expand the wave electric field to a Fourier series in the toroidal direction ϕ,

$$\begin{aligned}\tilde{\mathbf{E}}(r,\phi,z) &= \sum_{n=-\infty}^{\infty} \mathbf{E}_n(r,z)\mathrm{e}^{\mathrm{i}n\phi} \\ &= \sum_{n=-\infty}^{\infty} \{E_{rn}(r,z)\hat{\mathbf{r}} + E_{zn}(r,z)\hat{\mathbf{z}} + E_{\phi n}(r,z)\hat{\phi}\}\mathrm{e}^{\mathrm{i}n\phi}\end{aligned} \tag{1}$$

in a cylindrical coordinates (r,ϕ,z).

2.2 *Discretization*

We consider a particular toroidal mode n. In parallel direction, we can think permittivity invariant, so we use schalor interpolation function. As a result, we can

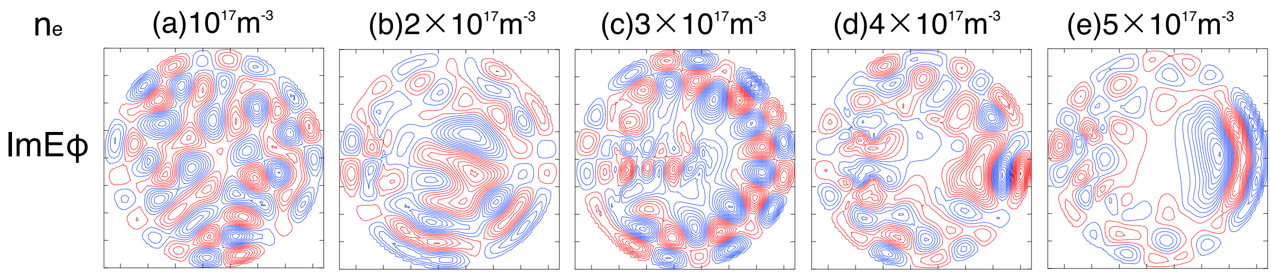

Figure 1: Density dependence of EC wave propagation

discretize electric field **E** in the element as

$$\mathbf{E}^e(r,\phi,z) = \left(\sum_{i=1}^{3} E^e_{rzi}\mathbf{w}^e_i(r,z) + \sum_{i=1}^{3} E^e_{\phi i}N^e_i(r,z)\hat{\phi}\right) \mathrm{e}^{in\phi} \tag{2}$$

where $\mathbf{w}^e_i$ is the vector interpolation function and $\mathbf{N}^e_i$ is the scholar interpolation function.

2.3 *Weak formulation*

We use weighted residual method. Maxwell equation becomes

$$\int_V (\nabla\times\mathbf{F})\cdot(\nabla\times\mathbf{E})\mathrm{d}\mathbf{V} + \int_S (\mathbf{F}\times(\nabla\times\mathbf{E}))\cdot\mathbf{n}\mathrm{d}\mathbf{S} = \frac{\omega^2}{c^2}\int_V (\mathbf{F}\cdot\overleftrightarrow{\varepsilon}\cdot\mathbf{E})\mathrm{d}\mathbf{V} + i\omega\mu_0\int_V \mathbf{F}\cdot\mathbf{J}_{\mathrm{ext}}\mathrm{d}\mathbf{V}. \tag{3}$$

We also use Galarkin's method, and weight function is chosen as

$$\mathbf{F}^e(r,\phi,z) = \left(\sum_{i=1}^{3} F^e_{rzi}\mathbf{w}^e_i(r,z) + \sum_{i=1}^{3} F^e_{\phi i}N^e_i(r,z)\hat{\phi}\right) \mathrm{e}^{-in\phi}. \tag{4}$$

Substituting eqs.(2) and (4) into (3), we get the large matrix equation. TASK/WF can use PETSc and MUMPS library and do parallel computing in order to solve large size matrix.

3. Numerical results

We analyzed density dependence of EC wave propagation using LATE's parameter. Used parameters are $R = 0.22$m, $a = 0.16$m, $f = 5$GHz, $B_0 = 0.072$T. Collisional cold plasma model dielectric tensor is used. Toroidal mode number is 0. Figure 1 shows numerical results. (c) shows mode conversion of EC wave to shorter wavelength wave. (d), (e) shows that there exists cutoff layer and EC waves cannot propagate to center of plasma region. Spatial resolution becomes higher.

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References

[1] T. Maekawa, et. al, Nucl. Fusion Vol. 45 1439-1445(2005)