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Highly Accurate Approximate Solutions of Stokes Equation for Polarimetry of ITER-like Plasmas

ITERをはじめとする核融合プラズマにおける偏光計測のためのストークス方程式の高精度近似解

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Polarimetry has been applied in many magnetic confinement fusion devices in order to measure the magnetic field and the electron density. The Faraday effect rotates the polarization ellipse, and the Cotton-Mouton effect changes the ellipticity of the polarization ellipse. In the dense plasmas like ITER plasma, these two effects interfere with each other, and the formulas for the two effects are no longer valid. In order to comprehend the plasma state from the polarimetric data, the Stokes equation should be solved. We have found the new equations equivalent to the Stokes equation and the highly accurate approximate solutions. Our solutions hold even in high density region (~10^{21} m^{-3}) and exhibit the highest accuracy among approximate solutions.

1. Introduction

Plasma polarimetric measurements have been installed in many fusion devices. Physical backgrounds of polarimetry are usually explained by the Faraday and the Cotton-Mouton effects. In the former effect the polarization ellipse (and, for linear polarization, the plane) rotates, while in the latter effect the ellipticity of the polarization ellipse changes. Although these two effects are usually treated as independent events, they interfere with each other in dense plasmas like ITER plasma. In such a case, formulas of the Faraday and Cotton-Mouton effects are not valid, and the Stokes equation expresses the change of the polarization state. In this study, we transform the Stokes equation to the new equations to obtain the highly accurate approximate solutions.

2. Faraday and Cotton-Mouton Effects

Polarization state can be defined by the polarization ellipse parameters; the orientation angle, ψ (with $0\leq\psi\leq\pi$), the ellipticity angle, χ (with $-\pi/4<\chi\leq\pi/4$), the auxiliary angle, α (with $0\leq\alpha\leq\pi/2$), and the phase shift angle, δ (with $0\leq\delta\leq2\pi$). Figure 1 shows the relation between the polarization ellipse and these parameters. When one of two effects is small enough, the Faraday and Cotton-Mouton effects are expressed as:

$$\Delta\psi = C_1 \int_{z_0}^{z_1} n_e B_{\parallel} dz\,, \tag{1}$$

$$\Delta\delta = C_2 \int_{z_0}^{z_1} n_e B_{\perp}^2 dz, \tag{2}$$

respectively. Here, z denotes the coordinate axis along propagation of electromagnetic radiation; n_e denotes the electron density; $B_{\parallel}$ denotes the component of the magnetic field parallel to z; $B_{\perp}$ denotes the component of the magnetic field orthogonal to z; and C_1 and C_2 denote the constant values.

3. Stokes Equation

The Stokes equation expresses the change of the polarization state and is written as

$$\frac{d\vec{s}}{dz} = \vec{\Omega} \times \vec{s}, \tag{3}$$

where the vector of $\vec{s}$ is the reduced Stokes vector and the vector of $\vec{\Omega}$ is the vector associated with the Mueller matrix representing the optical properties of the plasma[1]. The reduced Stokes vector, $\vec{s}$, is expressed as

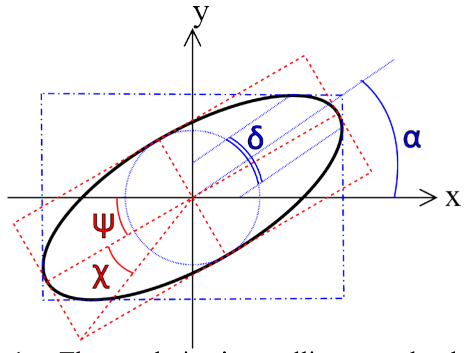

Fig. 1. The polarization ellipse and the polarization parameters (ψ, χ, α, and δ).

$$\vec{s} = \begin{pmatrix} \cos 2\chi \cos 2\psi \\ \cos 2\chi \sin 2\psi \\ \sin 2\chi \end{pmatrix} = \begin{pmatrix} \cos 2\alpha \\ \sin 2\alpha \cos\delta \\ \sin 2\alpha \sin\delta \end{pmatrix}. \tag{4}$$

The vector of $\vec{\Omega}$ is expressed as [1]:

$$\vec{\Omega} = \begin{pmatrix} C_{CM}\lambda^3 n_e B_\perp^2 \cos 2\beta \\ -C_{CM}\lambda^3 n_e B_\perp^2 \sin 2\beta \\ -2C_{FR}\lambda^2 n_e B_\parallel \end{pmatrix}, \tag{5}$$

where C_{FR} and C_{CM} denote the constant values, λ is the laser wavelength, β is the angle between the *y* direction and $B_\perp$.

4. New Expressions and Approximated Solutions of Stokes Equation

Although Stokes polarization parameters (components of $\vec{s}$) are observables, they are not as intuitive as the polarization ellipse parameters. We transformed the Stokes equation to more useful equations as follow:

$$\frac{d\chi}{dz} = \frac{C_{CM}}{2}\lambda^3 n_e B_\perp^2 \sin(2\psi + 2\beta), \tag{6}$$

$$\frac{d\psi}{dz} = -C_{FR}\lambda^2 n_e B_\parallel - \frac{C_{CM}}{2}\lambda^3 n_e B_\perp^2 \tan 2\chi \sin(2\psi + 2\beta), \tag{7}$$

$$\frac{d\alpha}{dz} = -C_{FR}\lambda^2 n_e B_\parallel \cos\delta + \frac{C_{CM}}{2}\lambda^3 n_e B_\perp^2 \sin 2\beta \sin 2\delta, \tag{8}$$

$$\frac{d\delta}{dz} = 2C_{FR}\lambda^2 n_e B_\parallel \frac{\sin\delta}{\tan 2\alpha} + C_{CM}\lambda^3 n_e B_\perp^2 \left(\cos 2\beta + \sin 2\beta \frac{\cos\delta}{\tan 2\alpha}\right). \tag{9}$$

When measuring objects are magnetic confinement fusion plasmas and the probing laser wavelength is in far-infrared range, the second term of RHS of eq. (7) is smaller than the first term. Assuming that the second term is negligible, we obtain the approximated solution related to the Faraday effect:

$$\Delta\psi = -C_{FR}\int_{z_0}^{z_1} \lambda^2 n_e B_\parallel dz. \tag{10}$$

Substituting eq. (10) into eq. (6) leads to the approximated solution related to the Cotton—Mouton effect:

$$\Delta\chi = \frac{C_{CM}}{2}\int_{z_0}^{z_1} \lambda^3 n_e(p) B_\perp(p)^2 \sin\left\{2\psi_0 + 2\beta(p) - 2\int_{z_0}^{q} C_{FR}\,\lambda^2 n_e(q) B_\parallel(q) dq\right\} dp. \tag{11}$$

5. Comparison among Approximated Solutions

Several approximated solutions to the Stokes equation have been proposed [2-5]. We have compared the error of our approximated solutions and the error of the approximated solution called as Type II [3]. The approximated solutions of Type II are expressed as:

$$\psi_1 = -\frac{1}{2}\arctan\left[\left\{\tan\left(C_{FR}\int_{z_0}^{z_1} \lambda^2 n_e B_\parallel dz\right)\right\}^{-1}\right], \tag{12}$$

$$\delta_1 = \arctan\left(\frac{F}{G}\right), \tag{13}$$

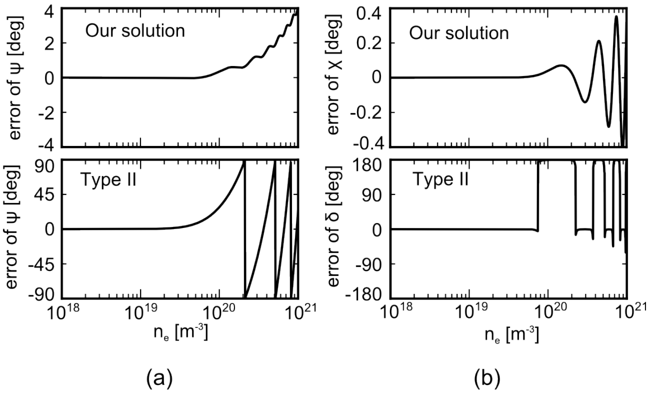

Fig.2. Errors of approximated solutions related to the (a) Faraday and (b) Cotton-Mouton effects as a function of density.

where

$$F = C_{CM}\lambda^3 \int_{z_0}^{z_1} n_e(p)\{B_y(p)^2 - B_x(p)^2\} \cos\left\{\int_{z_0}^{p} 2C_{CM}\lambda^3 n_e(q) B_\parallel(q) dq\right\} dp, \tag{14}$$

$$G = \cos\left\{2C_{FR}\int_{z_0}^{z_1} n_e(p) B_z(p) dp\right\}. \tag{15}$$

Eqs. (12) and (13) are related to the Faraday and Cotton-Mouton effects, respectively. Conditions for comparison of the error of the approximated solutions are $\lambda = 10^{-4}$ [m] $B_\parallel = 1$ [T], $B_\perp = 5$ [T], $z_1 - z_0 = 4$ [m], and $\beta = 0$. Figure 2 shows the differences between the true values calculated by Stokes equation and approximated solutions related to the Faraday and Cotton-Mouton effects as a function of electron density. Our new solutions are accurate even in dense plasmas and are more accurate than Type II. Comparing with other solutions [2-5], our solutions are the most accurate in the above condition and need the fewest assumptions.

6. Conclusions

We have transformed the Stokes equation to more useful equations using the polarization ellipse parameters. We have obtained the new approximated solutions of the Stokes equation for the Faraday and Cotton-Mouton effect. Our solutions hold even in high density region (~10^{21} m^{-3}) and exhibit the highest accuracy among approximated solutions.

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