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Electronic State Density and Average Ionization Based on the Plasma Microfield
プラズママイクロフィールドに基づく電子状態数密度と電離度

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In addition to numbers of bound state densities, free state density in a plasma is computed by the atomic model based on the plasma microfield. In the model, we assume that ions are immersed in the statistically distributed electric microfield in a plasma by Holtsmark and the large electric field components of which are due to a binary collision of two atoms. Continuous changes on state densities from bound to free states for various number densities of ions, and contour of average ionization degree of hydrogenic plasma are shown as typical results.

1. Introduction

Numerical computations of average ionization degree are old, but still a difficult problem on plasma physics. Once we choose a set of the statistical weight of bound state and the state density of free state, the average ionization degree seems to be able to be computed simply by the Saha-Boltzmann relation

$$\frac{N_{Z+1}N_e}{N_Z} = \frac{U_{Z+1}}{U_Z}\exp\left(-\frac{I_p}{k_BT_e}\right)\frac{\sqrt{2}}{\pi^2}\int_0^\infty \sqrt{E}\exp\left(-\frac{E}{k_BT_e}\right)dE$$
$$= \frac{U_{Z+1}}{U_Z}\exp\left(-\frac{I_p}{k_BT_e}\right)2\left(\frac{k_BT_e}{2\pi}\right)^{3/2} \qquad (1)$$

where N_Z and N_e are the number density of Z-charged ion and free electron, respectively. U_Z and I_p are the number of state and the ionization potential of Z-charged ion, respectively. T_e is the electron temperature. Atomic units are used through out in this article.

For hydrogenic case, number of bound state of which principal quantum number is n is $2n^2$ and state density of free state is proportional to the square root of energy of free electron. The problem is which maximum principal quantum number of the bound states has to be used in the computation. It is not simple. In Fig. 1, the problem is described using the simplest hydrogenic case.

All lines are contours of average ionization degree of 0.5, but maximum principal quantum number taken into account in the computations are different; maximum principal quantum numbers are up to n = 5, 10, 30, respectively. Generally, average ionization degree becomes smaller as the density becomes large, since free state density becomes relatively smaller to bound state. For low temperature cases, three curves converge into one. In the low temperature, almost all bound electrons exist in the ground state so that number of bound state does not affect in the average ionization degree computation. On the other hand, three curves spread out into two-order of magnitude range width in ion number density as the temperature becomes higher. In the high temperature, number of bound electron in excited states cannot be neglected in average ionization degree computation. As a result, three curves of the average ionization degree of 0.5 separate into. This shows the importance of choice of the maximum principal quantum number of bound state in the atomic process modeling.

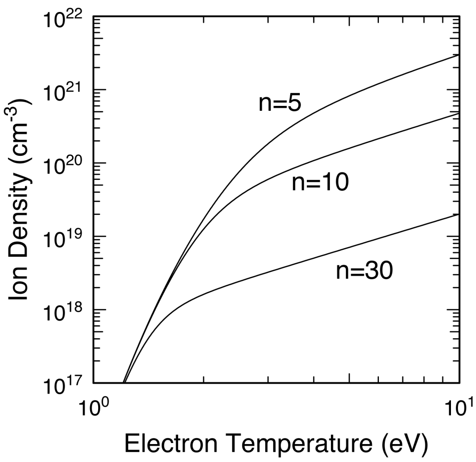

Fig. 1. Contour of average ionization degree equal to 0.5 for various choice of maximum principal quantum numbers of n = 5, 10, and 30 in the atomic process modeling.

2. Electronic State density by the plasma microfield and average ionization degree

Modeling on the bound state based in the plasma microfield by Holtsmark[2] had already been developed[1]. In the model, numbers of bound states are expressed simple analytical expression. In addition to the bound state, we have tried to make a simple expression of state density of free electron based on the plasma microfield. In the model, we rewrite the relation between the electron's energy and the number of state density of

$$p^2 = E + \frac{Z}{r} - Fr\cos\theta \qquad (2)$$

where F is the electric field at position (r, θ) inside the ion sphere R_0 defined by $4\pi R_0^{\,3} N_i / 3 = 1$. N_i is the number density of ions. The state densities are used those from quantum mechanics, while the potentials are from classical electro-magnetic theory. Sorry to say, the computed free state density by the simple state density has been shown inappropriate. Especially, the inappropriate overestimation in free state density has occurred in relatively large electric field. The values of such electric field are larger than what arises from the nearest neighbor atom set inside the ion sphere. In order to remove the difficulty, we assume that the large electric field components are due to binary collisions of two atoms. Therefore, the potential from the electric field has been computed as the binary symmetry on the saddle point as if two nucleus exist inside the ion sphere. Using the improvement, physically appropriate state densities of free has been able to be obtained. In Fig. 2, the state densities of free and bound states for various number densities of ions are shown. In order to show the continuous change of state densities from bound to free state, the bound states are shown as boxes of which integral values have $2n^2$. The boxes have spread over between intermediate energy levels of nearest larger and smaller levels. For low ion number density of $N_i = 10^{16}$ cm^{-3}, number of bound state almost fully exists up to n = 8 and gradually disappears as the energy of the state becomes larger. Alternatively, free state appear even the energy of electron is less than zero. As the number density of ion becomes larger, free state density becomes smaller as proportional to its number density and disappearance of the bound states density begins at the lower bound state.

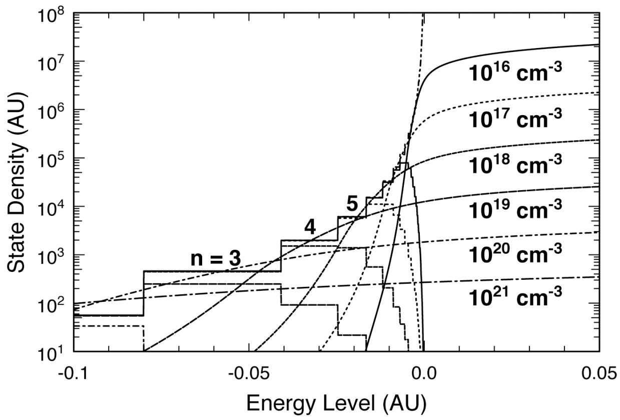

Fig. 2 Numbers of bound state densities and states densities of free states for various ion number densities.

Using the number of bound state and free state density as a function of electron energy described above, average ionization degree can be computed by eq. (1). Equation (1) only shows the relation between the ground states of different charge, contribution of the excited states has also been included in the computation. In Fig. 3, contour of average ionization degree of hydrogenic plasmas are shown. Solid curve shows the average ionization degree of 0.5. At the low-temperature and low-density case, the average ionization degree agrees with the result in Fig. 1. Even in the low temperature, average ionization degree becomes larger as the ion density becomes large due to the effect of the disappearance of the bound states. This effect has been called 'Pressure ionization'. The present model naturally can introduce the effect.

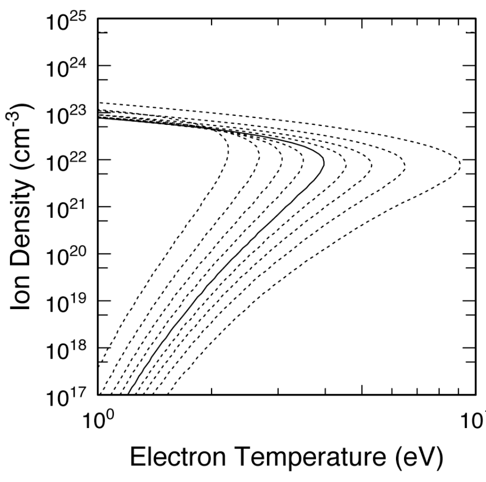

Fig. 3 Average ionization degree of hydrogenic plasma.

References

[1] T. Nishikawa: ApJ **532** (2000) 670.

[2] H. Griem: *Spectral Line Broadening by plasmas* (Academic Press, New York, 1974), Chap. II, p.17.