

## Momentum transport in full-f gyrokinetic simulations

### full-fジャイロ運動論シミュレーションにおける運動量輸送

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Momentum transport in full-f gyrokinetic simulations is discussed. Accuracy issues of momentum transport in gyrokinetic simulations are studied using a full-f gyrokinetic Eulerian code GT5D. Test calculations in the axisymmetric limit and in turbulent tokamak show that toroidal angular momentum is conserved and that momentum transport and related radial electric fields are correctly computed. Comparisons of ion temperature gradient driven turbulence with and without adaptive momentum sources, which cancel a flow generation, show that momentum transport tends to suppress heat transport by enhancing radial electric field shear, which is connected with rotation profiles through a force balance relation. An existence of residual stress is also identified in the latter simulation.

### 1. Introduction

Momentum transport induced by tokamak micro-turbulence is one of key issues in predicting the performance of ITER, and has been studied intensively using gyrokinetic simulations. However, based on an ordering argument, Parra and Catto [1] raised a serious concern about the accuracy of momentum transport and related radial electric fields  $E_r$  in gyrokinetic simulations, in particular, with so-called full-f approaches, where both momentum transport and rotation profiles develop self-consistently. On the other hand, a recent work [2] clearly demonstrated that toroidal angular momentum is conserved at any order, provided that the equation system is derived based on the modern gyrokinetic theory [3] with keeping an energetic consistency. In this work, we address accuracy issues of momentum transport in a full-f gyrokinetic Eulerian code GT5D [4] by studying basic properties of momentum transport in the axisymmetric limit and in turbulent tokamak. After clarifying the accuracy issues, we discuss influences of momentum transport on heat transport in the ion temperature gradient driven (ITG) turbulence.

### 2. Calculation model

GT5D is based on the modern gyrokinetic theory, in which the gyrokinetic equation is simply given as

$$\frac{\partial f}{\partial t} + \{f, H\} = C(f) + S, \quad (1)$$

where  $f$  is a full-f ion distribution function,  $\{, \}$  and  $H$  are the Poisson bracket operator and the

Hamiltonian in the gyro-center coordinates,  $C$  is a linear Fokker-Planck collision operator, and  $S$  is a source term, which involves fixed on-axis heating, a heat and momentum sink near the plasma boundary, and an adaptive momentum source. Turbulent fluctuations are determined by solving the gyrokinetic Poisson equation with linearized ion polarization density and adiabatic electron response. This equation system is derived from a single field Lagrangian and keeps an energetic consistency that leads to the energy and momentum conservation [2]. Eq. (1) is implemented using the toroidal angle and non-dissipative and conservative finite difference, which is a variant of centered finite difference, and therefore, the scheme keeps the toroidal symmetry numerically.

### 3. Accuracy of momentum transport

The first test case is in the axisymmetric and collisionless limit. If one initializes plasma with a local Maxwellian distribution  $f_{LM}$ , its non-equilibrium component excites initial electric fields, while plasma stays quiet when one uses a Vlasov equilibrium  $f_{CM}$  given as a function of the canonical angular momentum  $P_\phi$ . This shows a conservation of  $P_\phi$  in the axisymmetric limit. The second test case is still in the axisymmetric limit but with collisions, i.e., in the neoclassical case. In this case, the system initialized with  $f_{LM}$  shows a collisional relaxation towards a neoclassical quasi-steady state, where the particle flux vanishes due to the ambipolar condition, the heat flux converges to neoclassical levels, and  $E_r$  is determined to satisfy a force balance relation,

$$V_{\parallel} = \frac{T_i I}{m_i \Omega_i \psi'} \left[ (k-1) \frac{d \ln T_i}{dr} - \frac{d \ln n_i}{dr} + \frac{e}{T_i} E_r \right], \quad (2)$$

which shows a balance among  $E_r$ , the parallel flow  $V_{\parallel}$ , and the gradients of density  $n_i$  and temperature  $T_i$ . In this test, a coefficient  $k$  is estimated using  $E_r$  determined by the gyrokinetic Poisson equation, and other plasma profiles,  $n_i$ ,  $V_{\parallel}$ ,  $T_i$ , and it is found that  $k$  agrees with the neoclassical theory, which suggests that  $E_r$  is correctly computed when  $V_{\parallel}$  is given. We then examine the accuracy of momentum transport in turbulent tokamak. By taking  $P_{\varphi}$  moment of Eq. (1) and applying gyro-average and flux-surface average operators, one obtains a radial transport equation or a conservation law of toroidal angular momentum, which gives a balance among evolutions of flows, momentum transport, and momentum sources as shown in Fig. 1. It is noted that a conservative nature of GT5D, which leads to cancellation of all the terms related to the poloidal flux  $\psi$ , plays an important role in deriving the toroidal momentum conservation. In the core region, a bipolar flow generation (green) is balanced with momentum transport (blue), while in the edge region, momentum transport (blue) is canceled by co-current momentum injection from a sink term (magenta), which keeps a no-slip boundary condition. In the whole region, the momentum conservation is satisfied with good accuracy. In the final test case, we examine influences of higher order drift terms on heat and momentum fluxes by implementing higher order Hamiltonian [5]. The result shows that influences of higher order drift terms are negligibly small, as is expected from the gyrokinetic ordering. All the above test results suggest the correctness of turbulent momentum transport in GT5D.

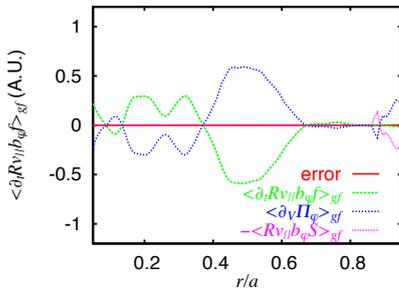


Fig.1 Momentum conservation in ITG turbulence.

#### 4. Impact of momentum transport on heat transport

In order to study influences of momentum transport on turbulent transport, we compare ITG turbulence with and without an adaptive momentum source, which cancels a flow generation due to

momentum transport. Numerical experiments with a fixed on-axis heating are performed for Cyclone-like parameters with plasma sizes of  $1/\rho^* = a/\rho_{ti} = 100, 150, 225$ , where  $a$  is the minor radius and  $\rho_{ti}$  is the thermal ion gyro-radius. Figure 2 (a) shows radial profiles of the heat diffusivity  $\chi_i$ , which shows worth-than-Bohm scaling [6]. In all the plasma sizes, it is shown that transport levels observed with the adaptive source (solid lines) exceed those without the adaptive source (dashed lines). In Fig. 2 (b), it is shown that a spontaneous flow generation due to residual stress enhances  $E_r$  shear through the force balance relation (2), which is approximately satisfied even with turbulent fluctuations. The enhancement of  $E_r$  shear leads to suppression of heat transport. Another important finding in the numerical experiment with the adaptive source is the observation of finite momentum flux induced at  $V_{\parallel} \sim 0$ . The result clearly shows an existence of residual stress, which is driven by the asymmetry of  $k_{\parallel}$  turbulent spectra. Although various symmetry breaking mechanisms such as  $E_r$  shear [7] and profile shear [8] have been proposed, the result show similar trend between radial profiles of momentum flux and  $E_r$  shear, which suggests that  $E_r$  shear is a dominant symmetry breaking mechanism in the present numerical experiment.

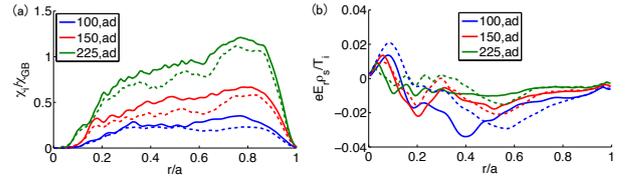


Fig.2 Radial profiles of (a) ion heat diffusivity  $\chi_i$  and (b) radial electric field  $E_r$  in ITG turbulence with  $1/\rho^* = 100, 150, 225$ . Solid (dashed) lines show results with (without) the adaptive momentum source.

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