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Theoretical Study of Nonlocal Effect of the Ponderomotive Force in High Intensity Laser Fields Using the Noncanonical Lie Perturbation Theory
非正準Lie摂動論を用いた高強度レーザー場中での動重力の非局所効果に関する理論研究

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We investigate the relativistic ponderomotive force in a tightly focused high intensity laser field which takes into account the higher order structure of the laser field amplitude. An extended formula for the ponderomotive force was derived by using the noncanonical Lie perturbation theory, which is based on the phase space Lagrangian formalism. The force is found to depend not only on the gradient of the field amplitude, but also on the curvature and its derivative in the higher order originating from the nonlocal particle motion during a laser period.

1. Introduction

The intensity of ultra-short high power lasers has reached the range of 10^{22}W/cm^2, where electrons irradiated by such lasers exhibit relativistic characteristics. Recently, efforts aiming at higher intensities of $10^{23\text{-}26}$W/cm^2 have been made, which is expected to open up an entirely new scientific regime [1]. In order to achieve such high intensities, reduction of the pulse length and/or the spot size is necessary so that the ponderomotive force (light pressure) becomes of critical importance in determining the laser-plasma interaction.

The ponderomotive force has been derived based on the averaging method [2] and is explained as the force proportional to the field gradient resulting from the first order perturbation to the uniform field. In this method, terms related to the higher-order derivatives have been neglected. However, they could become important in the case of strong focusing where the laser field amplitude varies within the excursion length of the oscillatory particle motion. Namely, since the particle experiences nonlocal field structure around the oscillation center, the average field strength and/or its gradient over the oscillation cycle varies in the existence of higher-order derivatives of the laser field amplitude. In order to investigate such nonlocal effects, here, we introduce the noncanonical Lie perturbation theory [3]. Based on the perturbed phase space Lagrangian, we derive the equations of motion of the particle taking into account the higher order field structure, i.e., the second and third derivatives of the field amplitude.

2. Noncanonical Lie Perturbation Theory

We introduce the extended phase space expressed by canonical variables, $z_c^{\mu} = (t; \mathbf{x},\mathbf{p}_c) = (t; x,y,z,p_{cx}, p_{cy}, p_{cz})$, and the corresponding covariant vector, $\gamma_{c\mu} = (-h; \mathbf{p}_c,\mathbf{0})$, where h is the relativistic Hamiltonian. In this paper, we use Latin indices that run from 1 to 6 whereas Greek run from 0 to 6. Using these notations, the variational principle is expressed as $\delta \int \gamma_{c\mu}\, dz_c^{\mu} = 0$, where $\gamma_{c\mu}\, dz_c^{\mu}$ is referred to as a fundamental 1-form. Under an arbitrary coordinate transformation $z_c^{\mu} \mapsto z^{\mu}$, the new covariant vector is obtained by the scalar relationship $\gamma_{c\mu} dz_c^{\mu} = \gamma_{\mu} dz^{\mu}$. Then, the equation of motion in an arbitrary noncanonical coordinate is given by the variational principle as $dz^i/dz^0 = \mathrm{J}^{ij}\, (\partial\gamma_j/\partial z^0 - \partial\gamma_0/\partial z^j)$, where J^{ij} is the Poisson tensor, $\mathrm{J}^{ij} = (\partial_i\gamma_j - \partial_j\gamma_i)^{-1}$. The Lie transformation is characterized by the near-identity transformation operator L as $z^{\mu} \mapsto z'^{\mu} = \exp(\varepsilon L)\, z^{\mu}$ [3]. The corresponding covariant vector is transformed as $\gamma_{\mu} \mapsto \gamma'_{\mu} = \exp(-\varepsilon L)\,\gamma_{\mu} + \partial_{\mu} S$, where S is the gauge function. In the Lie perturbation method, we utilize the coordinate transformation to simply describe the perturbed motion in each order by carefully choosing the generator of the transformation and the gauge function.

3. Preparatory Transformation

We consider a single particle motion in the relativistic regime irradiated by transversely non-uniform high intensity laser field. We express the laser field by the normalized vector potential, $\mathbf{a} = q\mathbf{A}/mc^2$, as $\mathbf{a} = a_x(x) \sin\eta\ \mathbf{e}_x$, where q and m are the charge and rest mass of the particle, respectively, c is the speed of light, $\eta \equiv \omega t - k_z z$ is the laser phase and $\mathbf{e}_x$ is unit vector in the x-direction. We introduce a smallness parameter, $l/L \sim \varepsilon$, $l^2/R \sim \varepsilon^2$, $l^3/T \sim \varepsilon^3$, where l is the transverse excursion length of the particle and $L^{-1} \equiv a_x^{-1}\partial_x a_x$, $R^{-1} \equiv a_x^{-1}\partial_x^2 a_x$ and $T^{-1} \equiv a_x^{-1}\partial_x^3 a_x$ are scaling factors of the field amplitude variation.

Since the particle in a uniform laser field exhibits the figure-eight motion in the period of η keeping the quantity $p_\eta \equiv p_z - \gamma mc$ constant [4], we at first transform the canonical coordinate z_c^μ to a new one, $z^\mu = (\eta; x,y,z,p_x,p_y,p_\eta)$, which is noncanonical. Here, γ is the relativistic factor and $\mathbf{p} = \mathbf{p}_c - mc\mathbf{a}$ is the mechanical momentum. By taking both η and p_η as coordinate variables, the new Hamiltonian K is found to be expressed in a simple form, $K = -(2k_z p_\eta)^{-1}(m^2c^2 + \mathbf{p}_\perp^{\,2} + p_\eta^{\,2})$.

4. Perturbation Analysis

Here, we expand the amplitude of the vector potential $\mathbf{a}$ around the oscillation center of the figure-eight orbit, X, as

$$a_x(x) = a_x(X)\left[1 + \varepsilon\frac{\tilde{x}}{L} + \varepsilon^2\frac{\tilde{x}^2}{2!R} + \varepsilon^3\frac{\tilde{x}^3}{3!T} + \cdots\right], \qquad (1)$$

where $\tilde{x} \equiv X - x$. Here, L, R and T are evaluated at X. In the zeroth order of ε, Eq. (1) denotes a uniform field. In this order, we treat $a_x = a_x(X)$ as a constant. From the zeroth order 1-form in the noncanonical coordinate z^μ, we can derive the equations of motion that yield the analytical solution denoting the unperturbed figure-eight motion. From the solution, the excursion length l is obtained as $l = a_x/k_z\zeta_0$, where ζ_0 is the constant defined by the initial condition, $p_\eta(\eta = 0) = -mc\zeta_0$.

Next, to investigate the motion of the oscillation center, we again transform the coordinate to the new one,

$$Z^\mu = (\eta; X, Y, Z, P_x, P_y, p_\eta), \qquad (2)$$

by using the relation $Z^\mu = z^\mu - [z^{(0)\mu}]_{\mathrm{os.}}$, where $[z^{(0)\mu}]_{\mathrm{os.}}$ is the oscillatory part of the zeroth order orbit. Note that hereafter we treat $a_x(X)$ not as a constant but as a variable depending on X. In the coordinate Eq. (2), we derive the perturbed equations of motion up to the third order of ε based on the Lie perturbation method. It is found that the covariant vector in the Lie-transformed coordinate Z'^μ does not depend on the variables Y' and Z' so that the corresponding variables P_y' and p_η' remain constant in all order. For the x-direction, we obtain the equation of motion,

$$\frac{dP_x'}{d\eta} = -\frac{mca_x}{2}\left[\varepsilon\frac{l}{L} + \frac{\varepsilon^3}{8}\left(\frac{3}{2}\frac{l}{L}\frac{l^2}{R} + \frac{l^3}{T} - \frac{3}{2}\frac{l^3}{L^3}\right)\right] + \varepsilon mca_x\frac{l}{L}\cos 2\eta. \qquad (2)$$

This equation determines the averaged motion of the oscillation center up to the order of ε^3. We see that, in addition to the first-order ponderomotive force proportional to the field gradient, the third-order terms related to R^{-1} and T^{-1} appear in Eq. (3). According to the equation, the ponderomotive force is modified to become larger when the second and/or third derivative is positive. This is because the positive second derivative increases the average value of the field strength so that the excursion length becomes larger, and positive third derivative gives steeper gradient at the oscillation center in average, respectively. Note that the second-order average force which depends on the second derivative, l^2/R, does not appear. This result indicates that, due to the symmetric nature of the curvature, the nonlocal effect to the field gradient is cancelled during single laser period. We confirmed that the same can be said for all the following even-order derivatives.

5. Summary

Based on the noncanonical oscillation-center coordinate, we derived the equations of motion up to the third order of ε by using the noncanonical Lie perturbation theory. We found that, in the third order, terms related to the second- and third-derivatives of the laser field amplitude are added as a modulation to the first-order ponderomotive force. These additional forces originate from the nonlocal particle motion around the oscillation center. The result suggests that the control of the higher order field structure is important in the case of tight focusing.

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