Velocity Field Estimation using Tomography in a Cylindrical Plasma

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This article presents a new method, called extended Rotational Movement Analysis (e-RoMA), to estimate the velocity field of a cylindrical plasma from its two-dimensional images based on Fourier-Bessel function expansion. The proposed method is applied to tomography images of a plasma produced in a linear cylindrical device PANTA and succeeded in obtaining the two- dimensional velocity field. The first results suggest an issue to be investigated, that is, the presence of azimuthal asymmetry in the velocity field.

Keywords: velocimetry, tomography, cylindrical plasma, plasma emission

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Various methods have been developed to infer local plasma velocity using imaging diagnostics [1]. A method called Rotational Movement Analysis (RoMA) is presented to successfully evaluate temporal evolution of plasma rotational (or one dimensional) velocity from tomography images of a cylindrical plasma in Plasma Assembly for Nonlinear Turbulence Analysis (PANTA) [2]. This paper presents a novel method, called e-RoMA, which is extended to two dimensional (2D) measurement, and its first application to PANTA plasma demonstrates its feasibility.

First, in the method, 2D velocity field is assumed to be expressed using two functions as,

$$v_r = \frac{\partial \Phi}{\partial r}, \quad v_\theta = \frac{1}{r}\frac{\partial \Phi}{\partial \theta} + \frac{\partial \psi}{\partial r}, \tag{1}$$

where Φ and ψ are the velocity potential function, and a function to describe vortex flows, respectively. Here, the latter called vortex function, ψ, can be regarded as the third element of a vector function $\vec{A} = (0, 0, \psi)$, then the velocity field is described as $\vec{v} = \nabla\Phi + \nabla \times \vec{A}$.

Second, the change of plasma emission can be predicted using the Eulerian equation as

$$\hat{\varepsilon}(\vec{r}, t + \Delta t) = \varepsilon(\vec{r}, t) - v_r(\vec{r}, t)\left(\frac{\partial \varepsilon}{\partial r}\right)\Delta t - \frac{v_\theta(\vec{r}, t)}{r}\left(\frac{\partial \varepsilon}{\partial \theta}\right)\Delta t, \tag{2}$$

where $\vec{r} = (r, \theta)$, and $\varepsilon(\vec{r})$ represents emission distribution. Then, the velocity field can be determined by minimizing the residual, χ^2, between the predicted and observed emission, which is defined as

$$\chi^2 = \sum_{k=1}^{N}\left(\eta(\vec{r}_k) + v_r(\vec{r}_k)\frac{\partial \varepsilon_k}{\partial r}\Delta t + \frac{v_\theta(\vec{r}_k)}{r}\frac{\partial \varepsilon_k}{\partial \theta}\Delta t\right)^2, \tag{3}$$

where $\eta(r, \theta, t) = \varepsilon(r, \theta, t + \Delta t) - \varepsilon(r, \theta, t)$ and the summation is carried out for different spatial points. The velocity potential and vortex functions are assumed to be expanded in the Fourier-Bessel function (FBF) [3] as

$$\Phi = \sum_m \sum_n a_{m,n} J_m(k_{mn} r)\cos(m\theta) + b_{m,n} J_m(k_{mn} r)\sin(m\theta), \quad \psi = \sum_l c_l J_{m=0}(k_{m=0,l} r), \tag{4}$$

where, $J_m(r)$, k_{mn} and m are the m-th order Bessel function, n-th zero point of $J_m(r)$, and azimuthal mode number,

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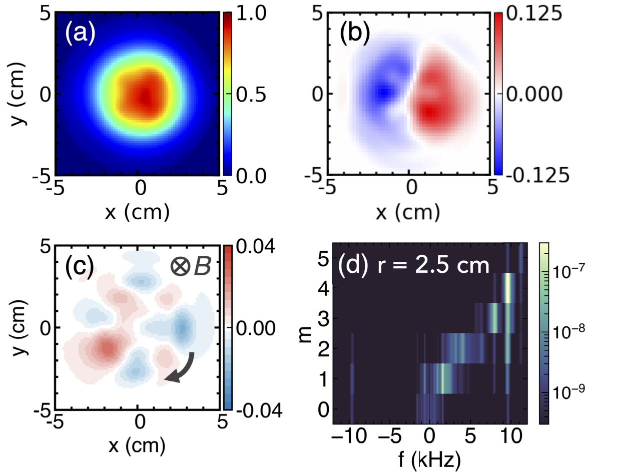

Fig. 1 (a) Temporally averaged emission profile reconstructed with Fourier Bessel function expansion. It shows azimuthal asymmetric structure. (b) The azimuthal asymmetry part of (a) reconstructed by removing the symmetric ($m = 0$) part. (c) A snapshot of fluctuation pattern. The arrow indicates the direction of rotation. (d) A dispersion relation or power spectra at $r = 2.5$ cm.

respectively. The coefficients $a_{m,n}$, $b_{m,n}$ and c_l are determined by minimizing the residual. By determining these two mother functions, e-RoMA can provide the velocity field continuously.

The method is applied to a PANTA plasma [2]. Figure 1 shows an overview of the target plasma measured with tomography that covers the entire plasma region in the following experimental conditions: magnetic field strength of 1300 G, filling Ar pressure of 3 mTorr, and RF input power of 6 kW. Reconstructed images of ArII emission are obtained in a sampling time of 1 µs. The temporally averaged emission profile has an azimuthally asymmetric structure as seen in Figs. 1 (a) and (b), and the fluctuation image in Fig. 1 (c) shows that the fluctuations are dominated by coherent modes. The dispersion relation (or power spectrum) in Fig. 1 (d) shows that the strongest mode is the azimuthal $m = 4$ mode, which should produce the sideband modes of $m = 3$ and $m = 5$ through the nonlinear coupling with the $m = 1$ mode.

A choice of fitting bases should change the results of velocity field estimation. Thus, the Akaike Information Criterion (AIC) is used to optimize the fitting bases, and an AIC minimum is found at $m = 0$ to 4, with $n = 0$ - 2 and $l = 0$ - 5, then the total number of bases functions is 33. Figure 2 shows the results of obtained velocity field using the optimized set of the basis functions: the temporal evolution of the radial and azimuthal velocities, temporally averaged ones, and their 2D images. Each plotted point in Figs. 2 (a) and (b) is evaluated from 4500 spatiotemporal points composed of 50 sequential temporal data, whose interval is 1 µs, at 90 plasma positions. From the longer temporal averages for 100 ms as a function of radius, the averaged radial and azimuthal velocities are found to have azimuthal asymmetry (Figs. 2 (c) and (d)). The azimuthally averaged velocity $\langle v_\theta \rangle$ in Fig. 2 (d) are consistent with the propagation velocity of $m = 4$ mode, obtained from the Fourier spectrum $\langle \omega/k \rangle = \sum \omega P(\omega)/\sum kP(k)$.

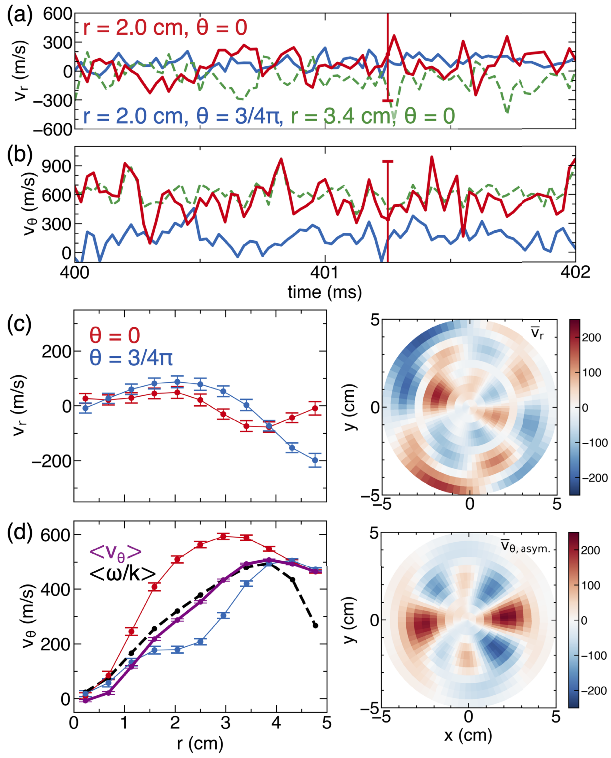

Fig. 2 (a) The temporal evolution of radial and (b) azimuthal velocity. (c) The radial profile of temporally averaged radial velocity (left), and the 2D image (right). (d) The radial profile of temporally averaged azimuthal velocity (left), and 2D image of its azimuthally asymmetric part (right). The azimuthal asymmetry is calculated in the mother function Φ without symmetric ($m = 0$) components.

To examine the reality of the velocity field asymmetry, the uncertainty of inferred velocity needs to be evaluated. The uncertainty can be evaluated from the well-known properties of chai-square (χ^2-) distribution: as the element number of the ensemble is sufficiently large, χ^2- distribution becomes closer to the Gaussian distribution that whose average and the standard deviation are N and $\sqrt{2N}$: that is, $\chi^2/\sigma^2 = N$ and $\delta\chi^2/\sigma^2 = \sqrt{2N}$. Thus, $\delta\chi^2/\chi^2 = \sqrt{2/N}$ is obtained, and a deviation of a fitting parameter, g_i, corresponding to ($a_{m,n}$, $b_{m,n}$, c_l), to cause the change of $\delta\chi^2$, can be estimated as $\delta\chi^2 = (\partial^2\chi^2/\partial g_i^2)\delta g_i{}^2/2$, where $\partial\chi^2/\partial g_i = 0$ is taken into account. Therefore, the deviation of the fitting parameters δg_i, to give a standard deviation of χ^2 is evaluated as

$$(\delta g_i)^2 = 2\sqrt{\frac{2}{N}} \frac{\chi^2}{(\partial^2\chi^2/\partial g_i^2)}. \tag{5}$$

Then, the uncertainty in the velocities shown in Fig. 2 is calculated as

$$(\delta v(\vec{r}))^2 = \sum_i \left(\frac{\partial v(\vec{r})}{\partial g_i} \right)^2 (\delta g_i)^2. \quad (6)$$

In the temporal evolution (Figs. 2 (a) and (b)), the uncertainty is larger than the fluctuations of velocities, while the uncertainty in the averaged velocities is much smaller than the asymmetry since the ensemble number is 2000 times larger. Thus, the results imply that the azimuthal asymmetry in the averaged velocity field could really exist. This may be caused by the presence of azimuthal asymmetry (see Fig. 1 (b)); the maximum fraction $|(\varepsilon - \varepsilon_{sym})/\varepsilon|$ is approximately 30% where ε_{sym} represents the local value of symmetric emission.

In summary, this article proposed a 2D velocimetry in which two mother functions for velocity fields are determined from the evolution of fluctuating emission patterns. The application to a cylindrical plasma finds, demonstrating the promising abilities, an azimuthal asymmetry in the averaged velocity field.

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