Perspective of Meta-Hierarchy Dynamics*)

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(Received 21 February 2023 / Accepted 1 May 2023)

The understanding of nature has been developed by separating and connecting elements in a reductive manner. For example, a *spatio-temporal scale* can provide us a clear picture of elemental separations. In biology, a *function* also gives us a useful picture to understand biological nature. Furthermore, a *physical model* also improves the outlook for understanding physical nature. These divided or connected elements, namely, scales, functions, and models form hierarchical structures in nature. On the other hand, fusion science explores various multi-scale and multi-physics phenomena, spreading over spatio-temporal scales from the microscopic to the macroscopic. In particular, collective motion causes structural formations not only in core plasmas but also material-facing ones. Therefore, fusion science has been an excellent subject for the application of the hierarchical approach. However, some problems have emerged with the progress of experimental and numerical research in fusion science. We often encounter phenomena that cannot be well understood by hierarchical separation. For these phenomena, beyond the conventional approaches for hierarchical systems, it is necessary to reconsider them with meta-perspectives, i.e., *meta-hierarchy dynamics*.

Keywords: hierarchy, multi-scale, multi-physics, nonlinear phenomena

DOI: 10.1585/pfr.18.2501057

1. Introduction

Hierarchical structure in nature is one of the most significant findings in natural sciences. The cosmic Uroboros [1] (Fig. 1), for example, well represents the hierarchical structure of the universe with several scales from the cosmic horizon of $> 10^{28}$ cm to the scale of a Grand Unified Theory (GUT) of 10^{-30} cm, using the serpent from its head to tail. The serpent covers whole scales of matter, i.e., quarks, nucleons, atoms, molecules, crystals, biological bodies, stars, nebula, and galaxies.

On the other hand, in biology, there is an alternative picture of hierarchy, *functions*. In this sense we can divide creatures into proteins, cell membranes, cells, tissues, individuals, species, and ecological systems. Furthermore, in terms of a physical path to understand nature, we have another hierarchical approach, i.e., *models* such as particle model, fluid model, turbulence model, bubble model and so on.

The point of view of the above hierarchical picture is that it can give us a clear understanding of nature. In plasma and fusion science, approaches based on the hierarchical picture have also provided significant insights such as the Macro-Micro Interlocked (MMI) method [2,3]. The basic idea of the method, which is a kind of connected hierarchy algorithm, is performed as follows, as shown in Fig. 2. At first, when a phenomenon occurring at the macroscopic scale reaches a situation that has a significant effect on the micro scale, the information is transferred

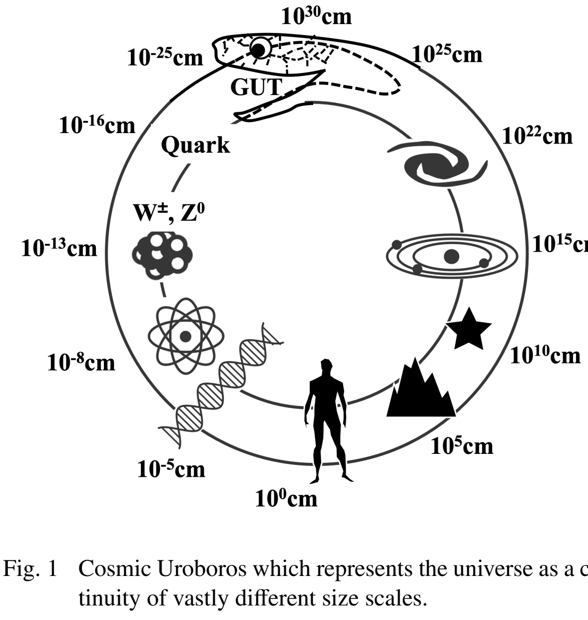

Fig. 1 Cosmic Uroboros which represents the universe as a continuity of vastly different size scales.

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*) This article is based on the presentation at the 31st International Toki Conference on Plasma and Fusion Research (ITC31).

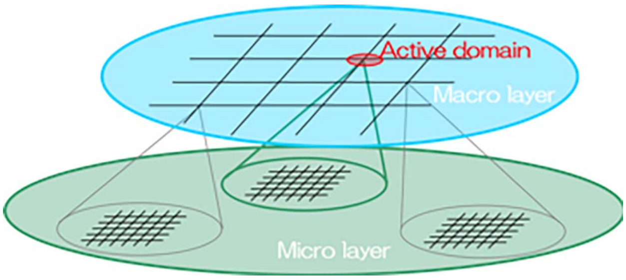

Fig. 2 Schematics of the Macro-Micro Interlocked (MMI) method. The information is transferred from the macroscopic to the microscopic scale. Then, a simulation using a microscopic equations is performed.

from macroscopic to microscopic. Then, a calculation using microscopic equations is performed. Next, the information derived from the calculation is transferred to the macroscopic scale. Finally, it is possible to simulate how the transformation develops into a global one.

However, we also know from our predecessors' studies that multi-scale and multi-physics phenomena involving complex processes and hierarchical structures cannot necessarily be solved or explained by conventional scale-based approaches. In order to study their physics, a meta-perspective approach to hierarchical structures, i.e., *meta-hierarchy dynamics* should be discussed. Therefore, we would like to share the issues among fields studied separately and develop research, including exploration of plausibility and conditions of the hierarchies and their static and dynamic characteristics. Although the themes that will be discussed in this paper are specific and concentrate on plasma and fusion science, almost all of them include complex and composite processes, and discussions can be applied to general features in this complex science.

This paper is organized as follows. In Sec. 2, conventional approaches to hierarchical systems in plasma and fusion sciences are introduced. In Sec. 3, we discuss the difficulty of the conventional approaches based on scale separations. Section 4 will be assigned to discussions of problems in the integration of the differences in scales and models. In Sec. 5, the paper will be summarized.

2. Conventional Approaches to Hierarchical System

The first approach to multi-scale and multi-physics phenomena is to try to understand each elemental part of the physics individually. In terms of elemental reduction, once the elementary processes are understood, it is expected that the next step is to understand the whole by integrating them. For example, in the case of plasma simulations, macroscopic phenomena are solved by magnetohydrodynamics (MHD) calculations, and microscopic motions are solved with particle calculations such as particle-in-cell (PIC) method. In order to integrate them, there are various levels of integration. A simple way is that the microscopic information from PIC is given to MHD as input parameters. As a higher level of the integration, the way may involve coupled hierarchical simulations in which PIC and MHD feed back into each other [4]. In the material research field, it has been also studied that macroscopic strain deformation and heat conduction in solids are solved using the finite element method and others based on continuum descriptions, while the behavior of atoms and molecules is solved using molecular dynamics (MD) and density functional theory (DFT) from a microscopic viewpoint. Nowadays, the concept and approach of multi-scales and multi-physics have become commonplace. At least, it is common practice to use different systems of physical equations for the macroscopic and microscopic physical pictures.

However, conventional elemental reduction is not necessarily effective for all problems in complex phenomena. In other words, there are still problems that cannot be successfully explained by understanding the elemental processes and their integration. The current trend in various fields is to seek new approaches to these problems. When the scale separation behind the phenomenon is sufficiently effective, the approach of understanding and integrating individual elementary processes works well. In this case, we regard the scale length of each elemental physical process as the characteristic one, and then we consider the effects of multiple different scale lengths for the complex process. It is probable that if the scale gap is large, the scale-separated description is a good approximation. Phenomena with a large scale gap and easy scale separation, however, are not necessarily easy to integrate. For example, in order to transfer microscopic information to the macroscopic scale, it is necessary to understand the correspondences that bridge the difference between microscopic and macroscopic descriptions. The question of whether macroscopic physical quantities and continuum pictures can be defined by microscopic physical quantities from particle pictures has been historically addressed since Irving-Kirkwood [5]. And in the case of large scale-gap problems, it is important to consider a statistical correspondence between macroscopic and microscopic degrees of freedom. In the research field of condensed matter, the construction of statistical correspondence often starts from assumptions of local equilibrium and the principle of detailed balancing. This is because their problems are targeted at relatively low temperatures. Modern plasma fusion research covers problems under strong non-equilibrium conditions, and the correspondence is not always satisfactorily developed.

Even if a method for transferring information from a microscopic physical process to a macroscopic one has been established, the opposite, a method for transferring information from a macroscopic to a microscopic process, may not have been established. Therefore, it is difficult to develop truly coupled simulations that provide feedback from the macroscopic to microscopic, as discussed above. Furthermore, even when macroscopic and microscopic descriptions are well established, a physical description of the *meso-scale* structure is not yet established. In solid materials, for example, the meso-scale structure is the grain boundary structure that lies between the macroscopic continuum description and the crystalline structure that results from the arrangement of atoms as a particle image.

In addition, the development of computer is also related to these problems. Since the rapid development of computers has made it possible to handle large degrees of freedom in simulations, there have been many studies with aspects of numerical algorithms. However, the evolution of computers has not necessarily progressed in the desired direction. Although the degree of parallelization has increased, the clock speeds and memory access speeds have not. Thus, although the number of degrees of freedom that can be treated has increased, the number of simulation steps has not increased much. Namely, the spatial scale has been increased, but the time scale has not been extended. In this situation, if the time scale gap is too large, the statistics needed to bring the microscopic information up to the macroscopic scale are not sufficient.

3. Difficulty of Scale Separation

The simplest approach to multi-scale / multi-physics problems is based on the assumption of scale separation and the perspective of their integration. However, as discussed in the previous section, the challenges of integration, such as with meso-scale structures, may be viewed as the essence of the difficulty, which is that the act of scale separation itself is difficult. This is also the case when the physical processes that are complex and elementary cannot be sufficiently identified. The difficulty in the scale separation means that the distance of the focused scales from microscopic to macroscopic is very close, with a small scale gap. In that case, we cannot establish a way to evaluate the phenomena with sufficient approximate expressions. Our problem is that in multi-scale and multi-physics phenomena, the description and understanding of scale-indivisible physical processes with small characteristic scale gaps is strongly needed for overall understanding.

Here, let us delve a little deeper into the issue of scale-indivisible physical processes. When the characteristic scales of macroscopic and microscopic physical processes are close to each other, what is the description of the macroscopic side? For example, if we consider the many-body distribution function as the starting point of the microscopic description of a continuum picture, the Boltzmann equation can be obtained as a one-step higher macroscopic description with the one-body distribution function, where the BBGKY (Bogoliubov-Born-Green-Kirkwood-Yvon) hierarchy [6–10] exists. In the establishment of a kinetic equation, the approximation of dropping higher order terms and being restricted to the collision terms of the two bodies is nothing more than an assumption of scale separation. For further approximation in constructing Euler's fluid equation from the Boltzmann equation, Maxwell-Boltzmann distribution is also assumed, with respect to the distribution in velocity space. This corresponds to taking out only slow motion, which is also a scale separation. In general, there may be cases in which such scale separation assumptions are not valid. For example, it is not sufficient to discuss magnetic confined plasma turbulence only by MHD equations. In that case, simulations using kinetic equations should be performed, even if they cost more.

Next, let us consider how a microscopic description is affected when the characteristic scales of the macroscopic and microscopic physical processes are close. Supposing that the microscopic side is described by a particle picture, the effect of the macroscopic physical process can be regarded as an "external field." In that case, the effect of the external field can be solved. In comparison, a situation with a smaller scale gap can be called a time-varying external field. In most of these cases, it is not realistic to obtain an analytical solution, except for special cases such as an external field that fluctuates very slowly or cyclically. Numerical simulation can be used to analyze time-varying external fields, but it does not always satisfy the objective.

3.1 Plasma turbulence

In plasma turbulence phenomena, the scale gaps and separations are quite significant. In plasma, light electrons and heavy ions are mixed together. The mass ratio of electrons to ions is in the order of 1000. Therefore, when the description of electrons and ions is separated, the distribution of ions, which is an external field for electrons, fluctuates at a speed that cannot be regarded as "very slow." In fact, magnetic confined plasmas also encounter this problem in understanding turbulence. The fluctuation spectra of general turbulence phenomena often exhibit power laws over a wide range of the wavenumber space, making scale separation inherently difficult. On the other hand, in the problem of microscopic turbulence developing in magnetic confined plasmas, scale separation has been considered possible because instabilities caused by the motion of ions driving the turbulence and those caused by the motion of electrons exhibit a gap between wavenumber scales on the high and low wavenumber sides of their Fourier space. However, with the recent development of kinetic turbulence simulations, it has become possible to solve the ion and electron scales simultaneously and collectively, compared to calculations daring to restrict the solution to the ion or electron scales only. Neither of the calculations agree with each other, and it becomes clear that the ion-scale and electron-scale motions affect each other through zonal flows and other nonlinear processes [11] as shown in Fig. 3.

A more complicated question is whether scale separation modeling is valid if the scale gap is sufficiently wide, which is not necessarily the case. As a specific example, in addition to the importance of the multi-scale in-

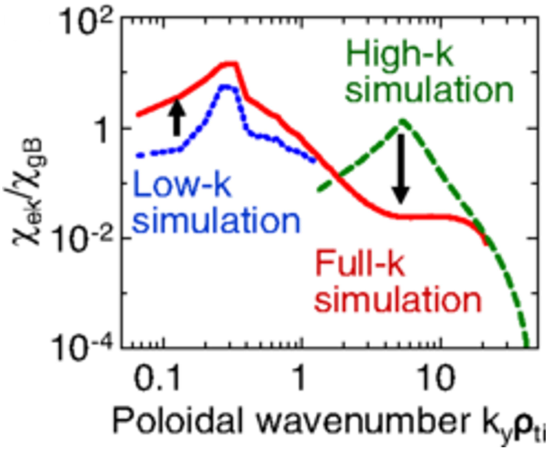

Fig. 3 Poloidal wave number spectra of the time-averaged electron energy diffusivity for electromagnetic (β = 2.0) turbulent plasma. The solid (red), dotted (blue), and dashed (green) curves plot the results obtained from the full-*k*, low-*k*, and high-*k* simulations, respectively [11].

teraction of ions and electrons in microscopic turbulence, the interaction of microscopic turbulence and macroscopic equilibrium distribution beyond scale separation is another important recent issue. There is a gap of more than 10^5 from the confinement scale to the electron scale. Phenomena such as localized turbulent fluctuations and transport fluxes propagating globally in the plasma like an avalanche, conversely trapped and localized by zonal flows and radial electric fields, and solitary structures caused by these fluctuations superimposed on the macroscopic temperature gradient distribution have been revealed by global kinetic simulations [12].

Furthermore, in the laboratory plasma experiments, an interesting result has been observed, slow transitions between two distinct turbulent states with different temperature dependences [13]. Such transitions among multiple states in magnetized plasmas, which are predicted based on the statistical theory of plasma turbulence [14], will be studied in terms of the meta-perspective approaches to understand the hierarchical structures in plasma turbulence physics.

3.2 Material science

The scale-indivisible problem associated with the previous subsection is also true for the analysis of atomic and molecular processes and materials. To explain the properties of molecules and materials, electrons should be solved according to quantum mechanics, and DFT is popular in recent research. Here, the nuclei play the role of an external field for the electrons and are basically fixed. If we want to track the movement of the nuclei, the Born-Oppenheimer approximation [15] is generally used, in which only the “static” electronic density is solved quantum mechanically on a stationary atomic configuration at a certain instant. The force received by each nucleus is calculated from the electronic density, and the nuclei are moved for a small amount of time. Namely, the state and the equation of the motion of nuclei are solved alternately.

However, there are phenomena that cannot be explained well by this method based on a scale or model separation. One such phenomenon is the quantum effect of nuclei in the motion of light atoms such as hydrogen and helium. In fusion reactor materials, the diffusion process of hydrogen and helium adsorbed in the first wall is an important issue in recycling, and it has been reported that the effective change of diffusion coefficient due to zero-point oscillation and a tunneling effect as a quantum effect of the nucleus takes effect even at room temperature [16]. Fusion research is a good area to investigate the quantum effect of nuclei for situations such as hydrogen isotopes, which chemically interact in exactly the same way but differ only in mass.

The other is the dynamics of electrons themselves in plasma-material interaction. Generally, the particles absorbed from plasma onto a material surface are charged ions, and then they are neutralized by the interaction on the material surface. The neutralization process is typically the dynamics of electrons. In simulations of plasma-material interaction such as MD, historically, this process is ignored and an incident particle is replaced by a neutral atom. These problems commonly cannot be presented by scale separation with the Born-Oppenheimer approximation.

In addition, we would like to mention that the mass ratio of light elements such as hydrogen and helium to metal elements such as tungsten, is in the order of 100. Even in such cases, scale separation may not hold. Here, let us consider a diffusion process. The diffusion of atoms has an aspect of statistical properties that can only be seen when the results of long time motion are observed. Therefore, dealing with hydrogen moving diffusively in a metallic material, we keep the problem as simple as possible and solve it only for hydrogen. Assuming that scale separation holds, we would make the metal atoms a stationary external field and treat the motion of hydrogen in that external field. If the external field is stationary, the motion of hydrogen can be modeled only by the barrier energy due to the external field and can be regarded as a jumping (hopping) motion between local minimum sites. However, such a model does not work well at high temperatures when the thermal vibrations of the metal atoms become non-negligible. This makes the approximation that the external field is stationary a bad one. The actual barrier energy and force acting on the hydrogen atoms fluctuates from moment to moment due to vibrations of the metal atoms. Moreover, it is not a linear or periodic change, but a “fluctuation,” which is difficult to handle. This can be regarded as a scale separation, which consists of a large mass ratio, becoming indivisible at high temperatures.

3.3 Generalized example

From here, we explore the more general issue of what happens when the scale gap is small in modeling while employing an "external field." For this issue, let us consider a particle moving in a potential or external field, as shown in Fig. 4 (a). The potential is assumed to originate from a physical element belonging to another physical process. If the gap between the scale of the particle of interest and the scale of the physical process from which the potential is derived is large, then the potential is time-independent for the particle. In this situation, let $\Delta E (= \Delta E_0)$ be the barrier energy that must be exceeded for the particle to move from local stable point A to local stable point B. If the scale gap is large, the problem can be considered in isolation, and the particle can be moved from point A to point B if it is given energy larger than ΔE.

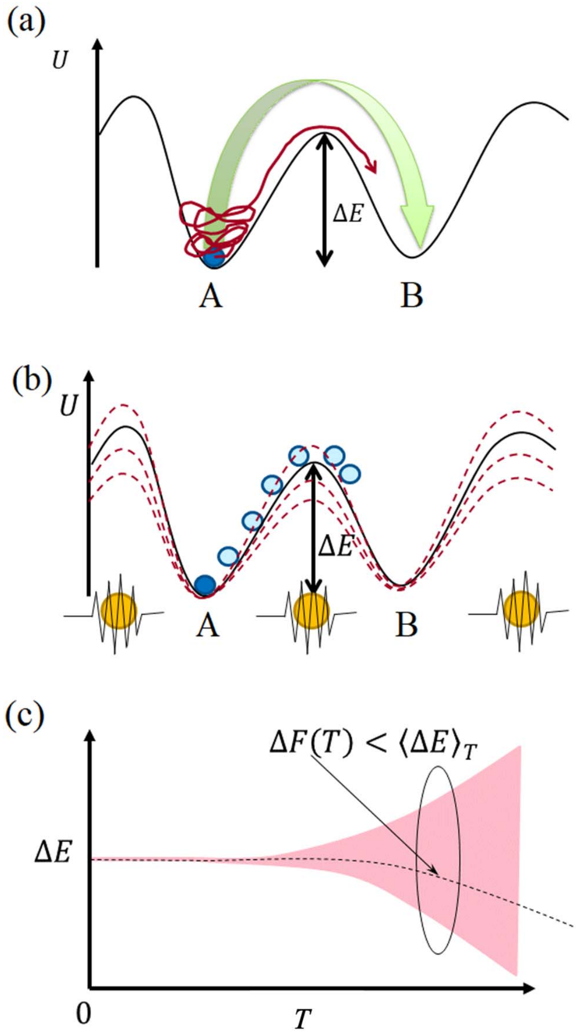

Fig. 4 (a) Motion of a particle (blue sphere) from point A to point B over an external field (solid curve). In contrast to the trajectory following the entire dynamical process of the solid red curve, there can be a Monte Carlo modeling by jump migration using the barrier energy ΔE. (b) The actual external field fluctuates as shown in the red dashed curve due to thermal vibrations of the surrounding matrix atoms (balloons) that create the external field. (c) Both barrier energies are distributed as shown in the pink colored region due to the fluctuations.

On the other hand, if the gap between the scale of the particle and the scale of the external system from which the potential is derived is small, the potential is no longer time-independent. Here, the time-dependence means that the particle will be able to feel how the potential changes from moment to moment in accordance with the motion of the external system. This is the so-called "fluctuation." However, since the scale gap is not zero, the potential will retain its original form to some extent, as shown by the dashed line in Fig. 4 (b). For the particle, the value of the barrier energy ΔE to be exceeded will change depending on the timing of its movement from point A to point B.

An example of this is the fluctuation of the potential for the hydrogen atoms in the diffusion process, due to the thermal vibration of the metal atoms, as mentioned earlier. The degree of fluctuation depends not on the scale but on the temperature parameter T. At low temperatures, there are almost no thermal fluctuations, and the motion of the potential is time-independent. At high temperatures, however, the thermal vibration of the metal atoms increases in amplitude and the fluctuation of the potential becomes larger, as shown in Fig. 4 (c).

3.4 Fluctuation

A consequence of the above discussions is that the influence of the scale gap and the indivisibility of the physical process can depend on parameters other than the scale. If this argument is applied to the case of plasma turbulence, for example, we may regard the distribution of electrons as the potential for ions. The degree of potential fluctuations should appear as the difference between a case where a high wavenumber is ignored and only a low wavenumber region is solved (low-k simulation), and one where a full wavenumber region is solved (full-k simulation). The high wavenumber component is expected to come into play as a fluctuation. The actual simulation results show that when plasma β is small, there is almost no difference between both simulations, and that the difference increases as plasma β increases [11]. Therefore, plasma β, like temperature in a case of diffusion in the material, can be the parameter that controls the influence of the scale gap.

If we assume that the physical process generating the potential is in thermal equilibrium, the potential that the particle perceives on average is often replaced by the Helmholtz free energy surface. When ΔF is the barrier of free energy that a particle perceives executively on the free energy surface, then $\exp[(-\Delta F/k_B T)] = \langle \exp[(-\Delta E/k_B T)] \rangle$. The averaging $\langle \cdots \rangle$ here is an ensemble averaging, but like path integrals, the averaging operation is performed at each time as the particles move. If we accept that the Jarzynski equality [17–19] holds, the averaging operation $\langle \cdots \rangle$ can be performed by a more realistic one, that is, we move from point A to point B by an external operation and measure the work on the system through the inequality at that time. More precisely, define a saddle point C, and let W_i be the work required to move from point A to point C. Here, i indicates the number of trials. By repeating N times trials, we have

$$\exp\left[-\frac{\Delta F}{k_B T}\right] = \left\langle \exp\left[-\frac{W_i}{k_B T}\right]\right\rangle = \lim_{N\to\infty}\frac{1}{N}\sum_i \exp\left[-\frac{W_i}{k_B T}\right]. \quad (1)$$

This can be interpreted as the barrier of free energy corresponding to its mean for the particle, as it travels back and forth between points A and B many times over a very long time period.

The Jarzynski equality can be derived naturally from the "fluctuation theorem." By the way, for the Jarzynski equality to be valid, the system must be in equilibrium and it must be a closed one. However, since most problems in plasma and fusion sciences are in non-equilibrium open systems, the Jarzynski equality cannot be applied as it is. It can also be said that the assertion of the Jarzynski equality refers to the correspondence between long-time and ensemble averaging. Therefore, the question is whether to take ensemble or long-time averaging in the correspondence between microscopic and macroscopic quantities in scale separation.

3.5 Scale and hierarchy structure

The concept of characteristic scales and their gaps for multi-scale / multi-physics is similar to that of *hierarchy* or *hierarchical structure*. However, the characteristic scales of physical processes discussed in previous sections are not necessarily limited to spatial scales. The separation of electron and ion motion in plasmas is an argument focused on the scale gap in wavenumber space, and the separation is caused by this difference between electrons and ions. However, the mass difference is not always synonymous with the scale gap in wavenumber space. In fact, if the effect produced by the mass difference is a quantum one, it can be more appropriate to bring the mass difference directly into the discussion for the gaps. Thus, if the physical process can be separated by the scale of physical variables, it is not necessary to be concerned with the spatial scale. Therefore, it is necessary to have a comprehensive understanding that covers not only space but also the hierarchical structure of physical processes separated by the physical variables.

What we have discussed above is a situation in which physical processes are indivisible due to a small scale gap. If we call one physical process that has a large scale gap and can be sufficiently separated from other processes, it can be separated and understood well enough, and can be described as a closed system of equations. When we focus on a physical process and it can be thus described, a space formed by the trajectories of the solutions will exist clearly, as shown in the left region of Fig. 5. For example, in the classical equations of motion, if we determine the range of

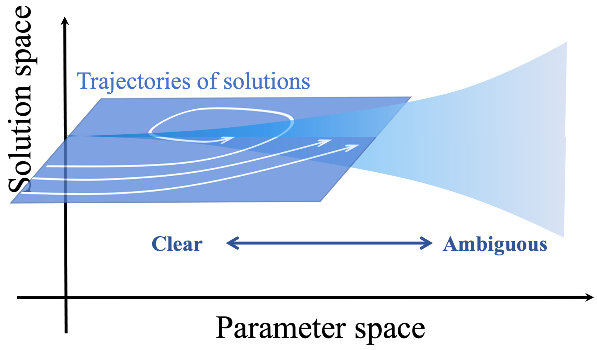

Fig. 5 Solution space against parameter space. When physical process in a system is clearly separable, the corresponding space formed by the trajectories of the solutions is also clearly separable. If the physical process becomes more complex and indivisible according to changes of physical parameters, the structure of the solution space becomes more ambiguous.

initial values, we can determine the trajectory of the solution. Due to conservation laws, the range of trajectories is bounded to a subspace in the space. Here, we can consider that a solution space is associated with a hierarchy.

On the other hand, in a case where the scale gap is small or the physical process is indivisible, due to changes of a certain parameter such as temperature, it should be considered that the solution space becomes ambiguous, as shown in the right region of Fig. 5. In that case, inseparable hierarchies coexist and make sense. If an inseparable, close-scale physical process acts as an external field to the physical process of interest, small but non-negligible mechanisms such as fluctuation will cause the closed system of equations to change from moment to moment. As a result, the trajectory of the solution is not determined only by the initial values, and solutions can reach regions that could not be before. In other words, the solutions that are degenerate in time-independent potential when the scale gap is large, begin to expand when it becomes small. Moreover, it is expected that the trajectory cannot move everywhere, but somewhat close to the original motion on the time-independent potential, and the degree of deviation from it may depend on other variables such as temperature or plasma β.

Thus, the solution space does not spread out completely but perhaps fuzzily, depending on the extent of the scale gap. Systems that form a hierarchical structure of scale-indivisible arrangements may be discussed in connection with such an extended solution space.

4. Problem of Integration

Until now, we have discussed the separability or indivisibility of physical processes at scale in an element-reductive manner, by integrating them for understanding the whole picture. And it has been mentioned that the activity of integration tends to assume that physical processes are separable. Therefore, if they are inseparable, their integration becomes an increasingly difficult task. One of the reasons why it is difficult to integrate physical processes, even if they can be understood by each other, this is that it is difficult to discuss the plausibility of the integration method. It is ironically possible that this method, with good reproducibility, may contain artificial tuning parameters, whereas the one with poor reproducibility may be a model created from a first-principles perspective.

In experimental observations and measurements used for verification, sufficient information cannot be measured due to technical limitations, even though we are well aware that this is a multi-scale, multi-physics process. In electron microscopy, for example, spatial resolution has increased, but information about time is not available. When we compare two micrographs taken before and after an experiment, it is often impossible to determine how much time has passed and what has happened in the experiment, making it difficult to compare the results with theory or simulations. Furthermore, there may be a case where the measurement principle used in the measurement device is actually based on the assumption of the separation of physical processes by scale. The pursuit of measurement principles that can be applied to scale-indivisible situations will be a necessary and interesting issue.

In simulation studies, development of an integration scheme is a specific and practical issue. There is also the issue of numerical algorithms. For example, in a coupled hierarchical simulation, let us consider a case where we want to obtain physical quantities from microscopic calculations for macroscopic ones in order to grasp the whole picture. It is almost impossible to capture long macroscopic behaviors using a scale with respect to the microscopic time scale. Conversely, if the time scale is based on the macroscopic one, the changes are too steep for the microscopic system. The issue of how to make microscopic changes plausible so that the macroscopic scale catches up with the macroscopic one is a problem that is shared not only in plasma, but also in other research fields. Through further cross-disciplinary discussions on the viewpoints of these methods, it is expected that new developments will emerge.

There is another challenge related to the development of computer performance, which has been slowing down in terms of clock and memory access speed for a long time, and it is expected that there will be no drastic improvement in the near future. Of course, the rise of Graphics Processing Units (GPUs), problem-specific advancements such as Field Programmable Gate Arrays (FPGAs), and newer technologies such as machine learning and quantum computing may contribute to different breakthroughs. In terms of mathematical and physical approaches, it will be important in the realization of the research to consider methods that find a balance between conventional methods based on physics and mathematics, and ones that can achieve the best performance in the current era.

Their resolution is a key issue in order to treat the large scale gap. One of the solutions in simulation studies is the Adaptive Mesh Refinement technique [20, 21] which can dynamically and locally change the resolution and can efficiently calculate scale differences of more than three orders of magnitude in massively parallel computers. Furthermore, as the next challenge, we have to consider the situation that a large difference of scale exceeds the scale range where the validity of the governing equations is guaranteed. How to deal with the situation, when the effective range of the governing equations is exceeded or the governing equations need to be switched, is an important issue. For example, in plasmas, as the resolution of MHD simulations is increased, the governing equations for turbulences can change. In the case of plasma-material interaction, a scale of nanometer is needed to understand the solid side, which is smaller than the Debye length of the plasma. Then a novel description of the plasma is required. Also, for these issues, an approach based on the renormalization group [22] is expected to be a useful method.

5. Summary

The hierarchy-based perspective is a powerful way to facilitate understanding of the natural sciences and provide a bird's-eye view of phenomena. However, as the accuracy of measurements and spatio-temporal resolutions in experiments and simulations has been improved, it has become possible to capture quite adjacent regions of the scales of interest and small gaps between phenomena. We have discussed how phenomena between such close proximity of scales necessitate a discussion of the nature of the hierarchy itself, rather than the conventional coupling or connection of separated hierarchies. It will be important to search for the physical quantities that govern the dynamics and to recapture how the hierarchical structure changes with *meta-perspective*. We believe that research on dynamics with meta-perspectives, i.e., *meta-hierarchy dynamics* research will make progress on this issue through various means of experiments, theories, simulations, and their combined studies.

Acknowledgments

The authors are highly grateful to Prof. Y. Kishimoto, Prof. H. Yamada, Prof. K. Ida, Prof. T. Sato, and Prof. Z. Yoshida for useful discussions.

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