Classical Cross-Field Self-Diffusion Due to Finite Larmor Radius

Tomonori TAKIZUKA, Satoshi TOGO[1)], Kenzo IBANO and Yuki HOMMA[2)]

Graduate School of Engineering, Osaka University, Suita 565-0871, Japan
[1)]*Plasma Research Center, University of Tsukuba, Tsukuba 305-8577, Japan*
[2)]*National Institutes for Quantum Science and Technology, Rokkasho 039-3212, Japan*

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It has been considered that the classical diffusion of plasma particles across the magnetic field is determined only by collisions between different species. Taking account of the finite Larmor radius ρ of an ion, the random walk of its guiding center (step size ρ per collision time τ) can result from collisions even with the same species. A resultant "self-diffusion" coefficient is $D_\perp \approx \rho^2/2\tau$. When there exists a radial electric field, the step size becomes asymmetric, and an "electric-field induced collisional displacement" of the guiding center is generated. In an inhomogeneous plasma, the collision time for an ion is varied during a gyration, and a "self-friction force" is induced. We propose these three collisional responses to be included to the ion fluid equations. We discuss that the ion cross-field self-diffusion becomes important in the edge plasma, where electrons are mainly lost along the magnetic field.

Keywords: classical cross-field diffusion, particle self-diffusion, finite Larmor radius, radial electric field, edge plasma

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It has been considered that the classical diffusion of plasma particles across the magnetic field is determined only by collisions between different species [1]. General fluid models so far do not directly include the classical particle diffusion term, while they include classical energy diffusion term [2]. We assume a single-ion-species plasma (charge number $Z = 1$) in a uniform magnetic field $\boldsymbol{B}$ along z direction. Plasma profiles (density n and temperature T) vary along x. Diamagnetic fluxes of ions and electrons along the homogeneous y direction are $nV_y^{\rm i} = (1/eB)\nabla(nT_{\rm i})$ and $nV_y^{\rm e} = -(1/eB)\nabla(nT_{\rm e})$, respectively. The friction force acts between ions and electrons, $F^{\rm i/e} = -F^{\rm e/i} = m_{\rm e}(V_y^{\rm e} - V_y^{\rm i})/\tau^{\rm e/i}$. The radial flux along x is obtained from the force balance relation along y, $0 = -eBV_{\rm x} + F^{\rm i/e}$. One obtains a formula of the collisional flux, $nV_{\rm x} = -(m_{\rm e}/\tau^{\rm e/i}e^2B^2)\nabla(nT_{\rm e} + nT_{\rm i}) \approx -(\langle\rho_{\rm e}\rangle^2/2\tau^{\rm e/i})(1 + T_{\rm i}/T_{\rm e})\nabla n$. This is similar to the diffusive flux with a diffusion coefficient $D_\perp{}^{\rm e} \approx \langle\rho_{\rm e}\rangle^2/2\tau^{\rm e/i}$. Here e is the elementary charge, $m_{\rm e}$ the electron mass, $\tau^{\rm e/i}$ the electron-ion collision time and $\langle\rho_{\rm e}\rangle = (2m_{\rm e}T_{\rm e})^{1/2}/eB$ is the electron Larmor radius.

Now we take account of the finite Larmor radius (FLR) $\rho_{\rm i}$ of an ion particle with charge Ze and mass $m_{\rm i}$. The random walk of its guiding center (GC) can result from the deflection collision even with the same species. The collisionless gyro-motion is given as

$$\boldsymbol{v}_x = \boldsymbol{v}_\perp \sin\varPhi,\ \boldsymbol{v}_y = \boldsymbol{v}_\perp \cos\varPhi \quad \text{and} \quad x = x_{\rm g} - \rho_{\rm i}\cos\varPhi, \tag{1}$$

where $\boldsymbol{v}_\perp$ is the perpendicular speed, $\varPhi = \varOmega t$ is the gyro phase with a cyclotron frequency $\varOmega = ZeB/m_{\rm i}$, and $x_{\rm g}$ is the GC position. Assuming that the above velocity (or gyro phase) is changed by a 90° scattering (deflection) collision,

$$\boldsymbol{v}'_x = \boldsymbol{v}_\perp \sin(\varPhi \pm \pi/2) \quad \text{and} \quad \boldsymbol{v}'_y = \boldsymbol{v}_\perp \cos(\varPhi \pm \pi/2), \tag{2}$$

the GC position along x is moved by this scattering;

$$x_{\rm g}{}' - x_{\rm g} = \rho_{\rm i}(\cos\varPhi' - \cos\varPhi) = 2^{1/2}\rho_{\rm i}\sin\varTheta, \tag{3}$$

where $\varPhi' = \varPhi \pm \pi/2$, and $\varTheta = \varPhi \mp \pi/4$. The mean square of the random walk step size is evaluated just $\rho_{\rm i}{}^2$. The correlation time is the deflection collision time $\tau_{\rm d}{}^{\rm i/i}$ given in [3]. Thus the classical "self-diffusion" across the magnetic field occurs, and a diffusion coefficient is given as

$$D_\perp{}^{\rm i/i} = \rho_{\rm i}{}^2/2\tau_{\rm d}{}^{\rm i/i}. \tag{4}$$

Throughout the present paper, the collision time is considered much longer than the gyration period, $\varOmega\tau \gg 1$.

Figure 1 illustrates gyro-orbits of two ions of the same species. We assume a binary collision of 90° scattering. Although the total momentum and the total energy are fully conserved, their GC positions are scattered.

The self-diffusion of ions is larger than that of electrons, $D_\perp{}^{\rm i/i} \sim Z^3(m_{\rm i}/m_{\rm e})^{1/2}D_\perp{}^{\rm e/e}$ for $T_{\rm i} \sim T_{\rm e}$. A radial electric field E can grow up immediately. This electric field drives the $E\times B$ drift of an ion along y, $\boldsymbol{v}_y = \boldsymbol{v}_\perp \cos\varPhi - E/B$. This time, a 90° scattering collision deflects the velocity as,

$$\boldsymbol{v}'_x = \boldsymbol{v}'_\perp \sin\varPhi' \quad \text{and} \quad \boldsymbol{v}'_y = \boldsymbol{v}'_\perp \cos\varPhi' - E/B. \tag{5}$$

Under an energy-conservation condition, $\boldsymbol{v}_x{}^2 + \boldsymbol{v}_y{}^2 = \boldsymbol{v}'_x{}^2 + \boldsymbol{v}'_y{}^2$, a post-collision perpendicular speed $\boldsymbol{v}'_\perp$ is altered as

$$\boldsymbol{v}'_\perp = \boldsymbol{v}_\perp + (E/B)(\cos\varPhi' - \cos\varPhi), \tag{6}$$

where the $E \times B$ drift speed is assumed much smaller than

author's e-mail: takizuka.tomonori@gmail.com

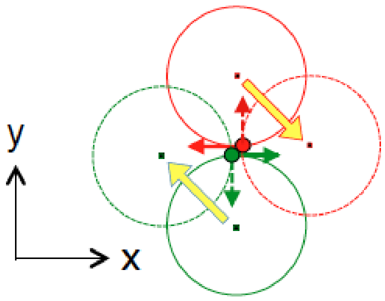

Fig. 1 Gyro-orbits of two ions (red and green particles). Solid and dashed lines denote pre-collision and post-collision orbits, respectively. Velocities are shown by thin arrows. Scatterings of GC positions are shown by block arrows.

$\boldsymbol{v}_\perp$. Since the Larmor radius, $\rho_i{}' = v'_\perp/\Omega$, is simultaneously altered, the GC position along x is moved as

$$x_g{}' - x_g = \rho_i{}' \cos\Phi' - \rho_i \cos\Phi$$
$$= 2^{1/2}\rho_i \sin\Theta + (E/B\Omega)(\cos^2\Phi' - \cos\Phi\cos\Phi'). \quad (7)$$

Gyro-average displacement, which we call the "electric-field induced collisional displacement" (EICD), is then obtained as $\langle x_g{}' - x_g\rangle_\Phi = E/2B\Omega$, and the resultant radial velocity is given by

$$V_{\rm EICD} = \langle x_g{}' - x_g\rangle_\Phi/\tau_d{}^{i/i} = E/2B\Omega\tau_d{}^{i/i}. \quad (8)$$

When ions are diffused outward, $\partial n_i/\partial t = D_\perp{}^{i/i}\nabla^2 n_i < 0$, an inward electric field can grow up, $\varepsilon_0\partial\nabla E/\partial t \approx ZeD_\perp{}^{i/i}\nabla^2 n_i$ (ε_0: permittivity of free space). The EICD flux, $\Gamma_{\rm EICD} = n_i V_{\rm EICD}$, is reverse to the diffusive flux, as expected.

We project the above responses of ion particles on the ion fluid equations. In the Boltzmann equation for an ion velocity distribution function f_i, $Df_i/Dt = C(f_i)$, the self-collision term is approximated as $C^{i/i}(f_i) \approx (1/\tau_d{}^{i/i})\partial^2 f_i/\partial\Phi^2$, for simplicity. Considering a collisionless relation, Eq. (1), this term can be replaced with $(\rho_i{}^2/2\tau_d{}^{i/i})\,\partial^2 f_i/\partial x_g{}^2$. Resultantly the "self-diffusion" term is directly introduced to the equation of continuity. Similar to this derivation, the "EICD" term is also included;

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i V_x) = \frac{\partial}{\partial x}\left(\langle D^i_\perp\rangle\frac{\partial n_i}{\partial x} - \langle V_{\rm EICD}\rangle n_i\right), \quad (9)$$

where $\langle D_\perp{}^i\rangle = \langle\rho_i\rangle^2/2\langle\tau_d{}^{i/i}\rangle$ is a diffusion coefficient, $\langle V_{\rm EICD}\rangle = E/2B\Omega\langle\tau_d{}^{i/i}\rangle$ is an EICD velocity, $\langle\rho_i\rangle = (2m_i T_i)^{1/2}/ZeB$ is the ion Larmor radius, and the collision time is that given in [2]

$$\langle\tau_d{}^{i/i}\rangle = 2.1\times10^{13}A^{1/2}T_{\rm i(eV)}{}^{3/2}/Z^4 n_i\lambda, \quad (10)$$

(A: mass number and λ: Coulomb logarithm). Units are τ in sec, $T_{(eV)}$ in eV, and n in m^{-3}. The "self-diffusion" coefficient $\langle D_\perp{}^i\rangle$ is a half of the thermal diffusivity $\chi_\perp{}^i = \kappa_\perp{}^i/n_i$ given in [2]. Note that the ion total radial flux consists of three parts as $\Gamma_x{}^i = n_i V_x - \langle D_\perp{}^i\rangle\nabla n_i + n_i\langle V_{\rm EICD}\rangle$.

The radial flow velocity V_x of ion fluid along x is almost balanced with a force along y as was described, $0 \approx -ZeBV_x + F_y$. In an inhomogeneous plasma, the collision time for an ion is varied during a gyration, and a "self-friction force" can be induced along the homogeneous y direction. Taking account of FLRs for both a test ion particle and field ion particles, we simply estimate the friction force along y to a test particle (gyro-phase Φ) from field particles (gyro-phase Ψ) as

$$F_{\rm tp} = m_i\{\boldsymbol{v}_\perp(1+\kappa_T\rho_i G_{\Psi-\Phi})\cos\Psi - \boldsymbol{v}_\perp\cos\Phi\}/(\tau_s{}^{i/i}K_{\Psi-\Phi})$$
$$\times(1+\kappa_n\rho_i G_{\Psi-\Phi})(1-3/2\kappa_T\rho_i G_{\Psi-\Phi}), \quad (11)$$

where $\kappa_n = n_i{}^{-1}\nabla n_i$, $\kappa_T = T_i{}^{-1}\nabla T_i$, $G_{\Psi-\Phi} = \cos\Psi - \cos\Phi$, and τ_s is the slowing-down time. A function, $K_{\Psi-\Phi} = \{1 - 2/3\cos(\Psi-\Phi)\}^{3/2}$, comes from the dependence of $\tau \sim |\boldsymbol{u}_r|^3$ ($\boldsymbol{u}_r$: relative velocity), and last terms from $\tau \sim T^{3/2}/n$. The gyro-average friction force is then obtained as

$$\langle F_{\rm tp}\rangle_{\Psi,\Phi} = 0.80 m_i\boldsymbol{v}_\perp(\kappa_n - \kappa_T)\rho_i/\tau_s{}^{i/i}. \quad (12)$$

Without considering the test-particle FLR, the friction force becomes $\langle F^*\rangle_{\Psi,\Phi} = m_i\boldsymbol{v}_\perp\rho_i(1/2\kappa_n - 1/4\kappa_T)/\tau_s{}^{i/i}$. This part of the force is canceled by reactions of field particles of the same species. The rest part, $\langle F^{\rm net}\rangle_{\Psi,\Phi} = m_i\boldsymbol{v}_\perp(0.30\kappa_n - 0.55\kappa_T)\rho_i/\tau_s{}^{i/i}$, can work as a net "self-friction force" on the ion fluid, which balances in a whole plasma system analogously to the diamagnetic flow. A resultant momentum equation of the ion fluid including the "self-friction force" is given by

$$m_i n_i\frac{dV_y}{dt} = -ZeBn_i V_x - n_e F^{e/i}$$
$$+\frac{1}{\Omega\langle\tau_S^{i/i}\rangle}\left(0.60T_i\frac{\partial n_i}{\partial x} - 1.10n_i\frac{\partial T_i}{\partial x}\right). \quad (13)$$

Neglecting a small $F^{e/i}$, a stationary radial flow velocity becomes $V_x = (\langle\rho_i\rangle^2/2\langle\tau_s{}^{i/i}\rangle)(0.6\kappa_n - 1.1\kappa_T)$. Consequently, a null condition for the ion total radial flux, $\Gamma_x{}^i = 0$, in an electric field is obtained from Eq. (9)

$$ZeE = (0.8\kappa_n + 2.2\kappa_T)T_i, \quad (14)$$

where $\langle\tau_s{}^{i/i}\rangle = \langle\tau_d{}^{i/i}\rangle$ is assumed for simplicity. If $\langle\tau_s{}^{i/i}\rangle \approx 1.2\langle\tau_d{}^{i/i}\rangle$ [3], the null-$\Gamma_x{}^i$ condition is $ZeE \approx (\kappa_n + 1.8\kappa_T)T_i$. Accurate numerical factors for collisional terms in the r.h.s. of Eqs. (9) and (13) have to be examined in future.

The ion self-diffusion becomes important in the edge plasma, such as scrape-off layer and divertor region. Electrons are mainly lost in parallel to $\boldsymbol{B}$, while ions can diffuse perpendicularly. The collision time is very small, $\tau^i < 10^{-7}$ s, in the high $n > 10^{20}$/m^3 and low $T < 5$ eV ($\langle\rho_i\rangle \sim 10^{-4}$ m for $B \sim 5$ T), and $\langle D_\perp{}^i\rangle \sim 0.1$ m^2/s. Competition between parallel flow and cross-field self-diffusion affects the plasma profiles in the obliquely-incident $\boldsymbol{B}$ divertor.

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