Viscous-Flux Approximation Modeling in Anisotropic-Ion-Pressure Fluid Scheme

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Anisotropic-Ion-Pressure (AIP) fluid model describes more accurately the characteristics of plasmas in open-magnetic-field systems, e.g., magnetic mirror, scrape-off layer, and divertor. AIP model combined with the virtual divertor model has a merit to be free from the boundary condition of flow speed at the end plate [S. Togo *et al.*, J. Comput. Phys. **310**, 109 (2016); Contrib. Plasma Phys. **58**, 556 (2018)]. In order to compare the AIP modeling with the conventional plasma fluid modeling by using only a single AIP code, we introduce a kind of "viscous-flux approximation (VFA)" into the AIP energy equations, where the anisotropic pressure relaxation term is modified artificially to equalize the pressure anisotropy with the parallel viscous stress. One-dimensional simulations are carried out with the original AIP modeling and with the VFA-AIP modeling, respectively. Expected numerical results are obtained for a simple mirror configuration.

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In future magnetic-confinement fusion reactors, the scrape-off layer (SOL) heat flux will be very huge. It is indispensable to reduce drastically the divertor heat load by the remote radiative cooling and plasma detachment. Numerical simulation studies have widely been carried out to find the effective heat control method. Conventional integrated simulation codes for SOL-divertor plasmas adopt mainly the Braginskii's fluid model [1]. The Anisotropic-Ion-Pressure (AIP) fluid model describes more accurately the characteristics of plasmas in open-magnetic- field systems [2]. Ion pressure components are treated separately as $p_{i\|}$ parallel to the magnetic field $\boldsymbol{B}$ and $p_{i\perp}$ perpendicular to $\boldsymbol{B}$. AIP model combined with the "virtual divertor" model has further a notable merit to be free from the boundary condition (BC) of the flow speed at the end plate [3, 4]. Anisotropic ion heating, such as ICRF heating, in the magnetic mirror can be studied by the AIP simulation [5]. Applications of the AIP modeling to tokamak SOL-divertor simulations have come to be seen recently [6, 7]. Benchmarking of our AIP code [3, 4] with the B2 code [8] was performed on a simple magnetic mirror configuration. Reasonable agreement between two models was obtained in the plasma profiles for a collisional case as expected, but a remarkable discrepancy was found for a rare-collision case [9].

At first, we shortly review the "viscous-flux approximation (VFA)" in the conventional modeling. The ion pressure force parallel to $\boldsymbol{B}$ is expressed by the parallel gradient of the parallel ion pressure, $-\nabla_\| p_{i\|}$, in AIP model, while in the conventional fluid model, it is given by the combination of the total pressure force and the viscous force, $-\nabla_\| p_i - \nabla_\| \pi_i$. The total ion pressure is defined as $p_i = (p_{i\|} + 2p_{i\perp})/3$, and the pressure anisotropy $\delta p_i = 2(p_{i\|} - p_{i\perp})/3$ is replaced with the parallel viscous stress π_i under the VFA. Considering the balance between collisional relaxation of pressure anisotropy and convective energy transport with the parallel flow velocity V, the parallel viscous stress is approximately given by

$$\pi_i = -\eta_0 B^{-1/2} \nabla_\| (B^{1/2} V), \tag{1}$$

where the classical viscosity is $\eta_0 = 0.96 p_i \tau_i$ (τ_i: ion collision time) [1]. Taking account of the restriction, $|\pi_i| \approx |\delta p_i| < p_i$, the viscosity η_0 is replaced with the following viscous-flux limited one,

$$\eta = \eta_0 / [1 + \beta^{-1} \tau_i |B^{-1/2} \nabla_\| (B^{1/2} V)|], \tag{2}$$

where the viscous-flux limiting factor β is set less than unity to maintain $|\pi_i|/p_i < 1$. The form, $B^{-1/2}\nabla_\|(B^{1/2}V)$, in Eq. (2) is essential, but a simpler form, $\nabla_\| V$, used in Ref. [9] was incomplete.

Because the viscous force is expressed by the second-order space derivative of V, the momentum equation requires BCs at the inlet and the outlet, respectively. Usual SOL-divertor codes apply a Bohm condition, $V \geq C_s$ (C_s: sound speed), at the outlet divertor plate. On the contrary,

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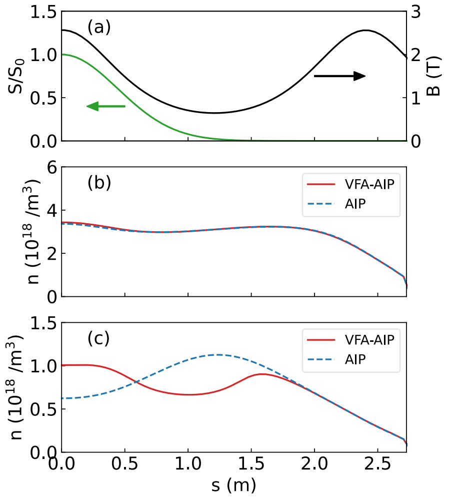

Fig. 1 (a) Profiles of the magnetic field strength (black) and the particle source (green). Density profiles along $\boldsymbol{B}$ are shown for (b) collisional and (c) rare-collision cases. Red solid lines are obtained by VFA-AIP model ($\alpha_{\mathrm{VFA}} = 10^5$ and $\beta = 0.5$) and blue dashed lines by AIP model ($\alpha_{\mathrm{VFA}} = 0$).

the AIP code does not require any BC at the outlet as noted earlier.

Comparison of the various physics models using multiple simulation codes is rather a troublesome work. On the other hand, it is practical to carry out model comparison by using a single code with multiple physics models. Now, we propose a new model of a kind of "VFA" applicable to the AIP scheme. We focus on the relaxation term of pressure anisotropy in the AIP energy equations,

$$Q_{\mathrm{rel}\parallel} = -(p_{\mathrm{i}\parallel} - p_{\mathrm{i}\perp})/\tau_{\mathrm{rel}} = -Q_{\mathrm{rel}\perp}, \tag{3}$$

where the relaxation time is $\tau_{\mathrm{rel}} = 2.5\tau_{\mathrm{i}}$. This term is modified so that the VFA condition, $\delta p_{\mathrm{i}} = \pi_{\mathrm{i}}$, is realized in the system with a characteristic length L,

$$Q_{\mathrm{rel}\parallel^*} = Q_{\mathrm{rel}\parallel} - (\alpha_{\mathrm{VFA}} C_{\mathrm{s}}/L)(\delta p_{\mathrm{i}} - \pi_{\mathrm{i}}). \tag{4}$$

The artificial enhancement factor for the VFA is set much larger than unity, $\alpha_{\mathrm{VFA}} \gg 1$. Since $Q_{\mathrm{rel}\perp *} = -Q_{\mathrm{rel}\parallel *}$, the invisible equation of total energy transport, i.e., summation of parallel and perpendicular energy transport equations, is not suffered by this modification. Nonlinear calculations of the AIP fluid equations including the above VFA modification can be advanced with reliable iterations during a time step.

One-dimensional simulations are carried out with the original AIP modeling ($\alpha_{\mathrm{VFA}} = 0$) and with the VFA-AIP modeling ($\alpha_{\mathrm{VFA}} = 10^5$ and $\beta = 0.5$), respectively. Target plasmas in a simple mirror configuration are the same as those of the previous benchmark [9]. Figure 1 shows the

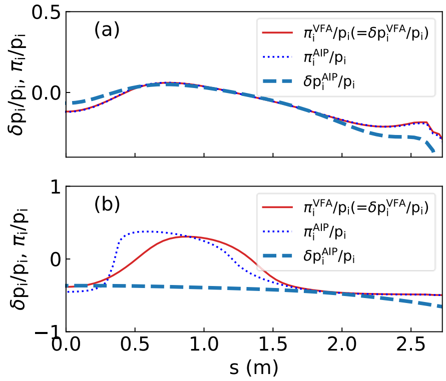

Fig. 2 Profiles of pressure anisotropy $\delta p_{\mathrm{i}}/p_{\mathrm{i}}$ and viscous stress $\pi_{\mathrm{i}}/p_{\mathrm{i}}$ for (a) the collisional and (b) rare-collision cases. Red solid line is $\pi_{\mathrm{i}}/p_{\mathrm{i}}$ ($= \delta p_{\mathrm{i}}/p_{\mathrm{i}}$) obtained by VFA-AIP model. Blue dotted line and blue bold broken line are $\pi_{\mathrm{i}}/p_{\mathrm{i}}$ and $\delta p_{\mathrm{i}}/p_{\mathrm{i}}$, respectively, obtained by AIP model.

density profiles along $\boldsymbol{B}$ for the collisional ($\tau_{\mathrm{i}} C_{\mathrm{s}}/L \sim 0.1$) and rare-collision ($\tau_{\mathrm{i}} C_{\mathrm{s}}/L \sim 3$) cases. It is found that the VFA-AIP modeling well reproduces the B2 results in Ref. [9], although a slight discrepancy is seen originating from the difference in the VF-limit model based on Eq. (2).

Figure 2 shows the profiles of δp_{i} and π_{i} normalized by p_{i}. Note that, even in the VFA-AIP calculation, both δp_{i} and π_{i} are individually evaluated and the VFA state ($\delta p_{\mathrm{i}} = \pi_{\mathrm{i}}$) is well satisfied, whose relative error (i.e. $|\delta p_{\mathrm{i}} - \pi_{\mathrm{i}}|/p_{\mathrm{i}}$) is less than 1% regardless of the collisionality. It is shown for the rare-collision case that the sign of $\pi_{\mathrm{i}}/p_{\mathrm{i}}$ ($= \delta p_{\mathrm{i}}/p_{\mathrm{i}}$) in VFA-AIP calculation in the upstream diverging-$\boldsymbol{B}$ region, except for $s < 0.5$ m with strong source, is opposite to the sign of $\delta p_{\mathrm{i}}/p_{\mathrm{i}}$ in AIP calculation (Fig. 2 (b)), which causes the drastic difference in the density profiles (Fig. 1 (c)).

In order to compare the AIP modeling with the conventional plasma fluid modeling by using only a single AIP code, we artificially modify the anisotropic pressure relaxation term in the AIP energy equations so that the pressure anisotropy is equalized with the parallel viscous stress. It is confirmed by test simulations that the present modeling realizes well the viscous-flux approximation.

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