Analysis of Turbulence Driven Particle Transport in PANTA by Using Multi-Field Singular Value Decomposition

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Multi-field singular value decompositions (SVDs) is applied to turbulence obtained in a cylindrical magnetized plasma, PANTA. This method enables us to obtain the spatial mode structures with common temporal evolution of different physical quantities, such as the fluctuations of density and flows. Turbulence driven particle transport is evaluated by the method. It is shown that only the coupling of the same mode drives the transport, which stems from the orthogonality of the SVD. Thanks to this characteristics, the number of degrees of freedom which plays roles for the transport dynamics could be significantly reduced.

Keywords: magnetized plasma, data-driven method, singular value decomposition, drift wave, particle transport

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Performance of magnetized plasmas is dominated by turbulence driven transport. Many degrees of freedom often appear in transport analysis using Fourier mode decomposition, because turbulence has generally broad spectrum [1]. Modal decomposition into a few degrees of freedom on real space is easier to understand detailed spatio-temporal dynamics. Analyses based on data-driven methods, such as dynamic mode decompositions (DMDs) [2, 3] and singular value decompositions (SVDs), have been applied to turbulence [4]. In particular, the SVD is possible to analyze interactions between modes, because it is based on orthogonal bases so that the energy of the mode can be defined. The SVD has been applied to a single physical quantity, where the turbulence driven Reynolds force has been evaluated from the decomposition of electrostatic potential [4,5]. In order to study the particle or heat transport, it is necessary to analyze the correlation among multiple physical quantities. Therefore, the extension of the SVD method is necessary.

In this study, we apply a newly proposed method to analyze the correlation among multiple physical quantities based on the SVD. A common temporal evolution with spatial mode structures of multiple physical variables can be derived by the proposed method. The method is applied to a set of experimental data obtained in a linear magnetized plasma device, PANTA [6, 7]. The analysis of turbulence driven particle transport driven is demonstrated.

Here, we describe the method to extract the common temporal evolution of different physical quantities. The spatio-temporal data, $\boldsymbol{X}(\theta, t)$, $\boldsymbol{Y}(\theta, t)$, is expressed as

$$\Phi = \begin{pmatrix} X(\theta_1, t_1) & X(\theta_1, t_2) & \cdots & X(\theta_1, t_n) \\ X(\theta_2, t_1) & X(\theta_2, t_2) & \cdots & X(\theta_2, t_n) \\ \vdots & \vdots & \ddots & \vdots \\ X(\theta_m, t_1) & X(\theta_m, t_2) & \cdots & X(\theta_m, t_n) \\ Y(\theta_1, t_1) & Y(\theta_1, t_2) & \cdots & Y(\theta_1, t_n) \\ Y(\theta_2, t_1) & Y(\theta_2, t_2) & \cdots & Y(\theta_2, t_n) \\ \vdots & \vdots & \ddots & \vdots \\ Y(\theta_m, t_1) & Y(\theta_m, t_2) & \cdots & Y(\theta_m, t_n) \end{pmatrix}, \qquad (1)$$

where θ is the spatial position and t_j stand for the measurement time. The SVD is applied to the matrix Eq. (1) to extract the common characteristics [8]. In this study, the ion saturation current N, and the radial flow V_r, are chosen, where V_r is estimated from the floating potential with the assumption of the E × B flow. The 64 channel azimuthal probe array placed at the fixed radial position $r = 4$ cm measures the fluctuations of the ion saturation current and the floating potential alternately [1]. The azimuthal position of the observation is denoted by θ_i ($i = 1{\sim}32$). The matrix is decomposed by the SVD as

$$\Phi_{ij} = \textstyle\sum_k U_{ik} \Sigma_{kk} V^T_{kj}. \qquad (2)$$

Here, Σ is the singular value matrix which is the diago-

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nal matrix, and the diagonal component σ_k is sorted in descending order ($\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_k \geq 0$). The matrix U corresponds to the spatial structure [4], and the matrix ΣV^T has the temporal evolution of turbulence common to the ion saturation current and the radial flow. The SVD modes N_α and $V_{r\beta}$ are defined as follows

$$N_\alpha(\theta_i, t_j) = U_{i\alpha}\sigma_\alpha V_{\alpha j}, \tag{3}$$

$$V_{r\beta}(\theta_i, t_j) = U_{i+32\,\alpha}\sigma_\alpha V_{\alpha j}. \tag{4}$$

This is the Multi-field SVD, where the common temporal characteristics $V_{\alpha j}$ can be deduced between the different physical quantities. It is noted that this method can be applied only for the physical quantities observed simultaneously.

Spatial structure U_{ik} and common temporal evolution ΣV^T_{kj} of the density and the radial flow fluctuations are evaluated. For the dominant mode $\alpha = 1$, $U_{\alpha=1}$ and $\Sigma V^T_{\alpha=1}$ are shown in Fig. 1. The density and the radial flow have a similar mode structure with clear spatial phase relations: the density fluctuation proceeds that of the radial flow, which is a typical characteristic of the resistive drift wave [9]. In this way, the systematic derivation of spatial phase relations is passible.

Then, the particle transport driven by the SVD mode is evaluated by

$$< \overline{\Gamma_r} > = \textstyle\sum_{\alpha,\beta} < \overline{N_\alpha V_{r\beta}} >, \tag{5}$$

where the subscripts α and β represent the mode number of the SVD mode. Here, $<\cdots>$ is the spatial average in the azimuthal direction, and $\overline{\cdots}$ denotes the time average. The transport driven by the coupling of each mode is shown in Fig. 2 (a). It can be seen that only the combination of modes with $\alpha = \beta$ can cause transport. This is due to the mode orthogonality of the SVD. Here, the positive/negative value corresponds to outward/inward flux, respectively. The dominant mode contributes to the outward flux. As a comparison, the transport matrix is evaluated by the conventional SVD, where each physical quantity is decomposed separately. By the conventional approach, all the mode coupling contributes to the transport (which stems from the non-orthogonality for each physical variable), as shown in Fig. 2 (b). In this way, the significant reduction of number of degree of freedom is possible by the Multi-field SVD.

In summary, we applied the Multi-field SVD to the fluctuation data obtained in PANTA. The spatial phase relation between density and radial E×B flow systematically deduced with common temporal evolution. Then, the turbulence driven particle flux is evaluated by using the decomposed data. The comparison of the Multi-field SVD and the conventional approach clearly demonstrates that the fluctuation driven flux can be analyzed with much less degrees of freedom by using the Multi-field SVD.

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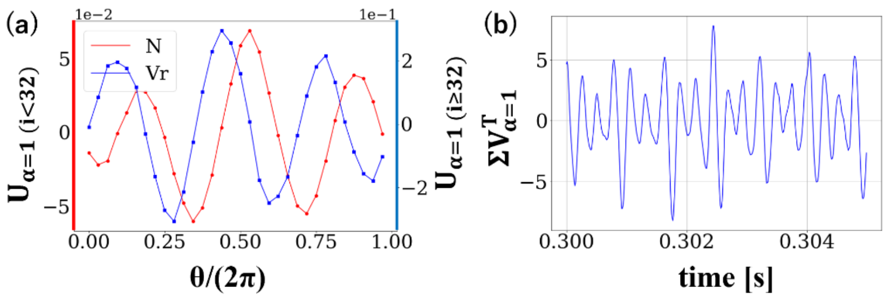

Fig. 1 (a) Spatial structure of the dominant mode $U_{\alpha=1}$. (b) Common temporal evolution of the corresponding mode $\Sigma V^T_{\alpha=1}$.

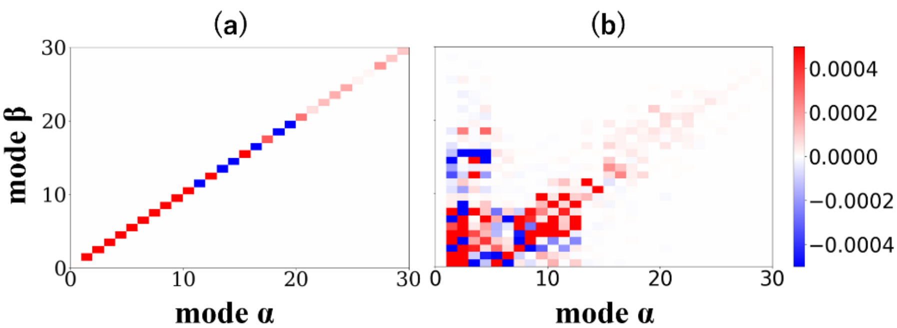

Fig. 2 Contribution of SVD mode on transport. (a) Multi-field SVD. (b) Conventional approach.

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