Wave Number Dependence on Ion Mass Number of Resistive Drift Wave Instabilities

Naohiro KASUYA and Makoto SASAKI[1)]

Research Institute for Applied Mechanics, Kyushu University, Kasuga, Fukuoka 816-8580, Japan
[1)]*College of Industrial Technology, Nihon University, Narashino, Chiba 275-8575, Japan*

(Received 14 February 2022 / Accepted 24 March 2022)

Ion mass dependence of the resistive drift wave instability is investigated to understand wave number spectra in magnetized cylindrical plasmas. Modes with larger axial mode numbers are linearly unstable in the case of smaller mass ions as helium. Analytical expression is obtained, which shows that not only the azimuthal mode number but also the axial mode number has the preferable one depending on the mass number. Therefore, 3-D observation of the spatial structure is important to capture the variation including this parameter.

Keywords: resistive drift wave instability, ion mass dependence, wave number spectrum, linear device

DOI: 10.1585/pfr.17.1201053

Variation of ion species in discharges affects performance of plasma confinement. Isotope effect is still the unresolved mystery in magnetic confinement fusion plasmas [1], and several kinds of impurities with larger mass numbers come into core plasmas [2]. Basic experimental devices with cylindrical configuration are used to investigate fundamental mechanism of plasma turbulence, such as formation mechanism of turbulence structures [3] and nonlinear interaction between plasma instabilities [4]. Effects of ion species have been also studied in the basic plasmas [5]. In simulations, variations of characteristic mode numbers and formed structures in plasma turbulence are predicted between different discharge gases [6]. This article represents the analytical expression of the resistive drift wave instabilities for the linear devices. There are preferable azimuthal and axial mode numbers corresponding to the ion mass number for this instability induced by density gradient.

Nonlinear simulations for PANTA linear device [7] using turbulence code NLD [8] have shown that a large number of modes with larger axial mode numbers become unstable in the case of smaller mass ions, and their nonlinear couplings gives difference in formed turbulent structures [6]. Figure 1 shows comparison of turbulent states in argon and helium plasmas. For the calculation of resistive drift wave instability in cylindrical plasmas with homogeneous magnetic fields in the axial direction, the following set of three-field reduced fluid equations is used [8];

$$\frac{dN}{dt} = -\nabla_{//}V - V\nabla_{//}N + \mu_{\mathrm{N}}\nabla_{\perp}^2 N + S, \tag{1}$$

$$\frac{d\nabla_{\perp}^2\phi}{dt} = \nabla N \cdot \left(-\nu_{\mathrm{in}}\nabla_{\perp}\phi - \frac{d\nabla_{\perp}\phi}{dt}\right) - \nu_{\mathrm{in}}\nabla_{\perp}^2\phi, \quad -\nabla_{//}V - V\nabla_{//}N + \mu_{\mathrm{W}}\nabla_{\perp}^4\phi \tag{2}$$

author's e-mail: kasuya@riam.kyushu-u.ac.jp

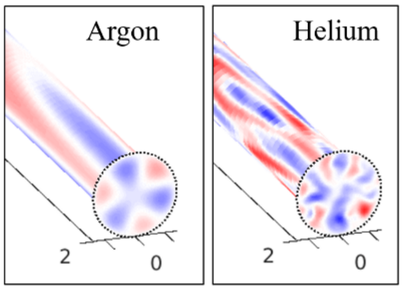

Fig. 1 Numerical simulations of potential perturbations in the saturated states of plasma turbulence in cylindrical plasmas. Comparison between argon and helium discharges are shown.

$$\frac{dV}{dt} = \frac{M}{m_{\mathrm{e}}}(\nabla_{//}\phi - \nabla_{//}N) - (\nu_{\mathrm{ei}} + \nu_{\mathrm{en}})V + \mu_{\mathrm{V}}\nabla_{\perp}^2 V, \tag{3}$$

where N is the density, ϕ is the electrostatic potential, V is the electron velocity in the magnetic field direction, S is the particle source, ν is the collision frequency, and μ is the viscosity. The time and distance are normalized by the ion cyclotron frequency Ω_{ci} and the effective Larmor radius $\rho_s = \sqrt{MT_e/eB}$ evaluated by using the electron temperature T_{e}, respectively. Neutral particles exist even in the center of this rather low temperature plasma, so the effect of neutrals is included by the ν_{in} terms. In addition, only charge number Z = +1 ions are considered with this range of the temperature.

Linearization with

$$\partial/\partial t \to -i\omega t, \partial/\partial r \to ik_r,$$
$$(1/r)\partial/\partial\theta \to ik_\theta, \nabla_\perp \to ik_\perp, \nabla_{//} \to ik_z$$

and simplification with $\phi_0 = \partial\phi_0/\partial r = V_0 = \partial V_0/\partial r = 0$ give the following set of equations;

$$\begin{bmatrix} -\omega - i\mu_N k_\perp^2 & \omega_* & k_z \\ 0 & T\left(\omega + i\nu_{in}\right) + i\mu_W k_\perp^4 & k_z \\ M_i k_z & -M_i k_z & -\omega - i\nu_e - i\mu_V k_\perp^2 \end{bmatrix} \begin{bmatrix} N \\ \phi \\ V \end{bmatrix} = 0, \tag{4}$$

where $\omega_* = k_\theta L_n^{-1}$ is the diamagnetic frequency, $M_i = Am_u/m_e$ is the ion mass ratio, A is the ion mass number, m_u is the atomic mass constant, $L_n^{-1} = -dN_0/dr$ is inverse of the density gradient length and $T = k_\perp^2 + ik_r L_n^{-1}$. Dependence of the linear growthrate is obtained as in Fig. 2 by solving Eqs. (4) numerically. The simulation parameters are the followings; magnetic field $B = 0.1\,\mathrm{T}$, electron temperature $T_e = 3\,\mathrm{eV}$, plasma radius $a = 7\,\mathrm{cm}$, device axial length $\lambda = 4\,\mathrm{m}$, electron-ion collision frequency $\nu_\mathrm{ei}/\Omega_\mathrm{ci} = 300$, electron-neutral collision frequency $\nu_\mathrm{en}/\Omega_\mathrm{ci} = 10$, viscosities $\mu_\mathrm{W} = \mu_\mathrm{V} = 10^{-4}$, $\mu_\mathrm{N} = 10^{-2}$ and $L_\mathrm{n} = a/5$. There are several (m, n) modes, which have positive growthrates (unstable modes) with a certain ν_in and A, and among them the mode with a maximum growthrate is picked up, which has mode number $(m_\mathrm{p}, n_\mathrm{p})$. Here, m and n are azimuthal and axial mode numbers, respectively. Larger ν_in makes modes stable, and heavier ion makes m_p and n_p smaller. $(m_\mathrm{p}, n_\mathrm{p}) = (15, 11)$ mode is the most unstable with helium plasma, though $(3, 1)$ mode is with argon plasma, when $\nu_\mathrm{in} = 0.04$.

An analytical expression for the linear growthrate is presented here to explain the mode number dependence with different ion species. In the drift wave range with $\omega \ll \nu_\mathrm{e}$, Eqs. (4) gives an eigen-equation

$$\omega^2 + \left[i\nu_{in} + iP\frac{T+1}{T}\right]\omega - P\left(\nu_{in} + \frac{i\omega_*}{T}\right) = 0, \tag{5}$$

where $P = M_i k_z^2/\nu_e$ is related to axial wave number k_z. Here viscosities are put to be zero $\mu_\mathrm{W} = \mu_\mathrm{V} = \mu_\mathrm{N} = 0$ for simplicity. The solution of Eq. (5) is

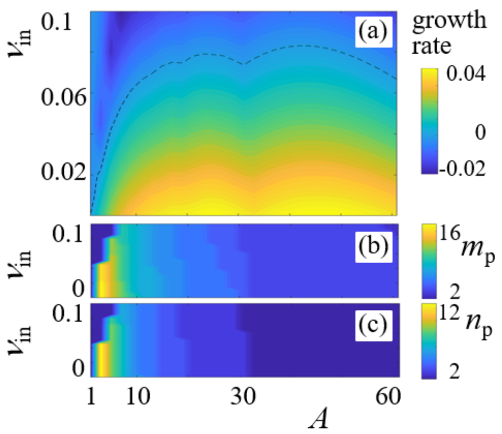

Fig. 2 (a) Dependence of the maximum linear growthrate on ion mass number A and collision frequency ν_in. The dashed line shows the unstable boundary where the growthrate is zero. The maxima are given with different (b) azimuthal m_p and (c) axial n_p, depending on the parameters.

$$\omega = \frac{-i\nu_{in} - iP\dfrac{T+1}{T} \pm \sqrt{-\nu_{in}^2 + 2P\left(\dfrac{T-1}{T}\nu_{in} + 2i\dfrac{\omega_*}{T}\right) - P^2\left(\dfrac{T+1}{T}\right)^2}}{2}. \tag{6}$$

In these parameters, ν_in and P have the same order of magnitude, so the expression is little bit complex. Figure 3 shows dependence of the imaginary part $\mathrm{Im}\,\omega$ on k_z. The curves have the maxima at $k_\mathrm{z} \sim 0.02$ with any ν_in smaller than 0.05. To evaluate the most unstable k_z, an analytical expression with $\nu_\mathrm{in} \ll P \ll 1$ is obtained as

$$\omega \sim -iP\frac{T+1}{2T} + \sqrt{iP\frac{\omega_*}{T}}. \tag{7}$$

Solution (7) has the maximum growthrate

$$\mathrm{Im}\,\omega = \frac{1}{4}\frac{k_\theta L_n^{-1}}{1+k_\perp^2}, \text{ when } P = \frac{1}{2}\frac{k_\perp^2}{(1+k_\perp^2)^2}k_\theta L_n^{-1}. \tag{8}$$

Then, $k_\theta^2 = 1 + k_\mathrm{r}^2$ with fixed k_r gives the maximum of Eq. (8) to be

$$\mathrm{Im}\,\omega = \frac{1}{8}\frac{L_n^{-1}}{(1+k_r^2)^{3/2}}, \text{ when } P = \frac{1}{8}\frac{1+2k_r^2}{(1+k_r^2)^{3/2}}L_n^{-1}. \tag{9}$$

For the radial direction, smaller k_r makes the mode unstable ($k_\mathrm{r} \to 0$), and the value is determined by the boundary condition, when the global mode structure is considered. From these expression, dependence of a spectrum peak on the ion mass number A is given to be

$$m_p \propto A^{-1/2} \text{ from } k_\theta \sim 1 \tag{10}$$

and

$$n_p \propto A^{-3/4} \text{ from } \frac{M_i k_z^2}{\nu_e} \sim \frac{1}{8}L_n^{-1}, \tag{11}$$

considering the normalization with ρ_s. In this way, preferable k_θ and k_z are obtained, which makes the plasma most

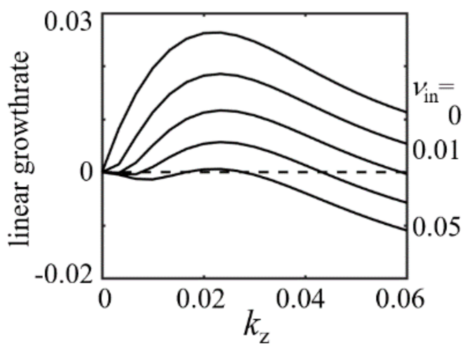

Fig. 3 Dependence of the linear growthrate on k_z with $\nu_\mathrm{in} = 0$ – 0.05. This is the case with $A = 4$, $k_r = \pi/(a/\rho_s)$, $k_\theta = 12/(a/\rho_s)$.

unstable. Perpendicular and parallel flow balance and delay of the response by collision is the cause of the resistive drift wave instability. In the balance, competition of the convective derivative term and the perpendicular drift (especially polarization drift) term in Eq. (2) is the key to determine the wave number spectrum.

In this research, the ion mass dependence of the resistive drift wave instability was investigated. The variation is related to change of Larmor radius, which gives the typical special length here. Not only k_θ but also k_z has the most unstable one depending on the mass number, which indicates 3-D features are important for understanding plasma turbulence. This is the result from the local linear analysis, but actual turbulent states must be determined with nonlinear processes. Global calculations including nonlinear relaxation of the background profile will be compared with experiments.

Authors acknowledge discussion with Dr. M. Yagi, Mr. Ishida and Mr. Todoroki. This study is partially supported by the Grant-in-Aid for Scientific Research (JP17H06089, JP20K03905) of JSPS, the collaboration program of NIFS (NIFS21KNST183) and of RIAM of Kyushu University.

[1] R.J. Hawryluk, Rev. Mod. Phys. **70**, 537 (1998).
[2] C. Angioni, Plasma Phys. Control. Fusion **63**, 073001 (2021).
[3] T. Yamada *et al.*, Nature Phys. **4**, 721 (2008).
[4] H. Arakawa *et al.*, Plasma Phys. Control. Fusion **53**, 115009 (2011).
[5] T. Kobayashi *et al.*, Phys. Plasmas **27**, 062309 (2020).
[6] N. Kasuya *et al.,* Plasma Fusion Res. **16**, 1201083 (2021).
[7] S. Inagaki *et al.*, Sci. Reps. **6**, 22189 (2016).
[8] N. Kasuya *et al.*, J. Phys. Soc. Jpn. **76**, 044501 (2007).