Electric Potential in Volume Produced Negative Ion Sources with Magnetic Field Increasing toward a Wall*)

Azusa FUKANO and Akiyoshi HATAYAMA[1)]

Monozukuri Department, Tokyo Metropolitan College of Industrial Technology, Higashi-Oi, Shinagawa, Tokyo 140-0011, Japan

[1)]*Faculty of Science and Technology, Keio University, Hiyoshi, Kohoku-ku, Yokohama, Kanagawa 223-8522, Japan*

(Received 9 November 2020 / Accepted 30 March 2021)

Distribution of electric potential near the wall in the volume produced negative ion source with the magnetic field increasing toward the wall such as the cusp magnetic field is investigated analytically. The plasma-sheath equation that gives the electric potential in the plasma region and the sheath region near the wall is derived analytically and the potential distribution near the wall is obtained by solving the plasma-sheath equation. Effects of the degree of increase of the magnetic field, the production amount of volume produced negative ion, and the ion temperature on the distributions of electric potential are shown.

Keywords: volume produced negative hydrogen ion, electric potential, magnetic field, plasma-sheath equation

DOI: 10.1585/pfr.16.2403067

1. Introduction

A neutral beam injection (NBI) is one of the most promising methods of heating plasma confined magnetically in Tokamak. In a hydrogen negative ion source for the NBI, plasma is confined by cusp magnetic field in order to suppress plasma loss on a wall. However, some plasma particles move along the magnetic field and are lost on the wall. The amount of negative ions produced in the ion source depends on the plasma density related to the particle loss on the wall and the plasma temperature related to the energy loss on the wall. It has been shown that a width of plasma loss region of the electron energy depends on a heat transmission coefficient that is the ratio of the heat flux to the particle flux multiplied by the electron temperature along the magnetic field [1]. Since the heat transmission coefficient is related to a sheath potential formed near the wall in the plasma [2], investigating the electric potential near the wall is important in the production of negative ions in the ion source. Emmert *et al.* have investigated formation of the electric potential considering both the plasma and the sheath regions by using a plasma-sheath equation [3]. Sato *et al.* have extended the method of Emmert *et al.* to a case of magnetized plasma with the magnetic field decreasing toward the wall such as the divertor plasma [4]. However negative hydrogen ion (H^-) has not been considered in both studies.

In this paper, we will investigate the distributions of the electric potential near the wall with the magnetic field increasing toward the wall such as the cusp magnetic field, where volume produced H^- ion is considered in addition to electron and positive hydrogen ion (H^+). The plasma-sheath equation is derived analytically and the distribution of the electric potential is obtained. The effects of the degree of increase of the magnetic field, the production amount of volume produced H^- ion, and the temperature of H^+ ion and volume produced H^- ion on the distribution of electric potential are shown.

author's e-mail: fukano@metro-cit.ac.jp

*) This article is based on the presentation at the 29th International Toki Conference on Plasma and Fusion Research (ITC29).

2. Plasma-Sheath Equation

2.1 Model and basic equations

The analysis model is shown in Fig. 1. In the analysis, walls on both sides are considered in order to maintain a conservation of particles. The problem is treated as one-dimensional model in z-direction. The electric potential $\phi(z)$ and the magnetic field $B(z)$ are assumed to be symmetric about $z = 0$ and $B(z)$ is B_0 at $z = 0$. Plasma is assumed to consist of H^+ ions, volume produced H^- ions, and electrons. It is also assumed that the magnetic field is perpendicular to the wall near the wall and an effect of the magnetic presheath is ignored. Total energies E of the H^+ ion and E_v of the volume produced H^- ion in the z-direction are

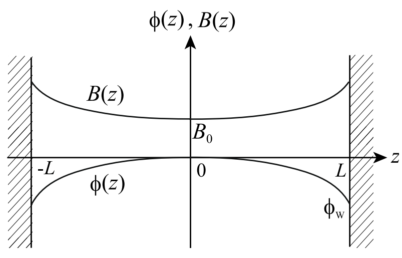


Fig. 1 Analysis model.

$$E = \frac{1}{2}M(v_\perp^2 + v_{//}^2) + q\phi(z), \tag{1}$$

$$E_\mathrm{v} = \frac{1}{2}M_\mathrm{v}(v_{\mathrm{v}\perp}^2 + v_{\mathrm{v}//}^2) - q\phi(z), \tag{2}$$

where M and M_v are the ion masses, $v_\perp$, $v_{\mathrm{v}\perp}$ and $v_{//}$, $v_{\mathrm{v}//}$ are the velocities perpendicular and parallel to the magnetic field, q and $-q$ are the charges of the H^+ ion and the H^- ion, respectively. The subscript "$_\mathrm{v}$" denotes value belonging to the volume produced H^- ion throughout this paper. The magnetic moments are given by

$$\mu = (1/2)Mv_\perp^2/B(z), \tag{3}$$

$$\mu_\mathrm{v} = (1/2)M_\mathrm{v}v_{\mathrm{v}\perp}^2/B(z). \tag{4}$$

The kinetic equations for the H^+ ion and the H^- ion in the phase space (z, E, μ) and $(z, E_\mathrm{v}, \mu_\mathrm{v})$ are described by

$$\sigma v_{//}(z,E,\mu)\frac{\partial f(z,E,\mu,\sigma)}{\partial z} = S(z,E,\mu), \tag{5}$$

$$\sigma v_{\mathrm{v}//}(z,E_\mathrm{v},\mu_\mathrm{v})\frac{\partial f_\mathrm{v}(z,E_\mathrm{v},\mu_\mathrm{v},\sigma)}{\partial z} = S_\mathrm{v}(z,E_\mathrm{v},\mu_\mathrm{v}), \tag{6}$$

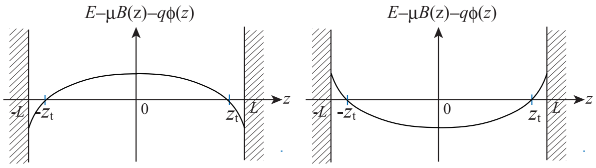
where $\sigma = \pm 1$ is the direction of the particle motion, $f(z, E, \mu, \sigma)$ and $f_\mathrm{v}(z, E_\mathrm{v}, \mu_\mathrm{v}, \sigma)$ are the distribution functions, and $S(z, E, \mu)$ and $S_\mathrm{v}(z, E_\mathrm{v}, \mu_\mathrm{v})$ are the source functions. We assume a symmetry about $z = 0$ for the distribution functions and the source functions. We also assume that particles are not reflected at the wall, then the boundary conditions of the distribution functions are $f(-L, E, \mu, +1) = f(L, E, \mu, -1) = f_\mathrm{v}(-L, E_\mathrm{v}, \mu_\mathrm{v}, +1) = f_\mathrm{v}(L, E_\mathrm{v}, \mu_\mathrm{v}, -1) = 0$.

2.2 Plasma-sheath equation

From Eqs. (1) - (4), the parallel velocities of the H^+ ion and the H^- ion are given by $v_{//} = [(2/M)\{E - \mu B(z) - q\phi(z)\}]^{1/2}$ and $v_{\mathrm{v}//} = [(2/M_\mathrm{v})\{E_\mathrm{v} - \mu_\mathrm{v}B(z) + q\phi(z)\}]^{1/2}$. The energy space of the particle is divided to some regions, which is based on the condition that $v_{//}$ and $v_{\mathrm{v}//}$ must be real number, that is, $E - \mu B(z) - q\phi(z) \geq 0$ for the H^+ ion and $E_\mathrm{v} - \mu_\mathrm{v}B(z) + q\phi(z) \geq 0$ for the H^- ion. The particle motion depends on its energy. The distribution functions $f(z, E, \mu, \sigma)$ and $f_\mathrm{v}(z, E_\mathrm{v}, \mu_\mathrm{v}, \sigma)$ for $\sigma = \pm 1$ are obtained by integrating Eqs. (5) and (6) for particle trajectory with the boundary conditions. The energy spaces of the ions are shown in Fig. 2 and Fig. 3, where z_t, and z_vt are the turning points of the H^+ ion and the H^- ion, respectively. The sum of the distribution functions about $\sigma = \pm 1$ for each energy region for H^+ ion becomes

(a) The case of energy space is upwards convex

$$\sum_\sigma f(z,E,\mu,\sigma) = \begin{cases} 2\displaystyle\int_0^L \frac{S(z',E,\mu)}{v_{//}(z',E,\mu)}\mathrm{d}z', \\ \qquad (\mu B(\pm L) + q\phi(\pm L) < E < \infty) \\ 2\displaystyle\int_0^{z_\mathrm{t}} \frac{S(z',E,\mu)}{v_{//}(z',E,\mu)}\mathrm{d}z', \\ \qquad (E_\mathrm{min} < E < \mu B(\pm L) + q\phi(\pm L)) \end{cases} \tag{7a}$$



(a) upwards convex (b) downwards convex

Fig. 2 Energy space of the H^+ ion.

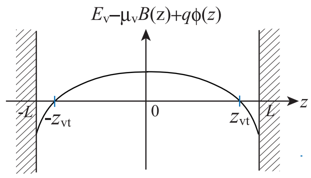


Fig. 3 Energy space of the H^- ion.

(b) The case of energy space is downwards convex

$$\sum_\sigma f(z,E,\mu,\sigma) = \begin{cases} 2\displaystyle\int_0^L \frac{S(z',E,\mu)}{v_{//}(z',E,\mu)}\mathrm{d}z', (\mu B_0 < E < \infty) \\ 2\displaystyle\int_{z_\mathrm{t}}^L \frac{S(z',E,\mu)}{v_{//}(z',E,\mu)}\mathrm{d}z', (E_\mathrm{min} < E < \mu B_0) \end{cases} \tag{7b}$$

and that for the H^- ion becomes

$$\sum_\sigma f_\mathrm{v}(z,E_\mathrm{v},\mu_\mathrm{v},\sigma) = \begin{cases} 2\displaystyle\int_0^L \frac{S_\mathrm{v}(z'_\mathrm{v},E_\mathrm{v},\mu_\mathrm{v})}{v_{\mathrm{v}//}(z'_\mathrm{v},E_\mathrm{v},\mu_\mathrm{v})}\mathrm{d}z'_\mathrm{v}, \\ \qquad (\mu_\mathrm{v}B(\pm L) - q\phi(\pm L) < E_\mathrm{v} < \infty) \\ 2\displaystyle\int_0^{z_\mathrm{vt}} \frac{S_\mathrm{v}(z'_\mathrm{v},E_\mathrm{v},\mu_\mathrm{v})}{v_{\mathrm{v}//}(z'_\mathrm{v},E_\mathrm{v},\mu_\mathrm{v})}\mathrm{d}z'_\mathrm{v}, \\ \qquad (E_\mathrm{v\,min} < E_\mathrm{v} < \mu_\mathrm{v}B(\pm L) - q\phi(\pm L)) \end{cases} \tag{8}$$

where $E_\mathrm{min} = \mu B(z) + q\phi(z)$ and $E_\mathrm{vmin} = \mu_\mathrm{v}B(z) - q\phi(z)$, and z' and z'_v are the generation positions of the H^+ ion and the H^- ion. As the source functions, we use the expression same as the Emmert *et al.* [3] and Sato *et al.* [4]

$$S(z,E,\mu) = S_0 h(z)\frac{M^2}{4\pi(kT_i)^2}v_{//}(z,E,\mu) \cdot \exp\left\{-\frac{E - q\phi(z)}{kT_i}\right\}, \tag{9}$$

$$S_\mathrm{v}(z,E_\mathrm{v},\mu_\mathrm{v}) = S_{\mathrm{v}0}h_\mathrm{v}(z)\frac{M_\mathrm{v}^2}{4\pi(kT_\mathrm{v})^2}v_{\mathrm{v}//}(z,E_\mathrm{v},\mu_\mathrm{v}) \cdot \exp\left\{-\frac{E_\mathrm{v} + q\phi(z)}{kT_\mathrm{v}}\right\}, \tag{10}$$

where k is the Boltzmann's constant, T_i and T_v are the temperatures, $h(z)$ and $h_\mathrm{v}(z)$ are the source strengths, and S_0 and $S_{\mathrm{v}0}$ are the average source strengths of the H^+ ion and the H^- ion, respectively. The density $n_\mathrm{i}(z)$ of the H^+ ion is given by integrating the Eqs. (7a) and (7b) over the $E - \mu$ space, and the density $n_\mathrm{v}(z)$ of the H^- ion are given by integrating the Eq. (8) over the $E_\mathrm{v} - \mu_\mathrm{v}$ space as [4].

$$n_{\rm i}(z) = \frac{2\pi B(z)}{M^2}\sum_\sigma \int dE \int d\mu \frac{f(z,E,\mu,\sigma)}{\upsilon_{/\!/}(z,E,\mu)}, \tag{11}$$

$$n_{\rm v}(z) = \frac{2\pi B(z)}{M_{\rm v}^2}\sum_\sigma \int dE_{\rm v} \int d\mu_{\rm v} \frac{f_{\rm v}(z,E_{\rm v},\mu_{\rm v},\sigma)}{\upsilon_{{\rm v}/\!/}(z,E_{\rm v},\mu_{\rm v})}. \tag{12}$$

By substituting Eqs. (7a) and (7b) into Eq. (11), and Eq. (8) into Eq. (12), and interchanging the order of integrations of them, the ion densities become

$$\begin{aligned} n_{\rm i}(z) = \frac{4\pi B(z)}{M^2}\Bigg\{&\int_0^L dz' \int_{\frac{-E_{\rm p}B_0}{B_{\rm p}-B_0}}^{\infty} dE \int_0^{\frac{1}{B(z)}(E-E_{\rm p})} d\mu\ \frac{1}{\upsilon_{/\!/}(z,E,\mu)}\frac{S(z',E,\mu)}{\upsilon_{/\!/}(z',E,\mu)} \\ &+ \int_0^L dz' \int_{E_{\rm p}}^{\frac{-E_{\rm p}B_0}{B_{\rm p}-B_0}} dE \int_0^{\frac{1}{B_{\rm p}}(E-E_{\rm p})} d\mu \frac{1}{\upsilon_{/\!/}(z,E,\mu)}\frac{S(z',E,\mu)}{\upsilon_{/\!/}(z',E,\mu)}\Bigg\}, \end{aligned} \tag{13}$$

$$n_{\rm v}(z) = \frac{4\pi B(z)}{M_{\rm v}^2}\left\{\int_0^L dz'_{\rm v} \int_{-E_{\rm p}}^{\infty} dE_{\rm v} \int_0^{\frac{1}{B_{\rm p}}(E_{\rm v}+E_{\rm p})} d\mu_{\rm v} \frac{1}{\upsilon_{{\rm v}/\!/}(z,E_{\rm v},\mu_{\rm v})}\frac{S_{\rm v}(z'_{\rm v},E_{\rm v},\mu_{\rm v})}{\upsilon_{{\rm v}/\!/}(z'_{\rm v},E_{\rm v},\mu_{\rm v})}\right\}, \tag{14}$$

where $E_{\rm p} = q\phi(z')$, $B_{\rm p} = B(z')$ for $z' < z$, $E_{\rm p} = q\phi(z)$, $B_{\rm p} = B(z)$ for $z' > z$, $E_{\rm p} = q\phi(z)$, $B_{\rm p} = B(z)$ for $z'_{\rm v} < z$, and $E_{\rm p} = q\phi(z'_{\rm v})$, $B_{\rm p} = B(z'_{\rm v})$ for $z'_{\rm v} > z$, and we considered a case that the increase rate of the magnetic field toward the wall is smaller than that of the electric potential. By substituting Eq. (9) into Eq. (13), and Eq. (10) into Eq. (14), and integrating them for μ, $\mu_{\rm v}$ and E, $E_{\rm v}$, we obtain

$$n_{\rm i}(z) = S_0\left(\frac{\pi M}{2kT_{\rm i}}\right)\int_0^L {\rm d}z' I(z,z')h(z'), \tag{15}$$

$$n_{\rm v}(z) = S_{\rm v0}\left(\frac{\pi M_{\rm v}}{2kT_{\rm v}}\right)\int_0^L {\rm d}z'_{\rm v} I_{\rm v}(z,z'_{\rm v})h_{\rm v}(z'_{\rm v}), \tag{16}$$

where

$$I(z,z') = \begin{cases} \exp\left\{\dfrac{q\phi(z')-q\phi(z)}{kT_{\rm i}}\right\}{\rm erfc}\left[\left\{\dfrac{q\phi(z')-q\phi(z)}{kT_{\rm i}}\right\}^{1/2}\right] + \dfrac{2}{\sqrt{\pi}}\left\{\dfrac{B(z)-B_0}{B(z')-B_0}\dfrac{q\phi(z')}{kT_{\rm i}} - \dfrac{q\phi(z)}{kT_{\rm i}}\right\}^{1/2} \\ \times\exp\left\{\dfrac{B(z')}{B(z')-B_0}\dfrac{q\phi(z')}{kT_{\rm i}}\right\} - \left\{\dfrac{B(z')-B(z)}{B(z')}\right\}^{1/2}\exp\left\{\dfrac{B(z')}{B(z')-B(z)}\dfrac{q\phi(z')-q\phi(z)}{kT_{\rm i}}\right\} \\ \times\Bigg\{{\rm erfc}\left[\left\{\dfrac{B(z')}{B(z')-B(z)}\dfrac{q\phi(z')-q\phi(z)}{kT_{\rm i}}\right\}^{1/2}\right] \\ -{\rm erfc}\left[\left\{\dfrac{B(z')\,(B(z)-B_0)}{(B(z')-B(z))\,(B(z')-B_0)}\dfrac{q\phi(z')}{kT_{\rm i}} - \dfrac{B(z')}{B(z')-B(z)}\dfrac{q\phi(z)}{kT_{\rm i}}\right\}^{1/2}\right]\Bigg\},\ (z' < z)\,, \\ \exp\left\{\dfrac{q\phi(z')-q\phi(z)}{kT_{\rm i}}\right\},\ (z' > z)\,. \end{cases} \tag{17}$$

$$I_{\rm v}(z,z'_{\rm v}) = \begin{cases} \exp\left\{\dfrac{-q\phi(z'_{\rm v})+q\phi(z)}{kT_{\rm v}}\right\},\ (z'_{\rm v} < z) \\ \exp\left\{\dfrac{-q\phi(z'_{\rm v})+q\phi(z)}{kT_{\rm v}}\right\}{\rm erfc}\left[\left\{\dfrac{-q\phi(z'_{\rm v})+q\phi(z)}{kT_{\rm v}}\right\}^{1/2}\right] \\ -\left\{\dfrac{B(z'_{\rm v})-B(z)}{B(z'_{\rm v})}\right\}^{1/2}\exp\left\{\dfrac{B(z'_{\rm v})}{B(z'_{\rm v})-B(z)}\dfrac{-q\phi(z'_{\rm v})+q\phi(z)}{kT_{\rm v}}\right\} \\ \times{\rm erfc}\left[\left\{\dfrac{B(z'_{\rm v})}{B(z'_{\rm v})-B(z)}\dfrac{-q\phi(z'_{\rm v})+q\phi(z)}{kT_{\rm v}}\right\}^{1/2}\right],\ (z'_{\rm v} > z) \end{cases} \tag{18}$$

where ${\rm erfc}(x)$ is the complementary error function. As the electron density $n_{\rm e}$, we use a Boltzmann distribution $n_{\rm e}(z) = n_0\exp[e\phi(z)/kT_{\rm e}]$ for simplicity, where n_0 is the electron density at $z = 0$, $-e$ is the electron charge, and $T_{\rm e}$ is the electron temperature. Substituting Eqs. (15), (16) and the electron density into Poisson's equation, the plasma-sheath equation is derived

$$\frac{{\rm d}^2\phi}{{\rm d}z^2} = \frac{n_0 e}{\varepsilon_0}\exp\left(\frac{e\phi(z)}{kT_e}\right) - \frac{qS_0}{\varepsilon_0}\left(\frac{\pi M}{2kT_i}\right)^{1/2}\int_0^L {\rm d}z' I(z,z')h(z') + \frac{qS_{\rm v0}}{\varepsilon_0}\left(\frac{\pi M_{\rm v}}{2kT_{\rm v}}\right)^{1/2}\int_0^L {\rm d}z'_{\rm v} I_{\rm v}(z,z'_{\rm v})h_{\rm v}(z'_{\rm v}). \tag{19}$$

The average source strengths S_0 and S_{v0} are derived from the equilibrium of the fluxes of the plasma particles at the wall. We consider $j_{ew} + j_{iw} + j_{vw} = 0$, where j_{ew}, j_{iw} and j_{vw} are the current densities of the electron, the H^+ ion and the H^- ion at the wall, respectively. The current densities are given by $j_{ew} = -en_0\{kT_e/(2\pi m_e)\}^{1/2}\exp\{e\phi_w/(kT_e)\}$, $j_{iw} = qS_0L$, and $j_{vw} = -qS_{v0}L$, respectively, where m_e is the electron mass and ϕ_w is the wall potential. Furthermore, we define a rate of production amount of the volume produced H^- ions to the H^+ ions to be $\beta_v = S_{v0}/S_0$. The average source strengths S_0 and S_{v0} are obtained as

$$S_0 = \frac{en_0}{qL(1-\beta_v)}\left(\frac{kT_e}{2\pi m_e}\right)^{1/2}\exp\left(\frac{e\phi_w}{kT_e}\right), \quad (20)$$

$$S_{v0} = \frac{en_0\beta_v}{qL(1-\beta_v)}\left(\frac{kT_e}{2\pi m_e}\right)^{1/2}\exp\left(\frac{e\phi_w}{kT_e}\right). \quad (21)$$

3. Numerical Solutions

Equation (19) is solved numerically. We introduce the normalized variables such as $\eta = (q/kT_e)(\phi_w - \phi)$, $R = B/B_0$, $s = z/L$, $\tau = T_e/T_i$, $\tau_v = T_e/T_v$, $Z = q/e$, where R is the mirror ratio and $Z = 1$ for the hydrogen plasma. The boundary conditions are $d\eta/ds|_{s=0} = 0$ and $\eta(s = 1) = 0$. We assume the mirror ratio with reference to the expression used by Sato *et al.* [4]

$$R(\eta) = \exp[\alpha\{\eta - e\phi_w/(kT_e)\}], \quad (22)$$

where α is a positive constant and indicates a degree of increase of the magnetic field toward the wall. As the value of the Debye length λ_D, we will use $\lambda_D/L = 5\times10^{-2}$ in all results of this paper. The profile of the normalized electric potential $\Phi(s) = -\eta$ for various values of α is shown in Fig. 4. As the degree of increase of the magnetic field toward the wall becomes large, the sheath width becomes small. The profile of the normalized electric potential $\Phi(s)$ for various values of the production amount of the H^- ion to the H^+ ion is shown in Fig. 5. As the production amount of the H^- ion becomes large, the sheath width becomes large. The profile of the normalized electric potential $\Phi(s)$ for various values of the ion temperature $\tau = \tau_v$ is shown in Fig. 6. It is shown that the electric potential strongly depends on the ion temperature. As the ion temperature becomes large ($\tau = \tau_v$ becomes small), the potential drop becomes small. Furthermore, the profile of the plasma density normalized by a sum of the H^+ ion density and the H^- ion density at $s = 0$ for $\alpha = 0.1$ and 0.3 are shown in Fig. 7. For the case of the degree of increase of the magnetic field

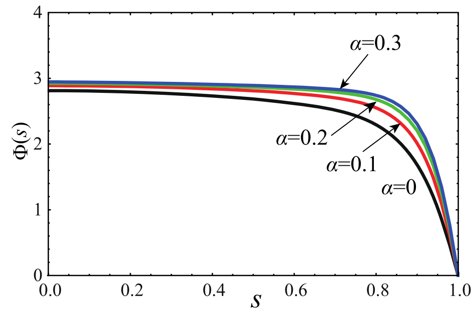


Fig. 4 Profile of the normalized electric potential $\Phi(s)$ for various values of α, with $\tau = \tau_v = 1$, $\beta_v = 0.2$.

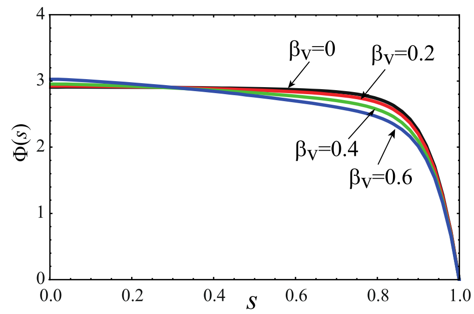


Fig. 5 Profile of the normalized electric potential $\Phi(s)$ for various values of β_v, with $\tau = \tau_v = 1$, $\alpha = 0.2$.

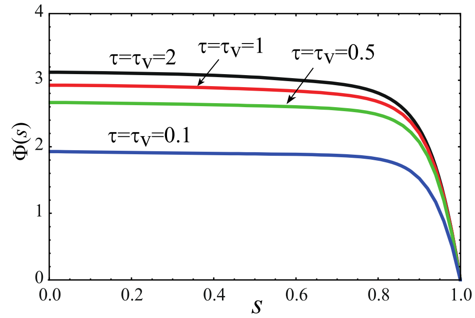


Fig. 6 Profile of the normalized electric potential $\Phi(s)$ for various values of $\tau = \tau_v = 1$, with $\beta_v = 0.2$, $\alpha = 0.2$.

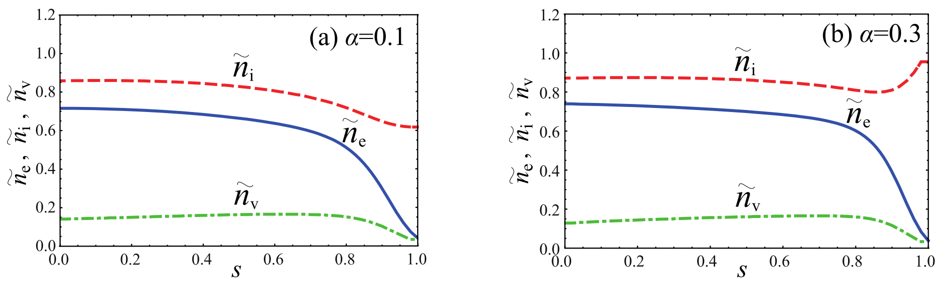


Fig. 7 Profile of the normalized particle density for $\alpha = 0.1$ and 0.3, with $\tau = \tau_v = 1$, $\beta_v = 0.2$.

toward the wall is large, the density of the H^+ ion near the wall increases. This seems because the H^+ ions are easy to move towards the wall under the steep potential gradient near the wall shown in Fig. 4.

4. Conclusions

The electric potential near the wall for the plasma that consists of the H^+ ion, the volume produced H^- ion and the electron with the magnetic field increasing toward the wall is investigated analytically. The profile of the electric potential is obtained by solving the plasma-sheath equation. Effects of the degree of increase of the magnetic field toward the wall, the production rate of the volume produced H^- ion, and the ion temperature on the distribution of the electric potential are shown. The effect of the ion temperature on the distribution of the electric potential is larger than the other effects. It is also shown that the density of the H^+ ion near the wall increases when the degree of increase of the magnetic field is large.

[1] A. Fukano, A. Hatayama and M. Ogasawara, JJAP **46**, No.4A, 1668 (2007).

[2] V.E. Golant, A.P. Zhilinsky and I.E. Sakharov, *Fundamentals of Plasma Physics* (John Wiley and Sons, New York, 1980).

[3] G.A. Emmert, R.M. Wieland, A.T. Mense and J.N. Davidson, Phys. Fluids **23**, 803 (1980).

[4] K. Sato, F. Miyawaki and W. Fukui, Phys. Fluids **B1**, 725 (1989).