Three-Dimensional Nonlinear Modeling of MHD Instabilities for Low-q Plasma on J-TEXT*)

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The characteristics of the magnetohydrodynamics (MHD) instabilities for low edge safety factor (low-q) plasma are investigated on J-TEXT with three-dimensional (3D) nonlinear MHD Infrastructure for Plasma Simulation (MIPs) code. The dynamics of $m = 2$ mode coupling with $m = 1$ mode is found in the linear growth phase with the initial $q_{axis} < 1$ equilibrium. And new 3D helical coils are designed for generating additional rotational transform based on the J-TEXT configuration, which is helpful to suppress the amplitude of the MHD instabilities from the modeling result.

Keywords: magnetohydrodynamics, nonlinear instability, tokamak, rotational transform, helical coil

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1. Introduction

Disruption and their consequences have long been an active area in tokamak research and the active control of disruption avoidance is very important for large tokamak devices like ITER. Well known reasons of disruption are the operation limits associated with the plasma density, pressure and plasma current [1]. For high current operation, there is a hard limit imposed by ideal kink modes which are unstable for $q_{edge} < 2$. In the experiment, the disruption behavior associated with kink or tearing mode instabilities could be observed in the low-q plasma when q_{edge} approaching the value of two [2, 3]. Recently researches which focus on disruption avoidance show one of the active feedback control techniques that using small non-axisymmetric magnetic field could suppress these unstable MHD instabilities [4]. In addition, the hybrid tokamak operation optimized by the external rotational transform with the 3D stellarator helical coils has shown the evidence of disruption avoidance and MHD activity suppression in the recently experiment [5].

The J-TEXT tokamak is a conventional iron core tokamak, which has a major radius $R_0 = 1.0$ m and a minor radius $r = 0.25$ - 0.29 m with a movable titanium-carbide coated graphite limiter. The main parameters of a typical J-TEXT discharge are toroidal field $B_T \sim 2.0$ T, plasma current $I_p \sim 200$ kA lasting for 800 ms, plasma density $n_e = (1-7) \times 10^{19}$ m^{-3} and electron temperature $T_e \sim 1$ keV. The main researches of J-TEXT focus on the impacts of 3D Magnetics Perturbations (MPs) field on MHD instabilities, plasma disruptions and plasma turbulence transport in the past several years [6]. Recently, the J-TEXT has the upgrade plan aiming at the 3D plasma physics.

In this work, the characteristics of MHD instabilities for low-q plasma are analyzed on J-TEXT by 3D nonlinear MHD Infrastructure for Plasma Simulation (MIPS) code [7] associated with the 3D equilibrium HINT code [8]. And two new 3D helical coils are superposed to investigate what extent strong additional rotational transform with $\iota_{coil}/\iota \sim 0.1$ ($\iota = 2\pi/q$) can be used to suppress these MHD instabilities at low-q or high current operation, which could be helpful to active control of the disruption avoidance.

In section 2 the physical model and the numerical methods are presented. In section 3 the result of turning of the rotational transform with 3D helical coils is shown. In section 4 the results of MHD instabilities analysis and the impact of external rotational transform on MHD properties are detailed. A summary and discussion follow in section 5.

2. Physical Model and Numerical Methods

The modeled equilibrium of a low-q plasma based on J-TEXT tokamak configuration is prepared with the HINT code. The HINT code uses two steps relaxation method to solve the 3D nonlinear equilibrium in the rectangular grids of the cylindrical coordinate $(R, Ø, Z)$, where R and Z are the horizontal and the vertical coordinates, respectively, and $Ø$ is the toroidal angle. In the first step, the pressure averaged (relaxed) along the field line is calculated to sat-

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isfy the equation,

$$\mathbf{B} \cdot \nabla p = 0, \tag{1}$$

with the fixed magnetic field, where $\boldsymbol{B}$ and p are the magnetic field and the pressure. The second step is a relaxation process of the magnetic field with fixed p given by the first step. The artificial dissipative MHD equations in the second step are:

$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \mathbf{J}_1 \times (\mathbf{B}_0 + \mathbf{B}_1), \tag{2}$$

$$\frac{\partial \mathbf{B}_1}{\partial t} = \nabla + [\mathbf{v} \times (\mathbf{B}_0 + \mathbf{B}_1) - \eta(\mathbf{J}_1 - \mathbf{J}_{net})] + \kappa_{divB} \nabla\nabla \cdot \mathbf{B}_1, \tag{3}$$

$$\mathbf{J}_1 = \nabla \times \mathbf{B}_1. \tag{4}$$

Where $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$. Here $\mathbf{B}_0$ is the vacuum field which is fixed in the calculation and $\mathbf{B}_1$ is the equilibrium response field. The $\mathbf{J}_{net}$ is the net toroidal current and κ_{divB} is small constant to keep the divergence free for the magnetic field. The η denotes the artificial resistivity. These two steps are iterated until $\partial \mathbf{v}/\partial t \to 0$ and $\partial \mathbf{B}_1/\partial t \to 0$, which satisfy the force balance equation, $\nabla p = \mathbf{J} \times \mathbf{B}$. Then the 3D MHD equilibrium can be obtained.

In J-TEXT, the initial pressure and current of 3D MHD equilibrium calculation for low-q plasma is given by the 2D EFIT code [9]. The initial magnetic field is the sum of the 2D magnetic field given by the EFIT and 3D vacuum field generated by the external helical coils when the coils superposed.

The dynamics of the plasmas in the low-q equilibrium by the HINT code is studied with the MIPS code. The MIPS code computes the following full MHD equations:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}), \tag{5}$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\rho(\mathbf{v} \cdot \nabla)\mathbf{v} + \mathbf{J} \times \mathbf{B} - \nabla p + \frac{4}{3}\nabla[\nu\rho(\nabla \cdot \mathbf{v})] - \nabla \times [\nu\rho\boldsymbol{\omega}], \tag{6}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \tag{7}$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot (p\mathbf{v}) - (\gamma - 1)p\nabla \cdot \mathbf{v} + \chi_\perp \nabla_\perp^2 (p - p_{eq}) + \chi_\parallel \nabla \cdot \left(\frac{\mathbf{B}}{B^2}\mathbf{B} \cdot \nabla p\right), \tag{8}$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta(\mathbf{J} - \mathbf{J}_{eq}), \tag{9}$$

$$\mathbf{J} = \nabla \times \mathbf{B}, \tag{10}$$

$$\boldsymbol{\omega} = \nabla \times \mathbf{v}. \tag{11}$$

Here, γ is the adiabatic constant, and $\mathbf{J}_{eq}$ is the equilibrium current density to make the MHD equilibrium consistent with the finite resistivity. These equations are solved in the cylindrical coordinate same as HINT. The viscosity ν, the resistivity η, the perpendicular and the parallel heat conductivities $\chi_\perp$ and $\chi_\parallel$ act as the dissipation parameters in the equations. These parameters are normalized by $v_A R_{cnt}$, where v_A is the Alfvén speed and R_{cnt} is the major radius of the magnetic axis, which is fixed as $R_{cnt} = 1.05$ m for J-TEXT calculation. The resistivity, the perpendicular and parallel heat conductivities are assumed to be $\eta/\mu_0 = 10^{-6}$, $\chi_\perp = 10^{-8}$, and $\chi_\parallel = 10^{-2}$, respectively. In the presented simulations, the numbers of the grid points are 64, 128, 64 for the direction of R, Ø and Z, respectively. The fourth-order Runge-Kutta method is used for the time integration of these equation and the dynamics of the MHD activity can be simulated by following the time evolution.

3. Turning of Rotational Transform with Helical Coils in J-TEXT

In this section, we focus on the additional rotational transform generated by the 3D external coils for low-q plasma in J-TEXT. Two simple 3D external coils with a set of $\iota = 2$, $m = 2$ continuous helical coils are designed to supply the rotational transform like the heliotron type device in the simulation. The major radius and the minor radius of the coil are $R_0 = 1.05$ m and $r = 0.3$ m, respectively. Figures 1 (a) and 1(b) show the shape of the coils. The safety factor profiles calculated by the HINT for the cases with and without helical coils are shown in Fig. 1 (c). The safety factor value at the magnetic axis is less than 1 for both cases. The objective of imposed external rotational transform ratio is $\iota_{coil}/\iota_0 \sim 0.1$ at the plasma edge, giving the magnitude of the coils current as $\boldsymbol{I}_{coit} = 30$ kA for the case with helical coils. For both cases, the plasma current is $I_p = 230$ kA and the toroidal magnetic field at the magnetic axis is $B_T = 2.0$ T for the initial of HINT calculation. The plasma beta at the magnetic axis is $\beta_0 = 0.38\%$ as a low-β plasma.

The difference in the q profile between without and

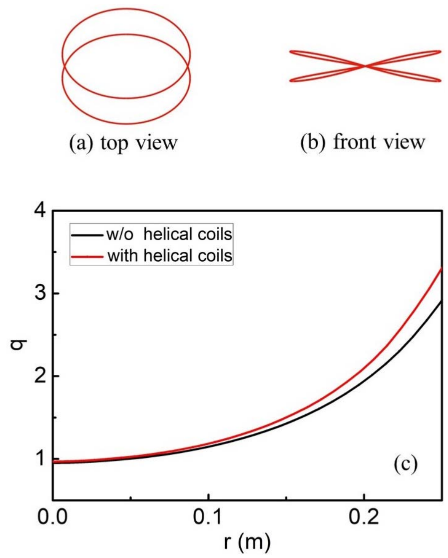

Fig. 1 (a) Top view of helical coils. (b) Front view of helical coils. (c) The q profiles of the equilibria in the cases without (black line) and with (red line) the external helical coils.

with the helical coils is attributed to several following factors. First, the helical coils generate negative poloidal field $B_{p,coil}$ and positive toroidal field $B_{T,coil}$ with respect to the original poloidal field $B_{p,plasma}$ and toroidal field B_{T0} given by the plasma current and toroidal field coils, respectively. As the poloidal component of helical coils current is in the opposite direction to plasma current and the toroidal component direction is the same as toroidal field coils. The ratio of $B_{p,coil}/B_{p,plasma}$ is much larger than $B_{T,coil}/B_{T0}$. And the major change of rotational transform in the plasma edge region is mainly due to the plasma density decreasing with minor radius. Second, the plasma elongation (as shown in Fig. 2 (b), the plasma shape becomes non-axisymmetric with helical coils) induced by a limit number of helical coils will change the plasma density profile in the 3D equilibrium calculation, which could also make contribution to the change of total rotational transform. In addition, the magnetic axis will be inward shifted with the helical coils superposed, which may also have the influence on the total rotational transform.

4. Nonlinear Dynamics of MHD Instabilities for Low-q Plasma

The initial 3D equilibrium for nonlinear MHD dynamics has been performed for the low-q plasma in J-TEXT. Note that the plasma pressure is very small while the current density is high, the equilibrium with low safety factor can be easily unstable against the current driven modes. Figure 3 shows the time evolution of the kinetic energy of MHD modes in the cases without and with external helical coils. The tendencies are significant different in both cases. For the case without helical coils, there are two different growth rates in the linear increasing phase. The growth rate could be increasing after $t = 20\tau_A$ as the slope (the green dotted line) change of the black line in Fig. 3. Here τ_A denotes the Alfvén time. The MHD modes will nonlinearly saturated after $t = 50\tau_A$. For the case with the helical coils, the growth rate is quite small in the linear phase and rapidly saturated, which indicate that the 3D external helical coils or the additional rotational transform can suppress the amplitude of the MHD modes for the low-q plasma.

To investigate the characteristics of the current driven modes in the low-q plasma, the mode patterns are analyzed by the time evolution of pressure perturbation. Figure 4 shows the contour plot of the perturbed component of the pressure in the cases without and with helical coils. The current driven instabilities evolve from $m = 1$ mode located at the $q = 1$ magnetic surface in the plasma core

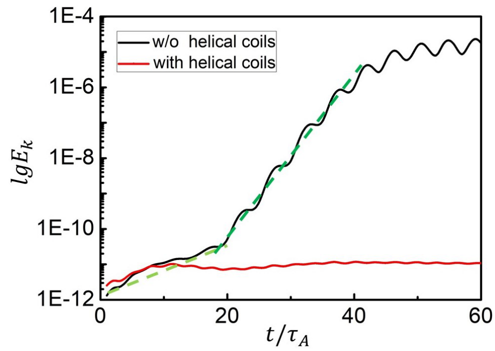

Fig. 3 Time evolution of the kinetic energy E_k of the MHD modes in the cases without (black line) and with (red line) the external helical coils. Here τ_A denotes Alfvén time.

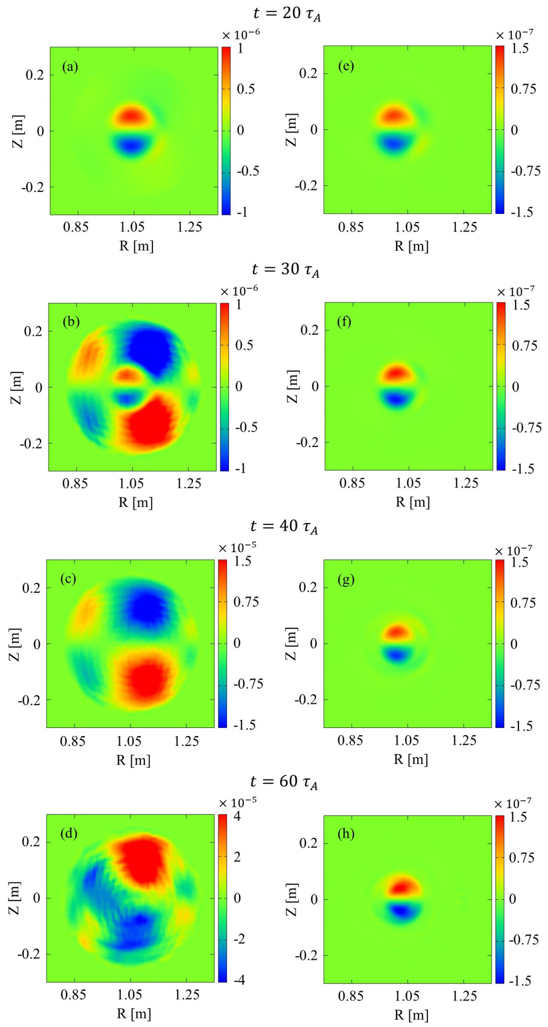

Fig. 4 Mode pattern of pressure perturbation at different times in the cases without (left column) and with (right column) external helical coils.

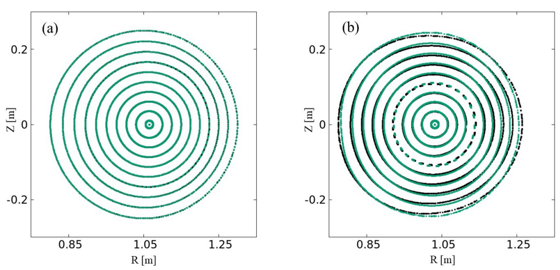

Fig. 2 Magnetic flux surface at Ø = 0° (black dots) and Ø = 45° (green dots) in the cases without (a) and with (b) the external helical coils.

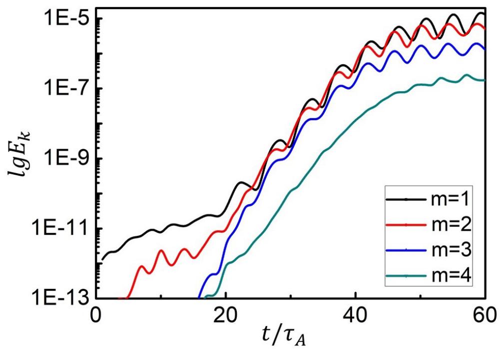

Fig. 5 Time evolution of the kinetic energy of poloidal mode components in the case without the external helical coils.

region for the both cases with $q_{axis} < 1$. For the case without helical coils, the $m = 2$ mode located at the $q = 2$ surface is unstable after $t = 20\tau_A$. The dynamics of $m = 2$ mode coupling with $m = 1$ mode will change the growth rate in the following linear growth phase, giving two different growth rates. The process of mode coupling also leads to the oscillation of the kinetic energy in the linear growth phase and nonlinear saturation phase as the black line shows in Fig. 3. The dominate mode is $m = 1$ like mode in the main region when the instabilities saturated. With the external helical coils superposed, the $m = 2$ mode is suppressed and the growth rate of $m = 1$ mode becomes smaller, and the mode will be always located at the $q = 1$ surface over time. The amplitude of the MHD instabilities is much smaller than the case without helical coils, showing the evidence of stabilization effects on low-q plasma. The mechanism in the difference of the characteristics of the MHD instabilities mainly depends on the $m = 2$ mode located at the edge, and the helical coils generate the additional rotational transform in the low-q plasma, which can change the magnetic shear and suppress the edge instabilities.

To analyze the mode coupling mechanism in detail, the time evolution of the poloidal mode components is calculated as shown in Fig. 5 for the case without the external helical coils. It shows clearly evidence of dynamics mode energy transformation between the $m = 1$ and $m = 2$ mode after $t = 20\tau_A$, as the time for the peak position of the kinetic energy are different in the periodic oscillation for the two modes. And the mode coupling can significantly trigger the $m = 3$ mode instability which is rapidly increasing after $t = 20\tau_A$.

5. Summary

In this study, the characteristics of MHD instabilities for low-q plasma are numerically analyzed with 3D nonlinear MIPS code in J-TEXT. And new helical coils are designed for the active control of the current driven instabilities. The mode structure changes depending on the stability of $m = 2$ mode located at the $q = 2$ surface with $q_{edge} < 3$. With helical coils superposed, the $m = 2$ mode can be significantly suppressed and the amplitude of the current driven instabilities become smaller. It can be inferred that the additional rotational transform generated by the external helical coil could improve the threshold of current limit for tokamak. This modeling result brings significant possible benefits in terms of disruption avoidance for low-q plasma operation that the hybrid tokamak combining with stellarator may improve the fusion performance. And it will be helpful to the J-TEXT upgrade in the future.

To calculate the initial equilibrium exactly, the nonlinear HINT code is utilized in the calculation. The 3D equilibrium of a low-q and low-β J-TEXT plasma with external helical coils is obtained. The helical coils mainly change the rotational transform at the plasma edge region where the plasma current density is small. Besides, the plasma shape and the magnetic axis position will also change a little with respect to the case without helical coils. In the current coil design, the pitch angle of magnetic field generated by the external helical coils is far from the original field. If we change the coils winding direction, or change the plasma current direction, the pitch angle of 3D external magnetic field is approximately to the original field, which can easily make the resonant perturbations on the rational surface.

The mechanism for the stabilization effects of external helical coils on low-q plasma is mainly due to the suppression of $m = 2$ mode, since the major changes of q profile located at the plasma edge with external rotational transform. The other factors like the plasma elongation and the magnetic axis shift may also have the impacts on the characteristics of MHD instabilities. More details investigation of the mechanism is a future work.

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[1] T.C. Hender *et al.*, Nucl. Fusion **47**, 128 (2007).
[2] C. Paz-Soldan *et al.*, Plasma Phys. Control. Fusion **61**, 054001 (2019).
[3] A. Cheetham *et al.*, Nucl. Fusion **27**, 843 (1987).
[4] P. Piovesan *et al.*, Phys. Rev. Lett. **113**, 045003 (2014).
[5] M.D. Pandya *et al.*, Phys. Plasmas **22**, 110702 (2005).
[6] Y. Liang *et al.*, Nucl. Fusion **59**, 112016 (2019).
[7] Y. Todo *et al.*, Plasma Fusion Res. **5**, S2062 (2010).
[8] Y. Suzuki, Plasma Phys. Control. Fusion **59**, 054008 (2017).
[9] L.L. Lao *et al.*, Nucl. Fusion **25**, 1421 (1985).