Proposal of Analysis Method for Pattern Recognition of Two-Dimensional Structure of Plasma

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Two-dimensional measurement becomes active to explore physics of plasma structure and dynamics. The paper proposes a set of new methods using the Fourier-Bessel function series expansion to characterize the two-dimensional images for plasma and illustrates an example of its application to a phenomenon of quasi-periodic oscillations observed with tomography in a linear plasma device PANTA. The analysis provides quantitative relations between the coherent structure and residual random fluctuations of the oscillations. The results suggest that similarity between their spatial patterns should be increased in the phase when both amplitudes become stronger.

Keywords: linear plasma, tomography, nonlinear oscillation, pattern recognition, similarity

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Two-dimensional (2D) measurement has shown its excellent ability to explore unsolved physics problems [1]. In a linear plasma device, Plasma Assembly for Nonlinear Turbulence Analysis (PANTA), tomography aiming at the entire measurement of plasma turbulence started to provide intriguing results [2, 3]. The trend needs to develop advanced tools to analyze the 2D plasma images. The paper proposes a set of methods for a 2D pattern recognition based on Fourier-Bessel function (FBF) series expansion, and illustrates its application to the tomography images of the quasi-periodic phenomenon called solitary oscillations in a PANTA plasma [4].

The solitary oscillations are observed in argon plasmas of 5 cm radius, when magnetic field and filling gas pressure are at 0.07 T and ~0.4 Pa, respectively. The typical evolution of the solitary oscillations is obtained in a newly proposed manner of conditional average called Correlation-estimated conditional averaged method (CECAME). The method separates the oscillations into two parts; the coherent (or deterministic) trend and the residual fluctuations [5].

Figure 1 shows the temporal evolution of tomography images of the coherent structure, $f_c(r,\theta,t)$, and the residual fluctuations, $f_r(r,\theta,t)$, with a temporal evolution of local emission and its residual fluctuation amplitude in the single cycle (~0.8 ms) of the solitary oscillations; note that the ensemble average of the residual fluctuation part, $f_r(r,\theta,t)$, is zero, which shows its randomness. However, the structure of the residual fluctuations becomes visible in their absolute value, $\hat{f}_r(r,\theta,t) = |f_r(r,\theta,t)| - \langle|f_r(r,\theta,t)|\rangle$, where $\langle\ldots\rangle$ represents temporal average. The images of $f_c(t)$ and $\hat{f}_r(t)$ are reconstructed with FBF series defined as

$$f(r,\theta) = \sum_{m,n} J_m(k_{mn}r)[\alpha_{mn}\cos(m\theta) + \beta_{mn}\sin(m\theta)], \qquad (1)$$

where $J_m(r)$ is the m-th order Bessel function, with m and

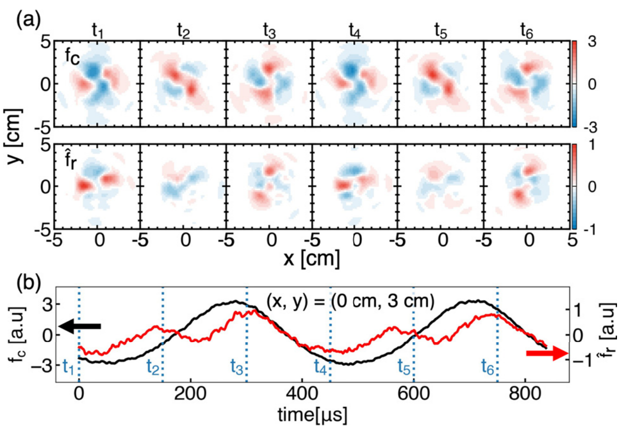

Fig. 1 (a) The spatiotemporal structure of the $f_c(t)$ and $\hat{f}_r(t)$ at each position 0, 150, 300, 450, 600, and 750 μs. (b) The black and red lines are $f_c(t)$ and $\hat{f}_r(t)$ of local emission for the position at $(x, y) = (0\,\text{cm}, 3\,\text{cm})$, respectively.

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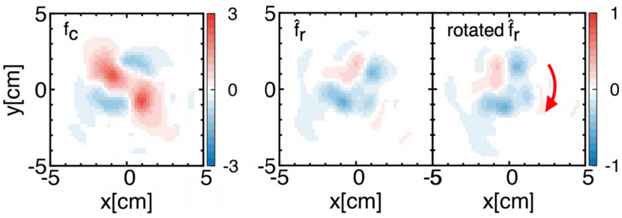

Fig. 2 Similarity analysis of two images at 140 µs. The maximum similarity is obtained by rotating the patterns of the residual fluctuation amplitude by 28°. The red arrow represents electron diamagnetic direction.

k_{mn} being the azimuthal mode number and the n-th zero point of the m-th order Bessel function, respectively. The waveforms in Fig. 1 (b) appear to be composed of two cycles (of a period of ~0.4 ms), corresponding to $m = 2$ structure, however, the Fourier analysis shows that a smaller fundamental component, corresponding to $m = 1$ structure, is included in the oscillations [5]. Moreover, the comparison between the temporal evolutions of the two images suggests that similar patterns appear as its degree changes temporally.

Assuming that $\varphi(r, \theta)$ and $\phi(r, \theta)$ are 2D images defined on the area, S, the inner product,

$$\langle \varphi, \phi \rangle = \int \varphi(r, \theta)\phi(r, \theta) dS, \tag{2}$$

is defined. Then, as is analogous to such as digital image correlation [6], the similarity index, σ, is introduced to evaluate the degree of similarity between the images, as

$$\begin{aligned}\sigma(\varphi, \phi) &= \frac{\langle \varphi, \phi \rangle}{\sqrt{\langle \varphi^2 \rangle \langle \phi^2 \rangle}} \\ &= \frac{\sum_{mn} (\alpha_{mn}\alpha'_{mn} + \beta_{mn}\beta'_{mn})}{\sqrt{\sum_{mn} (\alpha_{mn}^2 + \beta_{mn}^2) \sum_{mn} (\alpha'^2_{mn} + \beta'^2_{mn})}},\end{aligned} \tag{3}$$

where (α, β) and (α', β') are the Fourier-Bessel coefficients of φ and ϕ. The second reduction is allowed by using the orthonormal property of FBF. The ingenious use of the FBF series provides further methods of image quantification.

Figure 2 shows an example for evaluating the similarity index of coherent and residual fluctuation images at $t = 140$ µs. In evaluating the similarity, the pattern rotation should need to be taken into account, or the maximum similarity should be searched by rotating one of the patterns. In case of Fig. 2, the maximum similarity, $\sigma = 0.41$, is obtained at rotation angle $\theta_{rot} = 28°$. Since the structures rotate in electron diamagnetic direction, it suggests that the residual fluctuation may be ahead of the coherent structure. Moreover, the FBF coefficients can provide further image characterization. For instance, the spatially averaged azimuthal and radial wave numbers are defined as

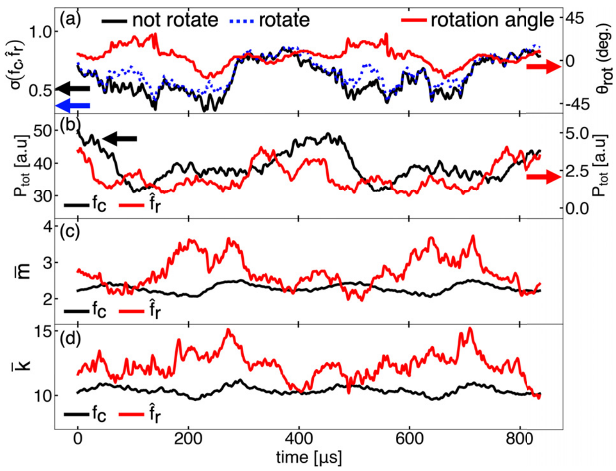

Fig. 3 (a) The similarity indices between the coherent structure and the residual fluctuation amplitude in case with and without pattern rotation. (b) The temporal evolution of total power of $f_c(t)$ and $\hat{f}_r(t)$. Total power is summation of all local power. (c) and (d) the temporal evolutions of averaged azimuthal or radial wavenumber of $f_c(t)$ and $\hat{f}_r(t)$.

$$\bar{m} = \frac{\sum_{mn} m \left(\alpha_{mn}{}^2 + \beta_{mn}{}^2\right)}{\sum_{mn} (\alpha_{mn}{}^2 + \beta_{mn}{}^2)}, \tag{4}$$

$$\bar{k} = \frac{\sum_{mn} \hat{k}_{mn} \left(\alpha_{mn}{}^2 + \beta_{mn}{}^2\right)}{\sum_{mn} (\alpha_{mn}{}^2 + \beta_{mn}{}^2)}, \tag{5}$$

where $\hat{k}(= k/a)$ is normalized with plasma radius, a. For the images in Fig. 2, the calculation gives $(\bar{m}, \bar{k}) = (2.21, 10.3)$ and $(2.51, 12.3)$ for the coherent and the residual fluctuation structure, respectively.

Figure 3 shows the temporal evolution of the similarity, the powers, the average azimuthal and radial wave numbers for the coherent and the amplitude of residual fluctuations structures. In Fig. 3 (a), it is found that the similarity index, $\sigma(f_c, \hat{f}_r)$ should be more than 0.4, and that the higher similarity is obtained in the period from t~300 to t~450 µs, when both powers become large (Fig. 3 (b)). The power increases are accompanied with the decreases of $\bar{m}$ and $\bar{k}$ in both structures (Fig. 3 (c) and 3 (d)). The facts can be interpreted as that the increases in the powers should increase their mutual couplings, and that larger scale 2D patterns, *i.e.*, low m (or k) modes, should result in enhancing their similarity.

Finally, it is expected that a nonlinear interaction between the coherent structure and the residual fluctuations should give a birth to the similarity in their spatial patterns. It is well known, in fact, that structured flows such as zonal flows can modulate envelop of small-scale fluctuations, through shearing, trapping and so on [7,8]. Although the mechanism needs to be identified in further studies, the similarity and its temporal variation found in the analysis suggests the presence of mutual interaction between the coherent structure and random fluctuations in the solitary oscillations. The presented results demonstrate the usefulness of the newly proposed method for investigation of structures and mutual relations in 2D plasma images.

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