Trajectory Calculation of Horizontally Injected Laser Fusion Energy Target in Residual Gas[*)]

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A calculation method is presented for the trajectory of a horizontally injected spherical laser fusion energy target in a reactor filled with residual gas. To evaluate the gas resistance and the placement error of the position measurement unit (PMU) set along the injection path, a test injection is successively made two times and the target positions in flight are measured by PMUs. Two unknown system parameters, gas resistance and placement error, are determined by solving an equation system. After determining the system parameters, the arrival position and arrival time of the newly injected fusion target at the reactor center can be calculated by simple arithmetic with the time data and position data of the injected target in flight.

Keywords: trajectory calculation, laser fusion energy target, Arago spot, Poisson spot, position measurement unit, placement error, gas resistance

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1. Introduction

A spherical fuel target is injected into the reactor chamber of a direct drive laser fusion energy reactor and the position and time of the injected target in flight are measured. However, the trajectory must be calculated in a few milliseconds to predict the arrival position and arrival time of the target at the reactor center. If the trajectory of the target is not controlled electrostatically [1] or magnetically [2], the focal point of the laser beam must be moved to the arrival position. Furthermore, the engagement error of the laser beam and the target at the main shot must be lower than 20 μm [3]. This criterion is based on a tolerable non-uniformity of the irradiation.

To meet the criterion of the engagement error, the error of the position measurement of the flying target as well as the calculation error must be lowered by an order of magnitude. Moreover, the position measurement units (PMU) are set along the target path [4, 5]. These PMUs measure the local coordinates of the flying target in each PMU and the time of the measurement, (x, y, z, t). If the placement errors of the PMUs are eliminated, the arrival position and arrival time of the target at the reactor center can easily be calculated.

In actual situations, two difficulties exist in the trajectory calculation. First, the placement error of PMU cannot be controlled. Besides, thermal expansion as well as thermal and mechanical stress of the structural materials in the operation phase of the reactor causes additional placement errors. For example, as the coefficient of linear thermal expansion of HT-9 is $11.8 \times 10^{-6}\,\mathrm{K}^{-1}$ at 400°C [6], a temperature increase of 10 K causes 1.18-mm linear expansion of the reactor of 10-m diameter. Thermal expansion of the building and earth tide [7] can be the cause of the placement error. Second, the residual gas in the reactor causes a frictional force on the injected target. The frictional force is a complex function of the radius of a spherical target, target velocity, and gas parameters. It is also difficult to estimate the friction term in the equation of the target motion.

In this paper, we propose a procedure for the trajectory calculation of an injected target in a reactor filled with residual gas. The procedure uses only the position and the time data of the injected target in the PMU. The setting and mechanism of PMU are described in Sec. 2. The idea of the procedure of the trajectory calculation of the target in vacuum is discussed in Sec. 3. The procedure is extended for the case where residual gas exists in Sec. 4. Generalization of equation of motion is shown in Sec. 5 and conclusion is given in Sec. 6.

2. Setting and Mechanism of PMU

In a laser fusion reactor, the reactor center must be defined optically. Ideally, the reactor center is the position where all the laser beams arrive at the same time. Figure 1 (a) shows that all the laser pulses, represented by the tips of the arrows, arrive at the center of the reactor at the same time. The target is injected to pass through the reactor center. If the injected target deviates from the ideal trajectory and does not pass through the center, then the target trajectory i.e. arrival time and arrival position at the reactor center plane where the target must be shot must be calculated. Based on the calculation, each laser beam must be

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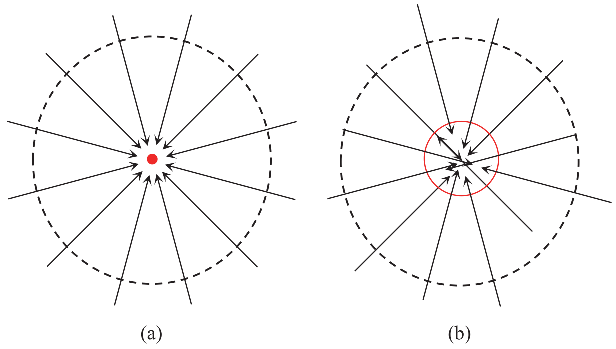

Fig. 1 Center (red point), shot region (broken circle) and reactor central region (red circle), (a) ideal case (b) practical case.

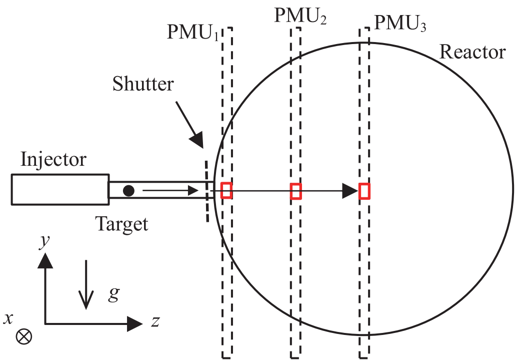

Fig. 2 Setting of PMU (broken rectangles). Red box represents the region of orthogonal irradiation.

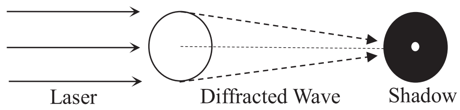

Fig. 3 Arago spot in the shadow.

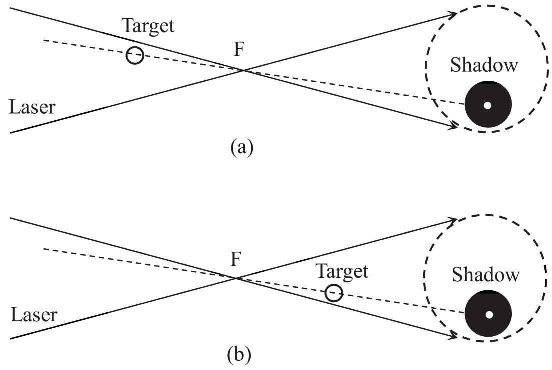

Fig. 4 Arago spot in the shadow. F, focal point (a) convergent beam illumination (b) divergent beam illumination.

moved to the arrival position. If the tolerance of the laser irradiation time on the target is less than 0.05 ns, the target must pass through a sphere of 15-mm diameter placed at the center, represented by the broken circle in Fig. 1 (a), and must be shot in this spherical region (shot region).

Practically, because of the placement errors of the optical elements, the arrival time of each laser beams varies and the tips of the arrows are distributed in a small sphere (reactor central region) represented by the red circle as shown in Fig. 1 (b). Thus, an ideal center of the reactor does not exist. However, a global coordinate system must be defined to calculate the trajectory. It is natural to set the origin of the global coordinate system in this reactor central region. A PMU is placed to locate the origin of the local XYZ coordinate system within the reactor central region. Moreover, the origin of the local XYZ coordinate system of this PMU is defined as the origin of the global XYZ coordinate system. By this definition, the placement error of this PMU is automatically eliminated when the target trajectory in the global XYZ system is calculated.

Let us consider the case where three PMU are set along the target path as shown in Fig. 2. Target flies in the vacuum tube and goes into the reactor chamber filled with residual gas when the shutter opens. The broken rectangles represent PMUs. If the flying target crosses the ray of light monitored by the optical sensor in the PMU, a trigger signal is emitted to the orthogonal (X and Y directions) pulsed laser system to irradiate the target in the central part of the PMU represented by the red box in Fig. 2. The XYZ coordinate system is introduced to the local coordinate system in all the PMUs and the global coordinate system as shown in Fig. 2. The corresponding axes are parallel.

Furthermore, a position measurement method using Arago spot is employed in the PMU. The position measurement method was proposed by Petzoldt *et al.* [8]. The Arago spot, also known as the Poisson spot, is a tiny bright spot that appears at the central portion of the geometrical shadow of the spherical object as shown in Fig. 3. This is because the diffracted laser waves from the edge of the sphere interfere constructively on the central axis of the sphere.

The accuracy of the position measurement method using Arago spot is less than 0.3 μm for a stationary spherical object [9]. However, a pulsed laser irradiation with a duration less than 10 ns enables us to measure the position of the injected target of 100 m/s within a measurement accuracy of 1 μm. This value satisfies the criterion of the position measurement error of the flying target. Thus, an orthogonal laser irradiation on the target enables 3-dimensional position measurement [4, 5]. Moreover, image compression using cylindrical lens can convert a 2D Arago spot image into a 1D image [5, 10]. This techniques reduces the amount of image data and enables real-time data processing for trajectory calculation.

Not only a parallel illumination but also convergent and divergent illuminations can give Arago spot in the shadow of the spherical target as shown in Fig. 4. The center of the spherical target, focal point F of the laser beam, and the Arago spot lie on a line [11]. If the spherical target is irradiated by a convergent or divergent laser beam, the shadow is magnified and the Arago spot of the spherical target can easily be observed at a large distance, thereby enabling the position measurement of the flying target from outside the reactor [4, 5].

The cross-section of the PMU is shown in Fig. 5. The procedure to obtain the local coordinate of the target in a PMU is as follows: First, the directions of the pulsed orthogonal divergent laser beams are calibrated in the PMU without target. Spherical objects (S_1 and S_2) and mark points (P_{F1} and P_{F2}) are placed to cross two base lines (S_1P_{F1} and S_2P_{F2}) at a designated point. In the case of a PMU at the reactor center, the designated point must be located in the reactor central region of 2-mm diameter. The cross point of two lines, S_1P_{F1} and S_2P_{F2}, defines the origin, O_{local}, of the local coordinate system of the PMU. Moreover, the position of O_{local} is defined as the position of PMU. Lines S_1P_{F1} and S_2P_{F2} define the x-axis and the y-axis, respectively. Two spherical objects, S_1 and S_2, placed outside of the reactor windows are then irradiated by the laser beams. To generate the Arago spot at P_{F1} and P_{F2}, the focal points of Laser1 and Laser2, F_1 and F_2, must be adequately adjusted (calibrated).

Consider that the target T is injected in the z-direction and irradiated by the lasers. Two Arago spots appear at P_1 and P_2. The cross point of lines F_1P_1 and F_2P_2 determines the position of the target T in Fig. 5. Therefore, we obtain the local coordinate of the target $(x, y)_{local}$ at laser irradiation. If the focal point moves from F_1 to F'_1, then the Arago spot of spherical object S_1 moves from P_{F1} to P'_{F1}. We can monitor the movement of the focal point in the operation phase and then calibrate the position coordinate.

In a 3-dimensional case, two Arago spots, P_1 and P_2, appear in the plane parallel to the YZ plane and other plane parallel to the XZ plane, respectively. We obtain a 3-dimensional local coordinate of the target $(x, y, z)_{local}$ at laser irradiation.

We defined the position of PMU as the cross point of two lines in Fig. 5. These lines were defined by spherical objects (S_1 and S_2) and mark points (P_{F1} and P_{F2}). If these physical parts of the PMU are separated from the laser fusion reactor chamber and reactor structure materials which can be deformed, then the stress distortion and thermal expansion of the reactor chamber do not affect the position of the PMU.

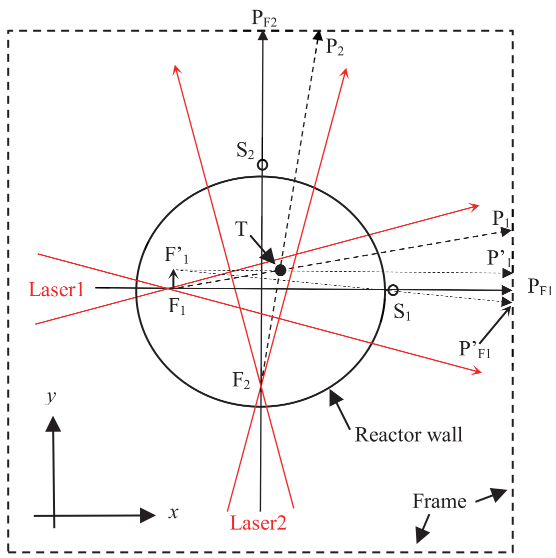

Fig. 5 Cross-section of PMU using Arago spot and pulsed orthogonal divergent laser beam. F_1 and F_2 indicate the focal points of Laser1 and Laser2, respectively; T indicates spherical target, and S_1 and S_2 indicate spherical objects.

3. Target Trajectory Calculation in Vacuum

We first consider the case where a test target is injected into vacuum under gravity, to evaluate the placement error of PMU. The analysis in this section can be applied to the injection module test in a dry wall chamber [12] and to the case of HIBALL-II [13] where high-vacuum is required. Figure 6 shows three PMUs and the trajectory of the test target which moves to the z-direction. For simplicity, we consider the case where three PMUs are expected to be placed along 5-m injection path with 2.5-m interval.

We set O_{local} of PMU_3 within the reactor central region and define it as the origin, O_{global}, of the global coordinate system (0, 0, 0) i.e. the reactor center. The placement error of PMU_3 is automatically eliminated for the trajectory calculation in this global coordinate system. Meanwhile, there are no material parts of PMUs in the reactor chamber. As shown in Fig. 5, the remote laser (Arago spot) measurements of the position of the injected target are performed.

The other PMUs are placed along the target path on a line. We define that the line between the origin of the global coordinate system (0, 0, 0) and the local origin of the most distant PMU as the z-axis. This procedure means that the x- and y-components of the placement errors of the most distant PMU, PMU_1, are automatically eliminated. In this section, we also assume that the z-component of the placement error is eliminated.

A remaining PMU, PMU_2, has its own unknown

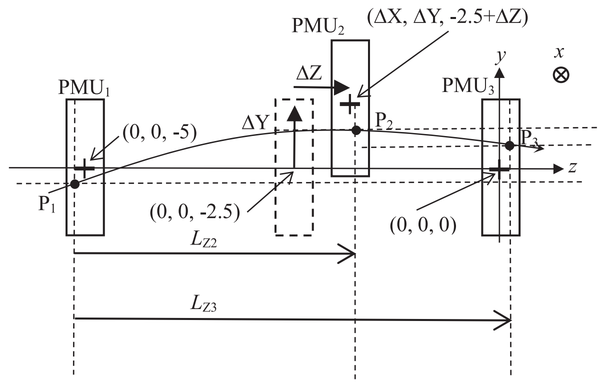

Fig. 6 PMU and test target trajectory.

Table 1 Local coordinate and global coordinate.

Point	Local coordinate	Global coordinate
P_1	(dx_1, dy_1, dz_1)	$(dx_1, dy_1, -5 + dz_1)$
P_2	(dx_2, dy_2, dz_2)	$(dx_2 + \Delta X, dy_2 + \Delta Y, -2.5 + dz_2 + \Delta Z)$
P_3	(dx_3, dy_3, dz_3)	(dx_3, dy_3, dz_3)

placement error which must be evaluated to calculate the trajectory of the injected target. First, we evaluate this placement error in the global coordinate system.

The local origin of each PMU, O_{local}, represented as crosses in the center of the boxes, are (0, 0, −5), (ΔX, ΔY, $-2.5 + \Delta Z$), (0, 0, 0) in the global coordinate, respectively. Here ΔX, ΔY, and ΔZ are unknown placement errors of the second PMU. The trajectory of the test target is plotted on a solid curve (parabolic curve) as shown in Fig. 6. When the test target is detected in the PMU (solid box), the target is irradiated by an orthogonal pulsed laser beams at points P_1, P_2, and P_3. The local coordinates of the points and the irradiation time measured in PMU are $(dx_1, dy_1, dz_1, 0)$, (dx_2, dy_2, dz_2, T_2), and (dx_3, dy_3, dz_3, T_3), where dx_{i}, dy_{i}, dz_{i} (i = 1, 2, 3) are the local coordinates of the target in the i-th PMU. We assume that $|dx_{\mathrm{i}}|$ and $|dz_{\mathrm{i}}|$ are smaller than 1 mm. The time variable can be a global variable by supplying common clock pulses to each PMU. The local coordinates can be transformed into global coordinates $(dx_1, dy_1, -5+dz_1, 0)$, $(dx_2+\Delta X, dy_2+\Delta Y, -2.5+dz_2+\Delta Z, T_2)$, and (dx_3, dy_3, dz_3, T_3) as shown in Table 1 if the placement errors ΔX, ΔY, and ΔZ are determined.

The equation of motion for the z-direction is:

$$F_z = m\frac{dv_z}{dt} = 0, \tag{1}$$

where m is the mass of the target. The z-component of the velocity and the position of the test target are:

$$v_z(t) = v_{z0}, \tag{2}$$

$$z(t) = v_{z0}t + (-5 + dz_1), \tag{3}$$

where v_{z0} is the z-component of the target velocity at $t = 0$. Similar equation holds for the x-direction. The displacement of the z-direction L_{z2} $(= z(T_2) - z(0))$ and L_{z3} $(= z(T_3) - z(0))$ are proportional to the flight time T_2 and T_3, we have:

$$\frac{L_{z3}}{L_{z2}} = \frac{(5 + dz_3 - dz_1)}{(2.5 + \Delta Z + dz_2 - dz_1)} = \frac{v_{z0}T_3}{v_{z0}T_2}. \tag{4}$$

We obtain an equation for ΔZ. Here, an unknown parameter i.e. placement error ΔZ, is determined by solving (4) to obtain:

$$\Delta Z = \frac{T_2(5 + dz_3 - dz_1) - T_3(2.5 + dz_2 - dz_1)}{T_3}. \tag{5}$$

Similarly, the ratio of the displacement of the x-directions is:

$$\frac{L_{x3}}{L_{x2}} = \frac{(dx_3 - dx_1)}{(\Delta X + dx_2 - dx_1)} = \frac{v_{x0}T_3}{v_{x0}T_2}. \tag{6}$$

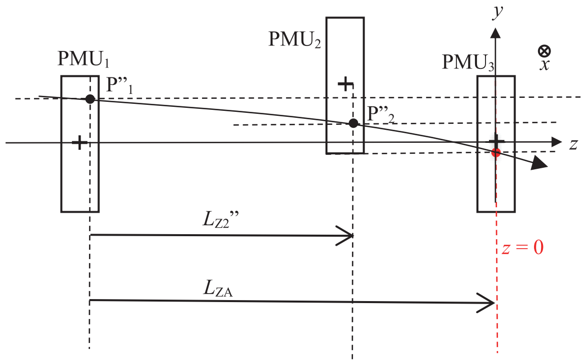

Fig. 7 Trajectory of newly injected target.

The parameter ΔX is determined by solving (6) to give:

$$\Delta X = \frac{T_2(dx_3 - dx_1) - T_3(dx_2 - dx_1)}{T_3}. \tag{7}$$

The equation of motion for the y-direction is:

$$F_y = m\frac{dv_y}{dt} = -mg, \tag{8}$$

where g is the gravitational acceleration. The y-component of the velocity and the position of the test target are:

$$v_y(t) = -gt + v_{y0}, \tag{9}$$

$$y(t) = -\frac{g}{2}t^2 + v_{y0}t + dy_1, \tag{10}$$

where v_{y0} is the y-component of the target velocity at $t = 0$. The displacement of the y-direction L_{y2} $(= y(T_2)-y(0))$ and L_{y3} $(= y(T_3) - y(0))$ are:

$$L_{y3} = -\frac{gT_3^2}{2} + v_{y0}T_3 = dy_3 - dy_1, \tag{11}$$

$$L_{y2} = -\frac{gT_2^2}{2} + v_{y0}T_2 = \Delta Y + dy_2 - dy_1. \tag{12}$$

The y-component of the initial velocity v_{y0} and the placement error ΔY are:

$$v_{y0} = \frac{g}{2}T_3 + \frac{dy_3 - dy_1}{T_3}, \tag{13}$$

$$\Delta Y = \frac{g}{2}T_2(T_3 - T_2) + \frac{T_2(dy_3 - dy_1) - T_3(dy_2 - dy_1)}{T_3}. \tag{14}$$

After determining the placement errors of the second PMU (ΔX, ΔY, ΔZ), we can calculate the trajectory of the newly injected target. If a target is newly injected, the data obtained in the PMUs are $(dx_1'', dy_1'', dz_1'', 0)$ and $(dx_2'', dy_2'', dz_2'', T_2'')$. We must calculate the arrival time and arrival position $(dx_3'', dy_3'', dz_3'' = 0, T_A)$ which is indicated by a red point in Fig. 7.

The local coordinates can then be transformed into global coordinates $(dx_1'', dy_1'', -5 + dz_1'', 0)$ and $(dx_2'' + \Delta X, dy_2'' + \Delta Y, -2.5 + dz_2'' + \Delta Z, T_2'')$ as shown in Table 2.

Table 2 Local coordinate and global coordinate.

Point	Local coordinate	Global coordinate
P''_1	(dx''_1, dy''_1, dz''_1)	$(dx''_1, dy''_1, -5 + dz''_1)$
P''_2	(dx''_2, dy''_2, dz''_2)	$(dx''_2 + \Delta X, dy''_2 + \Delta Y, -2.5 + dz''_2 + \Delta Z)$

The displacement of the z-direction of the target, $L_{z2''}$ ($= z(T''_2) - z(0)$) and L_{zA} ($= z(T_A) - z(0) = -z(0)$), are proportional to the flight time T''_2 and the arrival time T_A:

$$\frac{L_{zA}}{L_{z2''}} = \frac{(5 - dz''_1)}{(2.5 + \Delta Z + dz''_2 - dz''_1)} = \frac{v''_{z0} T_A}{v''_{z0} T''_2}, \tag{15}$$

where v''_{z0} is the z-component of the initial velocity at $t = 0$. Thus:

$$v''_{z0} = \frac{(2.5 + \Delta Z + dz''_2 - dz''_1)}{T''_2}. \tag{16}$$

Similarity, we have:

$$v''_{x0} = \frac{(\Delta X + dx''_2 - dx''_1)}{T''_2}, \tag{17}$$

$$v''_{y0} = \frac{g}{2} T''_2 + \frac{(\Delta Y + dy''_2 - dy''_1)}{T''_2}. \tag{18}$$

From (15), the arrival time of the target T_A at $z = 0$ (shot plane) is:

$$T_A = \frac{(5 - dz''_1)}{v''_{z0}} = \frac{T''_2 (5 - dz''_1)}{(2.5 + \Delta Z + dz''_2 - dz''_1)}. \tag{19}$$

The arrival positions of the target at the shot ($t = T_A$) are:

$$dz''_3 = v''_{z0} T_A + (-5 + dz''_1) = 0, \tag{20}$$

$$\begin{aligned} dx''_3 &= v''_{x0} T_A + dx''_1 = \frac{T_A(\Delta X + dx''_2 - dx''_1)}{T''_2} + dx''_1 \\ &= \frac{(5 - dz''_1)(\Delta X + dx''_2 - dx''_1)}{(2.5 + \Delta Z + dz''_2 - dz''_1)} + dx''_1, \end{aligned} \tag{21}$$

$$\begin{aligned} dy''_3 &= -\frac{g}{2} T_A^2 + v''_{y0} T_A + dy''_1 \\ &= \frac{g T_A (T''_2 - T_A)}{2} \\ &\quad + \frac{(5 - dz''_1)(\Delta Y + dy''_2 - dy''_1)}{(2.5 + \Delta Z + dz''_2 - dz''_1)} + dy''_1. \end{aligned} \tag{22}$$

If PMU_2 approaches PMU_1, then the trajectory is determined earlier. In practice, the position measurement error of the target exists and causes an error in the arrival time and arrival position. Therefore, the allowable calculation error restricts the minimum distance between PMU_1 and PMU_2.

4. Target Trajectory Calculation in Residual Gas

We consider the case where the target is injected into gas under gravity. In the laser fusion reactor concepts, liquid-metal wall is proposed to breed tritium and to protect the reactor wall against pulsed X-ray and plasma of a microexplosion [14]. The evaporation and condensation of the liquid metal occurs in the reactor chamber. Meanwhile, the vapor of the liquid metal exists as a residual gas. This residual gas acts as a frictional force on the target motion [15–17] and causes delay in the arrival time of the target [18, 19]. Although the temperature and number density of the residual gas changes instantaneously with time after microexplosion, repetitive target injection is scheduled after steady state of the gas is realized in the reactor. The temperature and number density of the steady-state residual gas are functions of the reactor wall temperature in the long term. In a reactor operation phase, the temperature of the reactor wall varies very slowly with time. Hence, we assume that the condition of the residual gas i.e. the temperature and number density do not change in each injection.

The measured data obtained in the PMUs are (dx_1, dy_1, dz_1, 0), (dx_2, dy_2, dz_2, T_2), and (dx_3, dy_3, dz_3, T_3), where dx_i, dy_i, dz_i (i = 1, 2, 3) are the local coordinates of the target in the i-th PMU, respectively. It was found experimentally that the frictional force for a small velocity is proportional to the velocity [20]. In this case, the z-component of the equation of target motion is:

$$F_z = m \frac{dv_z}{dt} = -k v_z, \tag{23}$$

where k is a gas friction coefficient. Gas friction coefficient is a function of temperature, number density of the residual gas, and radius of the target [15–17]. The velocity of the target is:

$$v_z(t) = v_{z0} \exp\left(-\frac{kt}{m}\right) = v_{z0} \exp(-Bt), \tag{24}$$

where $B = k/m$ and v_{z0} is the z-component of the initial velocity of the target at $t = 0$. Integrating (24), the position of the target is:

$$z(t) = \frac{v_{z0}}{B}[1 - \exp(-Bt)] + (-5 + dz_1). \tag{25}$$

The flight distance L_{z2} ($= z(T_2) - z(0)$) and L_{z3} ($= z(T_3) - z(0)$) are functions of the flight time T_2 and T_3:

$$L_{z3} = 5 + dz_3 - dz_1 = \frac{v_{z0}}{B}[1 - \exp(-BT_3)], \tag{26}$$

$$\begin{aligned} L_{z2} &= 2.5 + \Delta Z + dz_2 - dz_1 \\ &= \frac{v_{z0}}{B}[1 - \exp(-BT_2)]. \end{aligned} \tag{27}$$

However, two unknown parameters, ΔZ and B, must be determined. If we make two independent test injections (a and b), we obtain:

$$\begin{aligned} \frac{L_{z3a}}{L_{z2a}} &= \frac{(5 + dz_{3a} - dz_{1a})}{(2.5 + \Delta Z + dz_{2a} - dz_{1a})} \\ &= \frac{1 - \exp(-BT_{3a})}{1 - \exp(-BT_{2a})}, \end{aligned} \tag{28}$$

$$\begin{aligned} \frac{L_{z3b}}{L_{z2b}} &= \frac{(5 + dz_{3b} - dz_{1b})}{(2.5 + \Delta Z + dz_{2b} - dz_{1b})} \\ &= \frac{1 - \exp(-BT_{3b})}{1 - \exp(-BT_{2b})}. \end{aligned} \tag{29}$$

From these two equations, we can determine the two unknown parameters simultaneously.

Similarly, ΔX is obtained from following equation:

$$\frac{(dx_3 - dx_1)}{(\Delta X + dx_2 - dx_1)} = \frac{1 - \exp(-BT_3)}{1 - \exp(-BT_2)}. \tag{30}$$

We have:

$$\Delta X = \frac{1 - \exp(-BT_2)}{1 - \exp(-BT_3)}(dx_3 - dx_1) - (dx_2 - dx_1). \tag{31}$$

The y-component of the equation of motion, velocity, and position of the target are:

$$F_y = m\frac{dv_y}{dt} = -mg - kv_y, \tag{32}$$

$$\begin{aligned} v_y(t) &= \left(v_{y0} + \frac{mg}{k}\right)\exp\left(-\frac{kt}{m}\right) - \frac{mg}{k} \\ &= \left(v_{y0} + \frac{g}{B}\right)\exp(-Bt) - \frac{g}{B}, \end{aligned} \tag{33}$$

$$y(t) = \left(\frac{g}{B^2} + \frac{v_{y0}}{B}\right)[1 - \exp(-Bt)] - \frac{g}{B}t + dy_1. \tag{34}$$

Here v_{y0} is the y-component of the initial velocity of the target at $t = 0$. The displacement of the y-direction of the target, L_{y2} $(= y(T_2) - y(0))$ and L_{y3} $(= y(T_3) - y(0))$, are functions of the flight time T_2 and T_3. Thus:

$$\begin{aligned} L_{y3} &= \left(\frac{g}{B^2} + \frac{v_{y0}}{B}\right)[1 - \exp(-BT_3)] - \frac{g}{B}T_3 \\ &= dy_3 - dy_1, \end{aligned} \tag{35}$$

$$\begin{aligned} L_{y2} &= \left(\frac{g}{B^2} + \frac{v_{y0}}{B}\right)[1 - \exp(-BT_2)] - \frac{g}{B}T_2 \\ &= \Delta Y + dy_2 - dy_1. \end{aligned} \tag{36}$$

We have:

$$v_{y0} = \frac{gT_3 + B(dy_3 - dy_1)}{1 - \exp(-BT_3)} - \frac{g}{B}, \tag{37}$$

$$\begin{aligned} \Delta Y = &\frac{1 - \exp(-BT_2)}{1 - \exp(-BT_3)} \cdot \frac{g}{B}T_3 - \frac{g}{B}T_2 \\ &+ \frac{1 - \exp(-BT_2)}{1 - \exp(-BT_3)}(dy_3 - dy_1) \\ &- (dy_2 - dy_1). \end{aligned} \tag{38}$$

After determining ΔX, ΔY, ΔZ, and B, we can calculate the trajectory of the newly injected target. If a target is newly injected, the local coordinates of the injected target are transformed into global coordinates (dx_1'', dy_1'', $-5+dz_1''$, 0) and ($dx_2'' + \Delta X$, $dy_2'' + \Delta Y$, $-2.5 + dz_2'' + \Delta Z$, T_2''). The displacement of z-direction of the target is:

$$\begin{aligned} z(T_2'') - z(0) &= (2.5 + \Delta Z + dz_2'' - dz_1'') \\ &= \frac{v_{z0}''[1 - \exp(-BT_2'')]}{B}. \end{aligned} \tag{39}$$

We can determine the initial velocities as:

$$v_{z0}'' = \frac{B(2.5 + \Delta Z + dz_2'' - dz_1'')}{1 - \exp(-BT_2'')}, \tag{40}$$

$$v_{x0}'' = \frac{B(\Delta X + dx_2'' - dx_1'')}{1 - \exp(-BT_2'')}, \tag{41}$$

$$v_{y0}'' = \frac{B(\Delta Y + dy_2'' - dy_1'') + gT_2''}{1 - \exp(-BT_2'')} - \frac{g}{B}. \tag{42}$$

The arrival time T_A of the target at the shot is derived from following equation:

$$5 - dz_1'' = \frac{v_{z0}''}{B}[1 - \exp(-BT_A)]. \tag{43}$$

From this equation, we can determine the arrival time of the target T_A at $z = 0$ (shot plane) as:

$$T_A = -\frac{1}{B}\log\left[\frac{-B}{v_{z0}''}(5 - dz_1'') + 1\right]. \tag{44}$$

The arrival positions of the target are:

$$dz_3'' = \frac{v_{z0}''}{B}[1 - \exp(-BT_A)] + (-5 + dz_1'') = 0, \tag{45}$$

$$\begin{aligned} dx_3'' &= \frac{v_{x0}''}{B}[1 - \exp(-BT_A)] + dx_1'' \\ &= \frac{v_{x0}''}{v_{z0}''}\frac{v_{z0}''}{B}[1 - \exp(-BT_A)] + dx_1'' \\ &= \frac{(5 - dz_1'')(\Delta X + dx_2'' - dx_1'')}{(2.5 + \Delta Z + dz_2'' - dz_1'')} + dx_1'', \end{aligned} \tag{46}$$

$$\begin{aligned} dy_3'' = &\frac{(5 - dz_1'')(\Delta Y + dy_2'' - dy_1'')}{(2.5 + \Delta Z + dz_2'' - dz_1'')} + dy_1'' \\ &+ \frac{1 - \exp(-BT_A)}{1 - \exp(-BT_2'')} \cdot \frac{gT_2''}{B} - \frac{gT_A}{B}. \end{aligned} \tag{47}$$

We can extend results obtained in this section to the generalized case where the number N of PMUs is larger than 3. The placement errors of N-2 PMUs and gas friction coefficient are evaluated by two test injections experimentally as well.

5. Generalization of Equation of Motion

We assumed that v_{z0} is 100 m/s and radius of the reactor is 5 m. In order to pass through the shot region of a 15-mm diameter sphere, $|v_{x0}|$ and $|v_{y0}|$ must be smaller than 0.15 m/s. As v_{z0} is much larger than v_{x0} and v_{y0}, in some cases the friction term in the equation of motion may not be the same function form. Generally, a smooth function of velocity v such as frictional force is approximated locally by a line as $-k(v)v - S(v)$. Here, $k(v)$ and $S(v)$ are newly introduced constant parameters that can be determined experimentally. The equation of motion for the z-direction is not (23) but a general form, given by:

$$F_z = m\frac{dv_z}{dt} = -S - kv_z. \tag{48}$$

A similar analysis for (32) can be applied to (48). The velocity and position of the injected target are:

$$\begin{aligned} v_z(t) &= \left(v_{z0} + \frac{S}{k}\right)\exp\left(-\frac{kt}{m}\right) - \frac{S}{k} \\ &= (v_{z0} + D)\exp(-Bt) - D, \end{aligned} \tag{49}$$

$$\begin{aligned} z(t) = &\frac{1}{B}(v_{z0} + D)[1 - \exp(-Bt)] \\ &- Dt - 5 + dz_1, \end{aligned} \tag{50}$$

where $D = S/k$, $B = k/m$ and v_{z0} is the z-component of the initial velocity of the target at $t = 0$. The flight distances, L_{z2} and L_{z3}, are:

$$\begin{aligned} L_{z2} &= 2.5 + \Delta Z + dz_2 - dz_1 \\ &= \frac{(v_{z0} + D)}{B}[1 - \exp(-BT_2)] - DT_2, \end{aligned} \tag{51}$$

$$\begin{aligned} L_{z3} &= 5 + dz_3 - dz_1 \\ &= \frac{(v_{z0} + D)}{B}[1 - \exp(-BT_3)] - DT_3. \end{aligned} \tag{52}$$

Three unknown parameters, ΔZ, $B(= k/m)$, and $D(= S/k)$ are determined simultaneously by three equations from three independent injections (a, b and c) as:

$$\frac{(5 + dz_{3a} - dz_{1a}) + DT_{3a}}{(2.5 + \Delta Z + dz_{2a} - dz_{1a}) + DT_{2a}} = \frac{1 - \exp(-BT_{3a})}{1 - \exp(-BT_{2a})}, \tag{53}$$

$$\frac{(5 + dz_{3b} - dz_{1b}) + DT_{3b}}{(2.5 + \Delta Z + dz_{2b} - dz_{1b}) + DT_{2b}} = \frac{1 - \exp(-BT_{3b})}{1 - \exp(-BT_{2b})}, \tag{54}$$

$$\frac{(5 + dz_{3c} - dz_{1c}) + DT_{3c}}{(2.5 + \Delta Z + dz_{2c} - dz_{1c}) + DT_{2c}} = \frac{1 - \exp(-BT_{3c})}{1 - \exp(-BT_{2c})}. \tag{55}$$

After determining the unknown parameters, the initial velocity of the newly injected target can be determined. The flight distance is:

$$\begin{aligned} z(T_2'') - z(0) &= (2.5 + \Delta Z + dz_2'' - dz_1'') \\ &= \frac{(v_{z0}'' + D)[1 - \exp(-BT_2'')]}{B} - DT_2''. \end{aligned} \tag{56}$$

Thus, the initial velocity is:

$$v_{z0}'' = \frac{B(2.5 + \Delta Z + dz_2'' - dz_1'' + DT_2'')}{1 - \exp(-BT_2'')} - D. \tag{57}$$

The arrival time, T_A, is obtained by solving the equation:

$$5 - dz_1'' = \frac{(v_{z0}'' + D)}{B}[1 - \exp(-BT_A)] - DT_A. \tag{58}$$

The analysis in this section can be applied to the following examples.

Consider the case where the z−axis of the global coordinate makes an angle θ with the horizontal. The equation of motion for the z-direction is:

$$\begin{aligned} F_z = m\frac{dv_z}{dt} &= -S - mg\sin\theta - kv_z \\ &= -S_1 - kv_z. \end{aligned} \tag{59}$$

An adjusting constant parameter $S_1(= S + mg\sin\theta)$ is introduced. The effect of the acceleration can be included (treated) in the term S_1. This parameter can be also determined experimentally. A similar analysis for (48) can be applied to (59).

Consider again the case where the steady flow of a gas exists in the reactor chamber because of the pumping. The friction term is then represented as:

$$\begin{aligned} F_i &= -k_i(v_i - v_{\mathrm{wind(i)}}) = k_i v_{\mathrm{wind(i)}} - k_i v_i \\ &= -S_i - k_i v_i, \quad (i = x, y, z) \end{aligned} \tag{60}$$

where $v_{\mathrm{wind(i)}}$ is the velocity of the steady flow of the residual gas, $S_i(= -k_i v_{\mathrm{wind(i)}})$ is a newly introduced constant parameter which can be determined experimentally.

6. Conclusion

The setting and mechanism of PMU and its calibration method were presented. A calculation method was also presented for the trajectory of the horizontally injected spherical laser fusion energy target. In a residual gas, the placement error of PMU and gas friction coefficient were experimentally evaluated by two test injections. After determining the placement error and the gas friction coefficient, the arrival position and arrival time of the newly injected fusion target at the shot plane were calculated by a simple algebraic arithmetic with the local coordinate and time data in the PMU.

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