Anomalous transport is a paradigm for analysis of both Power Balance (PB) and Heat Pulse Propagation (HPP) in magnetically confined plasmas including so called non-local transport (NLT) phenomena (core heating as a reaction on the edge cooling). One of the alternative explanations of the NLT proposes to take into account the plasma motion caused by the perturbation of the force balance during HPP. First results of numerical modeling of the electron heat transport by the ASTRA code confirm the viability of this approach.

1. Introduction

Non-local transport (NLT) phenomena (core plasma heating appearing as a reaction on its cooling at the edge) attracts attention of the fusion society for decades [1–5]. Up to now there is no convincing explanation of this effect, though a number of models have been proposed, see reviews in [1, 2] and more recent approaches [3, 4]. The term NLT reflects the inability of simple diffusive models to reproduce the inversion of the heat/cooling pulse, while their potentials have not yet been exhausted. In such models reviewed in [1, 2] and briefly mentioned in [3–5] and in seemingly more advanced ones involving the turbulence etc. the energy exchange between the plasma and the magnetic field is completely ignored, while it can be essential [2, 6, 7] in tokamaks and stellarators. In such systems, the plasma-field interaction is the key element to provide the plasma confinement. Large external forces combined with small plasma motion can produce a significant energy transfer acting as an additional heat source and strongly affecting NLT [2, 6, 7]. This exchange has not been incorporated or even evaluated in [3–5] and similar studies [8, 9]. Here we consider this mechanism and take into account the plasma displacement caused by the perturbation of the force balance during Heat Pulse Propagation (HPP).

The goal of this work is modeling of NLT phenomena numerically for data observed after TESPEL injection into low-density LHD plasmas [5]. The results of simulations by means of the ASTRA transport code [10, 11] and their analysis are presented for the cold edge pulse propagation with account of the energy transfer [2, 6, 7] between magnetic field and plasma.

2. Formulation of the Problem

We consider such motions that the inertia in the force balance equation for the plasma

\[ m n \frac{dv}{dt} = -\nabla p + j \times B, \tag{1} \]

can be neglected. Then the reduced consequence of equation (1) can be used where the pressure expansion is counteracted by the Ampere’s force:

\[ \nabla p + j \times B. \tag{2} \]

The transition to a new equilibrium state can occur much faster (on the Alfven time scale, tens of μs) than characteristic times of plasma perpendicular transport.

A local drop in the pressure \( p \) due to the pellet-produced cooling at the plasma edge must be accompanied by the change in the current density \( \delta j \). Then a response \( \delta B \) in the magnetic field at a finite distance would be a completely natural event. It reaches the center with some delay, because the plasma, as a good conductor, prevents immediate penetration of \( \delta B \) into the plasma core. It would be rather slow due to the skin effect in a conductor that could not move. However, the plasma is a mobile gas, not a solid body. In tokamaks and stellarators, its mass is extremely small. Therefore, a slightest imbalance of the forces is sufficient for providing a finite velocity and the propagation of the plasma pulse.
The standard transport analysis [1, 3–5] postulates an absence of a plasma perpendicular velocity when the equilibrium is thus perturbed. This is partially justified by the presence of strong toroidal field preventing large radial displacement \( v \cdot \delta r \) in a flux-conserving plasma in tokamaks and stellarators. During the plasma evolution triggered by the pellet injection, a relative change in the plasma volume \( \Delta Vol/Vol \) within a given magnetic surface calculated by equation (1) in [10], must be, at best, of the order of \( 10^{-3} \) [2], which can be hardly detected in real experimental conditions. However, in [2, 6, 7] it was proved that such small deformations accompanying the plasma equilibrium perturbations should be taken into account in the energy balance equation for electrons

\[
\frac{3}{2} \frac{\partial nT_e}{\partial t} + \nabla \cdot \left( -\chi_e n \nabla T_e + \frac{5}{2} nT_e v \right) = S = j \cdot E + P_e.
\]

(3)

Here, \( n, T_e \) are the plasma density and electron temperature, \( \chi_e \) is the electron heat diffusivity, \( S \) is the source/sink term with \( P_e \) describing auxiliary heating/cooling. The plasma velocity \( v \) explicitly appears in the convective term, but it is also hidden in the source term \( S \). Indeed, for fast plasma motions when the magnetic flux conservation approach is only slightly violated, the Ohm’s law

\[
E + v \times B = \eta j.
\]

(4)

combined with equation (2) yields

\[
j \cdot E = \eta j^2 + v \cdot \nabla p.
\]

(5)

Here, along with the Joule heating due to \( \eta \cdot j^2 \) (essential in tokamaks) the additional contribution \( v \cdot \nabla p \) appears [2].

The plasma velocity \( v \) cannot be derived from equation (1) due to small plasma mass density \( m_i \cdot n \) and imprecise knowledge of other terms there. A validated model for \( v \) evaluation from (4) or similar relations with better description of \( j(E) \) has not yet been developed. Therefore, to make a step for the proof of principle [2] we analyze the possibility of reproducing the main features of the NLT phenomena during TESPEL injection into the LHD plasma shot #49708 (Fig. 1 in [5]) by simulations of the electron energy balance equation (3) with a source term (5) and ad hoc time and space evolution of the radial plasma velocity \( v(\rho, t) \).

In LHD, the NLT phenomena have been observed in a wide range of experimental conditions and plasma parameters: in ECR, NBI and NBI+ECR heated plasma, which demonstrated the independence of NLT on the plasma heating supply; with different sizes of TESPEL and corresponding level of the plasma perturbation at fixed plasma density prior to the injection [5, 8, 9]; with different plasma densities at fixed TESPEL size that allowed to see the tendency of the transition from the core temperature “reversal” to the diffusive picture of the cold pulse propagation

Fig. 1 Profiles of electron density (a), electron heat diffusivity (b), electron temperature (c), \( \mu(\rho) = 1/q(\rho) \) and \( i(\rho) \) (d), TESPEL ablation rate (e) on the normalized minor radius \( \rho \) used in ASTRA simulations. LHD shot #49708.
as the plasma density increased [8]. With powerful set of
diagnostics, the LHD data provide a good basis for analy-
sis.

In the absence of the essential net toroidal current in
LHD plasmas we can neglect the Joule heating term $\eta J^2$ in
the electron energy balance (3). It is also clear that the cold
pulse propagation and its amplitude reversal in the plasma
core cannot be attributed to the changes in this heating term
related to the current profile alterations [2, 3], which sim-
plies the analysis.

One of the main aims of the work was to simulate $\Delta \text{Vol}/\text{Vol}$ level at $v \neq 0$, to compare it with estimates in [2]
and see whether the $\delta T_e$ propagation and amplitude can be
affected by small radial displacements of the plasma. The
simulation model has to calculate the spatial and tempo-
ral evolution of $\Delta \text{Vol}/\text{Vol}$ during HPP. For this purpose, the
evolution of the magnetic equilibrium during HPP has to
be considered.

3. Model

From the latter reason, the ASTRA code (Automated
System for TRansport Analysis) is used here for analy-
sis based on equations (3) and (5) and solving equilib-
rium equation (2). Details of the algorithm, including
the full scheme summarized in the flow chart are published
in [10,11]. Some simplifications and assumptions have
been made prompted by experiments, and some to facili-
tate computations. The main elements of the model are:
(i) Density evolution is not simulated and the steady-state
density profile $N_i(\rho = r/a) = \text{const}(t)$ interpolated from
YAG Tomson Scattering (TS) experimental data prior to
TESPEL injection is used (Fig. 1a);
(ii) The NBI and ECRH input power profiles simulated in
Ref. [5] are used for calculation of the steady-state profile
of $N_e^{\text{inp}}(\rho)$ (Fig. 1b) to reproduce the interpolation of
electron temperature profiles (Fig. 1c) measured by TS and
2nd harmonic of Electron Cyclotron Emission (ECE) LHD
diagnostics prior to the TESPEL injection [5]. The ion
temperature was not measured in the LHD shot #49708
and is assumed $T_i(\rho) = T_e(\rho)/3 = \text{const}(t)$ according to
measurements of the core ion temperature in similar LHD
shots [12]. It is assumed that TESPEL injection do not dis-
turb the power heating terms in the plasma energy balance;
(iii) The ASTRA7 code is run in the prescribed (fixed)
boundary mode with the internal (not SPIDER) equilib-
rium solver (see [10, 11]. Since the LHD equilibrium is
not implemented in the ASTRA code, we used “equiv-
alent” magnetic equilibrium of tokamak with “effective”
plasma current of about 0.7 MA and with LHD parameters
in shot #49708 [5]: major radius at the magnetic axis,
$R = 3.5$ m; an average minor radius, $a = 0.58$ m; mag-
netic field at the axis, $B_t = 2.83$ T. The “effective” plasma
current value provides a close range of $\mu(\rho) = 1/q(\rho)$ in
the tokamak-like simulation model and the LHD rotational
transform $\mu(\rho)$ profiles shown in Fig. 1d. The profile of the
rotational transform will affect the distributions of the equi-
brum quantities, but cannot reverse the diffusion fluxes.
Therefore, if we will succeed in demonstrating the cold
pulse inversion, this should be attributed to other factors.
The “effective” Joule heating $\eta J^2$ term is not included in
the electron energy balance equations (3) and (5) because
actually it must be small in the current-free LHD plasma;
(iv) To simulate the electron temperature drop during
TESPEL injection we use the measured pellet ablation rate
profile $N(\rho)$ shown in Fig. 1e. According to calculations
in Ref. [13], the density rise and temperature drop forma-
tions within the helical domain close to the ablation region
during TESPEL injection are lasting about $\Delta t_{\text{inj}} \approx 2$ ms.
The cooling term in equations (3) and (5) is modelled by
$\nu_e = A N(\rho)$ during $\Delta t_{\text{inj}}$ and the coefficient $A$ is
adjusted to reproduce the measured electron temperature
drop after TESPEL injection (see Fig. 2a).
4. Results of Simulations and Discussion

At the first step, we performed simulations with \( v \equiv 0 \) to test our approach against the results of the standard transport model with the steady-state values of the electron heat diffusivity \( \chi_e^{PB} (\rho) \). A comparison of the measured (ECE) electron temperature evolutions with simulated ones is shown in Fig. 2a. First of all, one can see that the computed profiles reproduce the peripheral drop of the electron temperature just after the TESPEL injection. The subsequent diffusive propagation of the cold pulse also looks quite natural: the time of its arrival increases and the amplitude of the pulse decreases with distance from the ablation region. It is evident that the diffusive model without magnetic field as an energy source is not able to describe the rise of core electron temperatures, which is a well-known theoretical result [1, 2, 5]. Evolution of the relative volume inside the deformable magnetic surfaces of three different minor radii is shown in Fig. 2b. It demonstrates rather weak perturbations of the magnetic equilibrium during HPP with \( v \equiv 0 \).

In Fig. 3a, the simulation results are compared with experimental data when the adjusted ad hoc plasma velocity \( v(\rho, t) \) evolution shown in Fig. 4a, b is taken into account. Note that, because of the model essential assumptions and simplifications, we used rather simple function for \( v(\rho, t) = v_0(\rho) v(t) \). The radial dependence described by \( v_0(\rho) \) allows to get the electron temperature response maximal in the plasma core (inner half of minor radius). The temporal factor \( v(t) \) corresponds to the core electron temperature time evolution in experiments. One can see from Fig. 3a that although the simple function for \( v(\rho, t) \), the core electron temperature rise was obtained at plasma velocities below 3 m/s. The maximal \( \Delta \text{Vol}/\text{Vol} \) in the plasma core does not exceed \( 0.4 \times 10^{-3} \) in agreement with estimates in Ref. [2]. As yet, the true profile of the convective velocity is not known mainly because there are no experimental measurements of the core plasma motion with such velocities. This small amplitude is the most important outcome of our simulations.

Figures 3a, b show that the temporal behavior of \( \Delta \text{Vol}/\text{Vol} \) is similar to that of

\[
\Delta T_e/T_e = [(T_e(\rho, t) - T_e(\rho, t_{\text{inj}}))]/T_e(\rho, t_{\text{inj}}),
\]

in the plasma core. This is revealed more explicitly in Fig. 5 where \( \Delta T_e/T_e \) is shown versus \( \Delta \text{Vol}/\text{Vol} \) for several minor radii. There is a linear dependence of \( \Delta T_e/T_e \) on \( \Delta \text{Vol}/\text{Vol} \) in the plasma core when the plasma velocity is taken into account.

The convective term, \( (5/2) n \cdot \nabla T_e \cdot v \), and the source term, \( v \cdot \nabla p \), play different roles in the electron energy balance equation (3). To see their effect separately, we performed simulations by introducing the convective and the source terms separately. The results are shown in Fig. 6.

They confirm that the convective term stronger affects the plasma than the source term in the plasma core. This is because the source term, being proportional to the pressure gradient, cannot be significant near the center where \( \nabla p \) is small.

The results demonstrated in Figs. 5 and 6 can be qualitatively interpreted as follows. Due to appearance of plasma velocity in equations (3) and (5), the electron energy density increases, mainly by means of the convective term in plasma core, and it causes outward shift of the magnetic surfaces (increase of Vol(\rho)) there as can be expected from equation (2).

It should be noted that increase of the electron energy content in the inner part of the plasma is rather small in comparison with the magnetic reservoir available. To illustrate this, the time evolution of the simulated toroidal \( \beta \) and the ratio of the thermal plasma energy variation \( \Delta W_{th} = W_{th}(t) - W_{th}(t_{\text{inj}}) \) to the total energy of the magnetic
Fig. 4  The \( \text{ad hoc} v(\rho, t) = v_\rho(\rho) \cdot v_t(t) \) evolution used in simulations: a) \( v_\rho(\rho) \); b) \( v_t(t) \). LHD shot #49708.

Fig. 5  Correlation of simulated \( \Delta T_e/T_e \) and \( \Delta \text{Vol}/\text{Vol} \) after TESPEL injection for LHD shot. #49708.

field \( W_m \) within the plasma column are shown in Fig. 7. Here,

\[
W_{th} = \int_{\text{Vol}} \frac{3}{2} N_e (T_e + T_i) dW, \quad W_m = \int_{\text{Vol}} \frac{B^2}{2 \mu_0} dV.
\]

One can see a decrease of \( \Delta W_{th}/W_m \) just after TESPEL injection due to plasma cooling and then its subsequent rise, which reflects the core plasma heating. The maximal values of \( \Delta W_{th}/W_m \approx 0.7 \times 10^{-4} \) is only 10\% of small enough plasma \( \beta \approx 0.7 \times 10^{-3} \). It confirms the necessity of taking into consideration the energy exchange between the plasma and the magnetic field in analysis of the HPP phe-

Fig. 6  The measured (ECE – red) and simulated (ASTRA) electron temperature with \( \text{ad hoc} v(\rho, t) \neq 0 \): a) both terms (blue); b) convective term only (green); c) source term only (cyan). LHD shot #49708.
Fig. 7 Evolutions of simulated toroidal plasma $\beta$ (red) and $dW_{th}/W_m$ (blue) in LHD shot #49708.

5. Summary and Further Plans

This study was motivated by the LHD experimental results [5, 8] and ideas and estimates of Refs. [2, 6, 7]. The key element of the concept [2] is that the energy exchange between the plasma and the magnetic field should be incorporated into the energy balance for HPP such as described in [3, 5, 8]. Our study is the first attempt to verify such approach by comparing the simulation results with LHD data [5, 8].

Results of simulations without plasma velocity are close to those in Refs. [5, 8] and again confirm the inability of the standard diffusive model to reproduce the behavior of the measured electron temperature such as shown in Fig. 2. However, similar simulations of the electron heat balance, but with plasma radial velocity of small enough magnitude (meters/second) demonstrate that the temperature data can be reasonably reproduced. Maximal perturbations of the plasma volume $\sim 0.4 \times 10^{-3}$ are in the range of those estimated in [2]. It is clearly difficult to measure such small changes in experiment, but their effect is strong on the scale of the energies involved.

We demonstrated a linear correlation between $\Delta V_{ol}/V_{ol}$ and $\Delta T_e/T_e$ in the inner part of the plasma when the velocity was taken into account in equations (3) and (5). It is shown that the convective term in the electron energy balance (3) dominates in the plasma core producing the electron energy density increase. This causes outward shift of the magnetic surfaces (increase of $V_{ol}(\rho)$) there as can be expected from the magnetic equilibrium equation.

The increase of the electron energy content in the inner half of minor radius is rather small as compared with the magnetic reservoir available within the plasma. The maximal values of $dW_{th}/W_m \approx 0.7 \times 10^{-4}$ are one order of magnitude below small values of plasma $\beta \approx 0.7 \times 10^{-3}$. It confirms the conclusion of [2] that the energy exchange between the plasma and the magnetic field must be taken into account in analysis of the HPP phenomena.

Simulations of the measured electron density evolution in the LHD shot #49708 has to be done for proof of the principle. Experimental and simulated behavior of the ion temperature evolution would be helpful. The free boundary approach of ASTRA7 might influence the modeling results. The time-dependent magnetic equilibrium for LHD should be implemented in the ASTRA code. A physics based model for $v(\rho, t)$ is needed for incorporation into ASTRA simulations. The dependence of NLT phenomena on the plasma electron density has to be revealed. Simulations of NLT phenomena observed in other devices (both tokamaks and stellarators) with different perturbation techniques are expected.

Acknowledgements

This work is conducted while VS was a visiting professor at NIFS, and is performed with the support and under the auspices of the NIFS grant administrative costs (NIFS18ULHH012) and NIFS Collaboration Research program (NIFS18KLPH035). Numerical calculations were performed at the Polytechnic SuperComputer Center at Peter the Great St. Petersburg Polytechnic University.