Theoretical Study of Relation between Anomalous Resistivity and Toroidal Spin-Up of Field-Reversed Configurations*)

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In this paper, the relation between a magnetic flux decay and a spontaneous spin-up phenomenon of high-beta FRC plasma is outlined based on the research results so far. We propose a hypothesis that an anomalous resistivity produces a difference in the time change rate of the angular momentum of ions and electrons, resulting in the rotation of ions.

Keywords: field-reversed configuration, high-beta plasma, spontaneous toroidal spin-up, anomalous resistivity

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1. Introduction

The spontaneous toroidal rotation of a Field-Reversed Configuration (FRC) plasma [1, 2] is a phenomenon which is often observed in the conventional field-reversed theta-pinch experiment [3, 4] and it is known that the occurrence of rotational instability of $n = 2$ [4–7] due to this toroidal rotation leads to the collapse of the FRC plasma. Experimentally, it is indicated that the observation result of the line integral density has a sinusoidal waveform with respect to time [4]. Therefore, suppression of spontaneous rotation of FRC leads to suppression of rotational instability of $n = 2$, which is indispensable for maintaining configuration. In theoretical work for the FRC, understanding the origin of spontaneous toroidal spin-up is an important task. If the mechanism of spin-up is understood, it is possible to open the possibility of suppressing rotation by external control. Up to now, three rotation mechanisms have been considered.

One of them is that the FRC plasma rotates with particle loss [8]. Among the plasma ions that can move inside the separatrix, those can move to the end of the device have a different sign of angular momentum around the device axis from those having a closed accessible region inside the separatrix. That is, the selective loss may occur depending on the angular momentum among the ions inside the separatrix. For this reason, rotation occurs by ions trapped inside the separatrix. This mechanism can also be interpreted as the law of action-reaction. In the past, the FRC rotation due to particle losses has been explained by the "velocity space particle loss" model [9, 10]. However, if there is a rotation process accompanied by loss of particles having an accessible region open to the outside at the confinement device end, rotation will occur within the time to move half of the device length at the thermal speed; it is in faster time scale compared with the experiment.

There is another rotation mechanism. The second one is the inherent problem of an open-ended magnetic field plasma. An end-shorting effect has been considered as a possible mechanism of spin-up [11]. In an open field plasma outside the separatrix, it is assumed that the radial electric field and the pressure gradient relating to the ion fluid are balanced. The radial electric field is short-circuited through the conductive wall by the electrons movable along the magnetic line of force. Consequently, when the electric field disappears, the radial component of the Lorentz force needs to be balanced. Therefore, the ion flow velocity is generated in the toroidal direction so as to satisfy the radial force balance. According to this theory, the rotation of the plasma must be transmitted from the outside of the separatrix to the inside by the viscosity accompanying the shear flow velocity.

One of the authors proposed a different spin-up mechanism from the above two theories [12]. This idea is that the decrement of the poloidal magnetic flux is directly converted into the angular momentum and the rotation starts. This paper aims to look back on problems and deepen the discussion about our proposed spin-up model.

2. Direct Spin-Up Model by Flux Decay

Takahashi *et al.* proposed that the magnetic flux decay directly causes the toroidal spin-up of FRC [12]. This theory is as follows.

The FRC generated by the field-reversed theta-pinch method maintains relatively good axial symmetry until the subsequent rotation starts and the $n = 2$ rotational instability is induced. At this time, the canonical angular momentum

$$P_\theta = m_i v_\theta r + q_i \psi, \tag{1}$$

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conserves. Here, m_i is the ion mass, q_i is the ion charge, v_θ is the azimuthal velocity component for individual particles, and ψ is the poloidal flux function defined by

$$\psi(r,z) \equiv \frac{1}{2\pi}\int_0^r B_z(r,z)r'\mathrm{d}r'. \tag{2}$$

If magnetic flux decay is expressed as $\Delta\psi < 0$ (here, magnetic flux inside the separatrix is defined as $\psi \geq 0$), the deviation of canonical angular momentum becomes

$$\Delta P_\theta = \Delta\left(m_\mathrm{i}v_\theta r\right) + \Delta\left(q_\mathrm{i}\psi\right) = 0. \tag{3}$$

Therefore,

$$\Delta\left(m_\mathrm{i}v_\theta r\right) = -\Delta\left(q_\mathrm{i}\psi\right) > 0. \tag{4}$$

Hence the angular momentum of ions increases. Since the above holds for all ions in the separatrix, the FRC plasma can rotate directly. This is the direct rotation model.

Calculating the temporal evolution of plasma rotation with a magnetic field model giving artificial electrical resistivity and flux damping factor consistent with the experiment showed that it is relatively good agreement with the experiment [12–14].

However, some problems have also been pointed out with this model.

According to this model, electrons also obtain angular momentum in the opposite direction at the same time. This leads to an increase in the plasma current. This has the effect of canceling the magnetic flux decay which was supposed originally, and there is a possibility that neither rotation nor magnetic flux decay will occur if self-consistent calculation is carried out.

In addition, conservation of the canonical angular momentum for each particle species is a case where friction acting between the ion and the electron is not considered, and when the frictional force exists, the condition of the Eq. (3) cannot be satisfied. This can be shown from the θ component of the equation of motion considering the friction term with the electronic fluid:

$$\begin{aligned} m_\mathrm{i}\frac{\mathrm{d}v_\theta}{\mathrm{d}t} &= -m_\mathrm{i}\frac{v_r v_\theta}{r} + q_\mathrm{i}\left(E_\theta + v_z B_r - v_r B_z\right) \\ &\quad -m_\mathrm{i}\nu_\mathrm{ie}\left(v_\theta - u_{\mathrm{e}\theta}\right). \end{aligned} \tag{5}$$

Here, E_θ is the azimuthal electric field component, B_r, B_z is the radial and axial components of the magnetic field respectively, ν_ie is the slowing-down collision frequency of an ion against electrons, and $u_{\mathrm{e}\theta}$ is the azimuthal component of the electron flow velocity. The electric field E_θ can be written as

$$E_\theta = -\frac{1}{r}\frac{\partial\psi}{\partial t}, \tag{6}$$

using the surface integrated form of the Faraday's law

$$-\int_S \frac{\partial\vec{B}}{\partial t}\bullet\vec{n}\mathrm{d}S = \int_S\left(\nabla\times\vec{E}\right)\bullet\vec{n}\mathrm{d}S, \tag{7}$$

so substituting this into Eq. (5) gives

$$\frac{\mathrm{d}P_\theta}{\mathrm{d}t} = -m_\mathrm{i}\nu_\mathrm{ie}r\left(v_\theta - u_{\mathrm{e}\theta}\right). \tag{8}$$

Therefore, it is necessary to compare the rotation time scale with the collision time.

Let us discuss the sum of the canonical angular momentum of ion fluid and electron fluid, not individual particle species like ions and electrons. We define the ensemble averaged canonical angular momentum for the ion fluid as

$$\begin{aligned} &\frac{1}{n_\mathrm{i}}\int_{-\infty}^{\infty} P_\theta f_\mathrm{i}(\vec{v})\mathrm{d}\vec{v} \\ &= \frac{1}{n_\mathrm{i}}\int_{-\infty}^{\infty}\left(m_\mathrm{i}v_\theta r + q_\mathrm{i}\psi\right)f_\mathrm{i}(\vec{v})\mathrm{d}\vec{v} = m_\mathrm{i}u_{\mathrm{i}\theta}r + q_\mathrm{i}\psi, \end{aligned} \tag{9}$$

where n_i is the number density of ions and $f_\mathrm{i}(\vec{v})$ is the ion distribution function. From the action-reaction law

$$\begin{aligned} &R_{\mathrm{ie}\theta} + R_{\mathrm{ei}\theta} = 0, \\ &R_{\mathrm{ie}\theta} = -m_\mathrm{i}\nu_\mathrm{ie}r\left(u_{\mathrm{i}\theta} - u_{\mathrm{e}\theta}\right), \\ &R_{\mathrm{ei}\theta} = -m_\mathrm{e}\nu_\mathrm{ei}r\left(u_{\mathrm{e}\theta} - u_{\mathrm{i}\theta}\right), \end{aligned} \tag{10}$$

in the case of classical electrical resistivity, we can estimate as follows:

$$\begin{aligned} &\frac{\mathrm{d}P_{\mathrm{i}\theta}}{\mathrm{d}t} + \frac{\mathrm{d}P_{\mathrm{e}\theta}}{\mathrm{d}t} \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\left(m_\mathrm{i}u_{\mathrm{i}\theta}r + q_\mathrm{i}\psi\right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(m_\mathrm{e}u_{\mathrm{e}\theta}r - e\psi\right) \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\left(m_\mathrm{i}u_{\mathrm{i}\theta}r + m_\mathrm{e}u_{\mathrm{e}\theta}r\right) \approx \frac{\mathrm{d}}{\mathrm{d}t}\left(m_\mathrm{i}u_{\mathrm{i}\theta}r\right) \\ &= -m_\mathrm{i}\nu_\mathrm{ie}r\left(u_{\mathrm{i}\theta} - u_{\mathrm{e}\theta}\right) - m_\mathrm{e}\nu_\mathrm{ei}r\left(u_{\mathrm{e}\theta} - u_{\mathrm{i}\theta}\right) \\ &= 0. \end{aligned} \tag{11}$$

Here, we assume $q_\mathrm{i}\psi - e\psi = 0$.

In the plasma, not only Coulomb collisions but also magnetic field fluctuations exist as causes of the electrical resistivity. We refer to processes other than the Coulomb collisions as anomalous resistivity, but electrons with small mass are susceptible to magnetic field fluctuations as their inertial force is much smaller than that of ions. Therefore, it can be considered that the angular momentum of electrons is selectively lost. Suppose that a frictional force acting on electrons is assumed to be A times the classical resistance. When the so-called anomaly factor is A, the time change of the angular momentum is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(m_\mathrm{i}u_{\mathrm{i}\theta}r\right) = (A-1)\,m_\mathrm{i}\nu_\mathrm{ie}r\left(u_{\mathrm{i}\theta} - u_{\mathrm{e}\theta}\right). \tag{12}$$

Therefore, when the anomaly factor exceeds 1, an increase in angular momentum is caused.

This is also shown from self-consistent numerical calculation. Figure 1 shows the time evolution of the toroidal flow by three-dimensional hybrid simulation. The plasma and device parameters for this calculation are as follows: the device radius is 0.17 m, the half-length of the device is 1.0 m, the external magnetic field is 0.4 T, the ion temperature is 100 eV, the electron temperature is 50 eV and

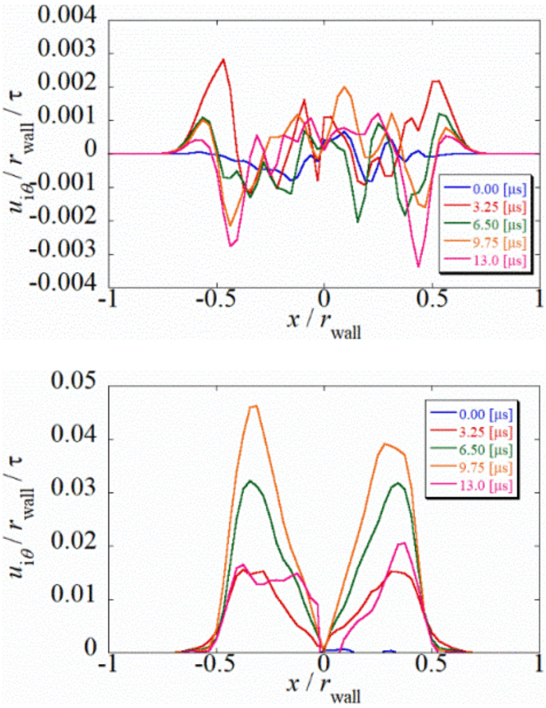

Fig. 1 Ion flow velocity profile calculated by 3-D hybrid simulation for (top) classical resistivity case and (bottom) anomalous resistivity case. Here $r_{\mathrm{wall}} = 0.17\,\mathrm{m}$ and $\tau = 1.3 \times 10^{-7}$ sec (i.e. the typical gyration time).

the ion and electron densities are both $2.68 \times 10^{21}\ \mathrm{m}^{-3}$, respectively. The parameters are basically the same as the NUCTE-III device [7]. The horizontal axis is taken on the x axis and the vertical axis is taken for the toroidal flow velocity. However, in the negative region of the x coordinate, the sign of the toroidal flow velocity is reversed. When the frictional force is classical, the change in the flow velocity fluctuates little, whereas in the case of the anomalous resistivity the flow velocity increases with time.

3. Electron Fluid Fluctuation Field

The electron current is dominant in an FRC plasma. Therefore, it is reasonable to think that a magnetic flux decay is induced by phenomena dominated by the electron current. When considering this fact to the previous section, it can be thought as follows.

1. Electrons lose their angular momentum under an influence of magnetic field fluctuations, while ions have large Larmor radii and are not affected by the magnetic field fluctuations. Therefore, the angular momentum loss for ions is small.
2. The loss of the angular momentum of the electrons leads to a decrease in the electron current and decay of the magnetic flux.
3. A toroidal rotation is caused by the difference in the time change rate between the ion and electron angular momentum.

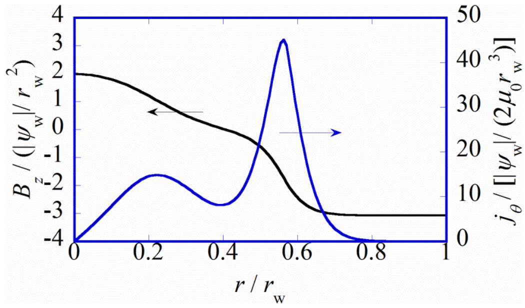

Fig. 2 Radial profiles of (black) the axial magnetic field and (blue) the current density.

Therefore, it can be considered an anomalous decrease of the angular momentum of electrons is linked to the ion fluid rotation. A steady-state fluctuating fields of the electron fluid satisfies the following set of equations:

$$\frac{\partial n_{\mathrm{e}}}{\partial t} + \nabla \bullet (n_{\mathrm{e}} \mathbf{u}_{\mathrm{e}}) = 0, \tag{13}$$

$$m_{\mathrm{e}} n_{\mathrm{e}} \left[\frac{\partial \mathbf{u}_{\mathrm{e}}}{\partial t} + (\mathbf{u}_{\mathrm{e}} \bullet \nabla) \mathbf{u}_{\mathrm{e}} \right] = -e n_{\mathrm{e}} (\mathbf{E} + \mathbf{u}_{\mathrm{e}} \times \mathbf{B}) - \nabla p_{\mathrm{e}}, \tag{14}$$

$$\frac{3}{2} \left[\frac{\partial p_{\mathrm{e}}}{\partial t} + (\mathbf{u}_{\mathrm{e}} \bullet \nabla) p_{\mathrm{e}} \right] + \frac{5}{2} p_{\mathrm{e}} (\nabla \bullet \mathbf{u}_{\mathrm{e}}) = 0, \tag{15}$$

$$\mu_0 \mathbf{j} = \nabla \times \mathbf{B}, \tag{16}$$

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E}, \tag{17}$$

$$\mathbf{j} = e n_{\mathrm{e}} (\mathbf{u}_{\mathrm{i}} - \mathbf{u}_{\mathrm{e}}) = -e n_{\mathrm{e}} \mathbf{u}_{\mathrm{e}}. \tag{18}$$

From the top, it is the equation of continuity for electron fluid, the electron equation of motion, the equation of electron thermal energy, Ampere's law, Faraday's law, and the definition of current density. Here, n_{e} is the electron density, $\mathbf{u}_{\mathrm{e}}$ is the electron flow velocity, p_{e} is the electron pressure, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, and $\mathbf{j}$ is the current density. Here, we assume the ion flow velocity is zero.

Here we consider the electron fluctuating field in a high-beta plasma. The distribution of the equilibrium quantity (zero-th order quantity) of the electron current and the axial direction magnetic field is given as shown in Fig. 2. This corresponds to the midplane distribution by the solution of the Grad-Shafranov equation to the FRC plasma. Since a steep pressure gradient can be generated in the vicinity of the separatrix, a peak of current can be generated. On the other hand, in the vicinity of the field-null circle, the current has a local minimum value.

The electron current is diamagnetic one accompanying the electron density gradient. Therefore, density fluctuations cause those of the current density and the magnetic field.

Consider fluctuating fields that can be written in

$$f_1(r, \theta, z, t) = \delta f(r) e^{\mathrm{i}\ell\theta} e^{\mathrm{i}kz} e^{-\mathrm{i}\omega t}. \tag{19}$$

Here, ℓ is the toroidal mode number, k is the wave number in the z direction, and ω is the angular frequency of the fluctuations. Since the amplitude of the wave depends on the distribution of the zero-th order quantity, it is assumed to be a function of r. In this case, we find the condition

$$\omega = \frac{\ell u_{\theta 0}}{r}, \tag{20}$$

to lead a resonating feature of wave field.

In the future, it is our task to clarify the anomalous mechanism by studying particle transport in electron fluid fluctuation fields, and to demonstrate the relevance to spontaneous spin-up phenomena.

4. Summary

We have investigated the relation between magnetic flux decay and spontaneous spin-up phenomenon of high-beta FRC plasma. We have proposed a hypothesis that an anomalous resistivity produces a difference in the time change rate of the angular momentum between ions and electrons, resulting in the rotation of ions. Investigation on anomalous mechanisms by studying particle transport in electron fluid fluctuation fields and demonstration of their relevance to spontaneous spin-up phenomena remain as a future study.

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