Stability of the Drift-Cyclotron Loss-Cone and Double-Humped Modes in Multispecies Plasmas*)

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Stability of the flute-like electrostatic Drift-Cyclotron Loss-Cone and Double-Humped modes in a mirror trap is critically revisited. The isotopic effect is taken into account as well as the anisotropy of warm ions used to stabilise these modes.

Keywords: mirror trap, microinstability, drift-cyclotron loss-cone mode, double-humped mode, nuclear fusion

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1. Introduction

It has been long known that the Drift-Cyclotron Loss-Cone (DCLC) oscillations can be unstable in a mirror trap because of the empty loss cone in the distribution of hot ions which are produced in the course of neutral-beam injection [1]. It is also known that the instability can be suppressed by addition of a small amount of warm ions with isotropic distribution. However increasing the fraction of warm ions provokes the Double-Humped (DH) microinstability [2] so that the conditions for stabilizing the DCLC and DH modes are contradictory. With an empty loss cone, the DCLC oscillations are unstable if the radial gradient of the plasma density in the direction across the magnetic field exceeds a certain critical value, whereas the DH oscillations can be unstable even in a homogeneous plasma. A review of early papers on the problem of stability of DCLC and DH was made in the article by Richard Post [3].

Recent resumption of the interest in this problem is connected with the discussion of the reactor prospects of the gasdynamic trap. The facts are such that even a small decrease in the fraction of warm ions leads to a significant improvement in the integral parameters of the plasma and an increase in the thermonuclear power [4]. An essential feature of such a device is that it has a population of hot ions created by inclined injection of powerful neutral atomic beams into the target warm plasma. Hot ions are confined in an adiabatic regime, which is characterized by an empty loss cone. On the contrary, relatively cold target plasma is confined in the gas-dynamic regime, where the loss cone is filled. In Ref. [5], studying the case of a purely deuterium plasma, we have shown that it is possible to simultaneously stabilize the DCLC and DH modes with a warm-to-hot ion ratio of the order of 10% if the hot ion temperature exceeds 1% of the neutral-beam injection energy. With such parameters it is difficult to imagine that the distribution of warm ions remains isotropic (as was assumed in [5]) because of intensive heating of warm ions by the hot ions. In addition, it should be taken into account that in a real reactor the plasma contains different species of ions, for example, deuterium and tritium ions. In the present article we report the results of studying the isotopic effect and the effect of anisotropy of warm ions on the stability of the DCLC and DH modes.

2. Dispersion Equation

In Ref. [5], we have derived anew the dispersion equation of electrostatic microvibrations with $k_\parallel = 0$. In this equation, the summation of an infinite series of the products of Bessel functions with integer indices was carried out using identities independently established by Lerche and Newberger [6,7]. The result of summation is expressed in terms of the product of Bessel functions with indices that depend on the frequency ω:

$$1 = \sum_a \frac{\omega_{pa}^2}{k^2} \int \left(2\frac{\partial f_a}{\partial v_\perp^2} + \frac{k_\perp \eta}{\Omega_a \omega} f_a \right) \left(1 - \frac{\pi\omega}{\Omega_a} \times \frac{\mathrm{J}_{\omega/\Omega_a}(k_\perp v_\perp/\Omega_a)\, \mathrm{J}_{-\omega/\Omega_a}(k_\perp v_\perp/\Omega_a)}{\sin(\pi\omega/\Omega_a)} \right) \pi\, \mathrm{d}v_\perp^2 . \quad (1)$$

Index a in this dispersion equation labels different particle species such as electrons, deuterium and tritium ions. The equation involves the integration over the distribution function $f_a(v_\perp)$, which is already has been integrated over the longitudinal velocity $v_\parallel$. The function is normalized so that $\int_0^\infty f_a(v_\perp)\, 2\pi v_\perp\, \mathrm{d}v_\perp = 1$. Parameters $\omega_{pa} = \sqrt{4\pi e_a^2 n_a/m_a}$ and $\Omega_a = e_a B/m_a c$ have their conventional meaning of the plasma and cyclotron frequencies for the species a, the parameter $\eta = (\partial n_a/\partial r)/n_a$ is the same for particles of all species (including electrons) and is associated with the radial gradient of the density, $k_\perp$ denotes the azimuthal wave number, finally $k = |k_\perp|$.

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The contribution of electrons to the sum over the index a is evaluated as $\sum_{a=e} \approx -\omega_{pe}^2/\Omega_e^2$. To calculate the contribution of hot ions, we approximate their distribution by the Gerver function, which is expressed through the difference between two Maxwellian functions with distinct effective temperatures [8]

$$f_h(v_\perp) = \frac{R_h + 1}{\pi v_h^2 (R_h - 1)} \times \left(e^{-(R_h+1)v_\perp^2/R_h v_h^2} - e^{-(R_h+1)v_\perp^2/v_h^2} \right). \tag{2}$$

It depends on two parameters: parameter v_h approximately represents velocity of injected fast ions before they lose their energy, and parameter R_h is related to but not equal to the mirror ratio $B_{\max}/B_{\min}$ (see Fig. 2 in Ref. [5]). We have modeled the distribution of warm ions in one case by an isotropic Maxwellian function

$$f_m = \left(1/\pi v_m^2\right) \exp\left(-v^2/v_m^2\right), \tag{3}$$

with a thermal velocity v_m, and by the Gerver function f_w with the parameters v_w and R_w in the other case. The result of calculating the integral in Eq. (1) for such distribution functions is expressed in terms of the generalized hypergeometric function ${}_2F_2$. A method of finding the roots of the dispersion equation in the complex plane of the frequency ω is described in Ref. [9].

To represent the results of our calculation, we introduce the dimensionless frequency $\nu = \omega/\Omega_p$ normalized to cyclotron frequency of protons Ω_p, dimensionless velocity $u = v_\perp/v_p$, normalized to a certain value of the velocity v_p, exceeding all the characteristic values of the proton velocity in the problem under consideration. Parameter v_p was chosen in Ref. [5], where it was equal to the maximum proton velocity on the grid, which was used for numerical evaluation of the distribution function of hot ions.

We also take into account that the populations of both hot and warm ions can be composed of various isotopes. Assuming that any isotope can be ionized only once, we characterize every ion species by its atomic weight $A_i = m_i/m_p$ defined as the ratio of the ion mass m_i to the proton mass m_p. Its cyclotron frequency $\Omega_i = \Omega_p/A_i$ is then expressed through the fraction of the cyclotron frequency Ω_p of a proton. The density of a species i composes a fraction $\beta_i = n_i/n_e$ of the total ion density $\sum_i n_i$, which is equal to the density n_e of electrons, so that $\sum_i \beta_i = 1$. Finally, we introduce ratio $\alpha_i = n_{iw}/n_{ih}$ of the density of warm ions n_{iw} of the species i to the density n_{ih} of hot ions of the same species and dimensionless variables $\gamma = \Omega_e^2/\omega_{pe}^2$, $\varkappa = k\rho_p$, $\overline{\eta} = \eta\rho_p$, where $\rho_p = v_p/\Omega_p$. Sometimes we also refer to the integral warm-to-hot ion density ratio which is formally defined as $\alpha = n_w/n_h = \left[\sum_i \alpha_i\beta_i/(1+\alpha_i)\right] / \left[\sum_i \beta_i/(1+\alpha_i)\right]$. In all cases described below, the distribution of hot ions f_h was modeled by the Gerver function (2) with effective mirror ratio $R_h = 6$ and effective thermal velocity $u_h = 0.5$; as shown in Ref. [5] such a value of R_h corresponds to a mirror trap with the mirror ratio $R = 16.31$.

3. Isotopic Effect

Discussion of the results of our calculations we begin with the case when warm ions have a Maxwellian distribution, but the isotopic composition of warm and hot ions can differ. In Fig. 1 the atomic weight of the first (warm) component was fixed at $A_1 = 3$ (tritium), the weight of second (hot) component at $A_2 = 2$ (deuterium). We plotted imaginary part of the roots of dispersion equation $\nu(\varkappa)$ with a positive imaginary part for zero value of the density gradient $\overline{\eta}$ and relatively small fraction of warm ions $\beta_1 = 1/11$, which corresponds to the ratio $\alpha = n_w/n_h = 1/10$ of warm-to-hot ion density. The resonances at the harmonics of cyclotron frequency of deuterium correspond to the values $\nu = \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\ldots$, and the resonances at the harmonics of cyclotron frequency of tritium correspond to $\nu = \frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}, 1\frac{2}{3}, 2, 2\frac{1}{3}\ldots.$ Thus, the spectrum of cyclotron frequencies of warm ions does not completely overlap the spectrum of cyclotron harmonics of hot ions. As can be seen from the figure, the instability develops even in a homogeneous plasma on those harmonics of the hot ions that are absent in the spectrum of warm ions. This observation allowed us to formulate a spectral rule that was confirmed by further calculations: for the stabilization of the DCLC and DH oscillations, it is necessary that all the harmonics that exist in the spectrum of cyclotron oscillations of hot ions are also present in the spectrum of cyclotron oscillations of warm ions.

The ratio $n_w/n_h = 1/10$ guaranties existence of stable regimes in case when both species belong to same isotope. We repeat the calculations for many other ratios n_w/n_h ranging from 0 to 10 and different combination of warm and hot isotopes. Figure 2 shows the dependence of maximal increments on the ratio n_w/n_h at fixed radial gradient $\overline{\eta} = \frac{1}{8}$ in the case of hot deuterium for three variants of warm species: hydrogen, deuterium and tritium. In case of warm deuterium no unstable modes were found for

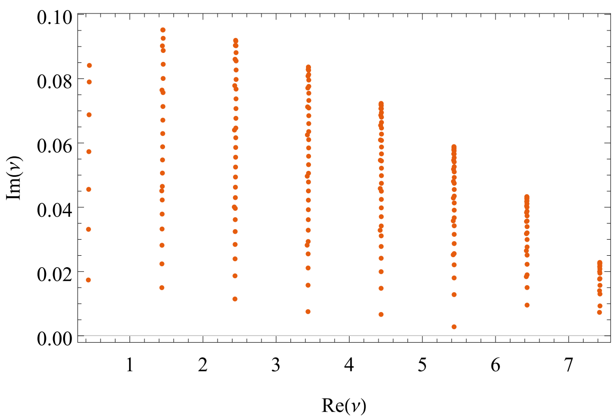

Fig. 1 Roots of the dispersion equation with a positive imaginary part in case of warm tritium and hot deuterium at $\overline{\eta} = 0$, $\gamma = 1$, $A_1 = 3$, $A_2 = 2$, $\alpha_1 = \infty$, $\alpha_2 = 0$, $\beta_1 = 1/11$, $\beta_2 = 10/11$, $u_{w1} = 0.1$, $R_2 = 6$, $u_{h2} = 0.5$; unstable modes were found for $5 < \varkappa < 69$.

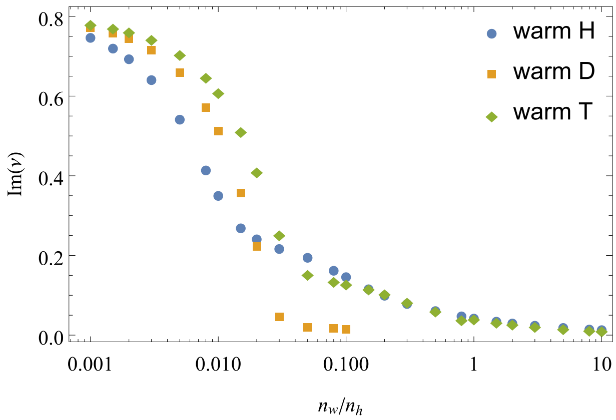

Fig. 2 Increments of most unstable modes in case of hot deuterium ions and three variants of warm ions: hydrogen, deuterium, and tritium; $\overline{\eta} = 0.125$, $u_{w1} = 0.1$, $R_2 = 6$, $u_{h2} = 0.5$.

$n_w/n_h > 0.1$ since warm and hot ions have common spectrum of cyclotron harmonics. In case of warm hydrogen and warm tritium unstable modes were found for any ratio n_w/n_h, since the spectrum of warm ions does not cover all cyclotron harmonics of hot deuterium ions; note however that the maximal increment clearly decreases as the ratio n_w/n_h grows.

4. Effect of Warm Ions Anisotropy

To elucidate how the devastation of the loss cone in the distribution of warm ions affects the stability boundary of the DCLC and DH modes, we found numerical solution of the dispersion equation for the following two extreme cases.

In the first case, the loss cone of warm ions was completely filled and their distribution was modeled by isotropic Maxwell distribution f_m with the thermal velocity $u_m = 0.1$. In the second case, the distribution of warm ions f_w has an empty loss cone, which is simulated by the Gerver function (2) (with the index w instead of h) for effective mirror ratio $R_w = 22$ and thermal velocity $u_w = 0.11$; these values were found by fitting a solution of the kinetic equation for a mirror trap found by Gersh Budker [10]. To avoid hiding the effect of the warm ion anisotropy by the isotopic effect in this section we assume that both warm and hot species are composed of ions of deuterium ($A = 2$).

Figure 3 (a) shows the dependence of the critical density gradient $\overline{\eta} = \eta\rho_p$ on the ratio of warm to hot ion density $\alpha = n_w/n_h$. Plasma is unstable if the actual value of the dimensionless gradient $\left(\rho_p/n_e\right)(\partial n_e/\partial r)$ exceeds the critical value indicated on the graph. Note that the critical value exists because the temperature of warm ions (which is determined by the values of the parameters $u_m = 0.1$ and $u_w = 0.11$) in Fig. 3 is high enough for the stabilization of the DH instability at $\overline{\eta} = 0$, as described in Ref. [5]. At the chosen temperature, the instability disappears at any value of α if the density gradient is less than the critical value. As can be seen from the figure, in the case of warm ions with the distribution function f_w (with empty loss cone), the critical density gradient is noticeably lower than in the case of the Maxwell distribution f_m (with filled loss cone). Thus, the shape of the distribution function of warm ions significantly affects the stability boundary.

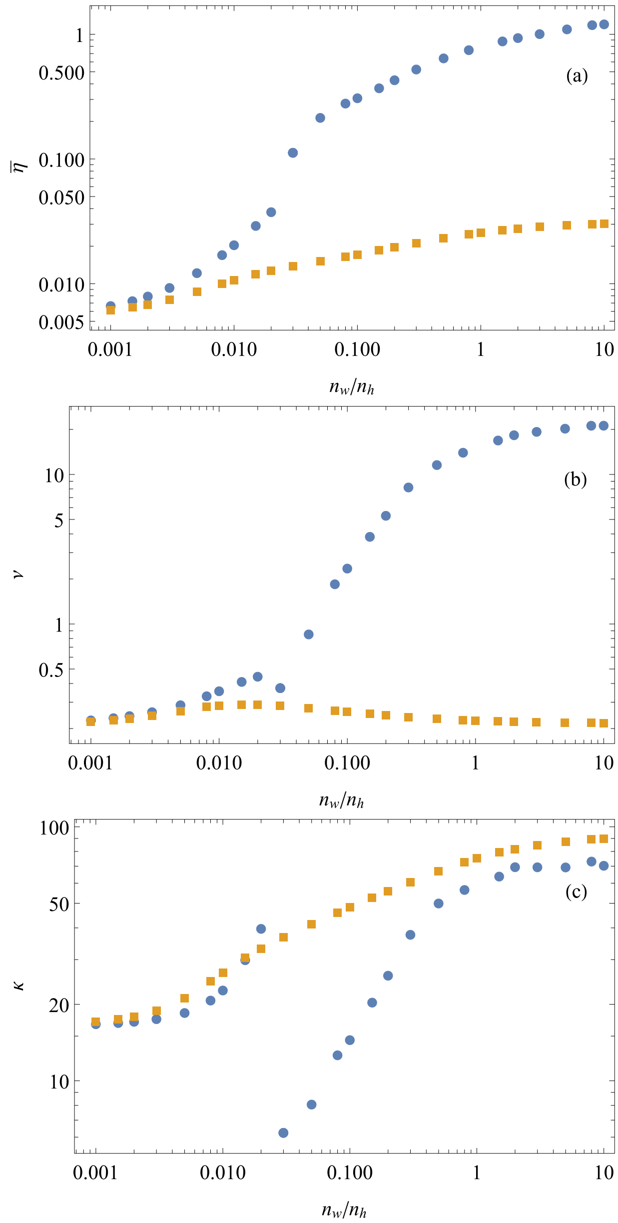

Fig. 3 Critical gradient $\overline{\eta} = \eta\rho_p$ (a), frequency $\nu = \omega/\Omega_p$ (b) and wavenumber $\varkappa = k\rho_p$ (c) of the most unstable perturbations as a function of the ratio $\alpha = n_w/n_h$ in the limit of the Maxwellian distribution f_m of warm ions at $u_m = 0.1$ (blue circles) and the distribution f_w of warm ions with an empty loss cone for $u_w = 0.11$ and $R_w = 22$ (orange squares). Other parameters are: $\gamma = 1$, $u_h = 0.5$, $R_h = 6$, $A = 2$; actual mirror ratio $R = 16.31$.

Dimensionless frequency $\nu = \omega/\Omega_p$ and wave vector $\varkappa = k\rho_p$ are shown in Fig. 3 (b) and 3 (c) for the most unstable oscillations at the critical value of the gradient; such

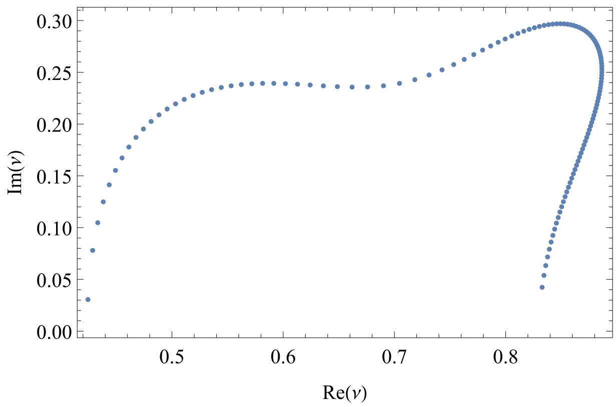

Fig. 4 Low-frequency instability in a plasma with anisotropic warm ions: $R_w = 22$, $u_w = 0.1$, $n_w/n_h = 0.1$.

oscillations acquire a positive increment if the actual density gradient even slightly exceeds the critical value shown in Fig. 3 (a).

Figure 4 illustrates the case when the DCLC and DH instabilities would be suppressed in a plasma with a Maxwellian distribution f_m of warm ions, but they exist if the loss cone in the distribution of warm ions is empty as described by the Gerver function f_w. In this case, an instability of low-frequency oscillations arises. Their phase velocity $\nu/\varkappa$ is much less than the characteristic velocity u_h of hot ions, so we can hope that these oscillations will not cause a catastrophic degradation of the distribution of hot ions.

We also note that the results of the calculations presented in Fig. 4 correspond to the extreme case when the loss cone is absolutely empty even in the region of the lowest energies. Under real conditions, the loss cone will be at least partially filled, so that the distribution function of warm ions will be something average between f_m and f_w. Therefore, it should be expected that the instability will be suppressed for the real distribution of warm ions or instability increment will be smaller than that shown in Fig. 4.

5. Discussion

We numerically studied the stability of the DCLC and DH modes in a multispecies plasma with two different isotopes of hydrogen confined in a mirror trap. We concluded that multispecies plasma is less stable as compared to single-species plasma, and the instability increments are significantly larger, although stable regimes can be found.

Our analysis showed that in order to suppress the instabilities of both DCLC and DH modes a spectral rule should be satisfied. It means that the set of harmonics of the cyclotron frequencies of warm ions must overlap the entire set of harmonics of the cyclotron frequencies of hot ions. This requirement is easily met if the isotope composition of the population of warm ions coincides with the isotope composition of the population of hot ions. We found that both modes can be stabilized by addition of warm ions provided that the concentrations of different isotopes in warm ion population are proportional to those in the population of hot ions and the temperature of warm ions is not too small. We also found that the use of heavy isotopes, such as argon, to create a population of warm ions does not lead to stabilization of the instability of light hot ions, even if the spectral rule is satisfied.

We have also left behind the framework of this article important effects of the inhomogeneity of the magnetic field and the finite plasma pressure. The study of these effects seems to be a difficult task, which can not be solved within the framework of single article. However, as noted in Ref. [5], it is widely believed that these effects extend the stability region of DCLC and DH oscillations. Thus, the stability conditions found in the present paper are sufficient, but not necessary.

We also considered the effects associated with the transition of the warm ions confinement from the gasdynamic regime to the kinetic regime. We have established that the devastation of the loss cone in the distribution of warm ions can provoke a kind of DCLC low-frequency instability that develops due to the disequilibrium of warm rather than hot ions. At not too small differences in the temperatures of warm and hot ions, unstable perturbations are characterized by a short wavelength, a low phase velocity (much lower than the thermal velocity of warm ions), and small increments (of the order of fractions of the cyclotron frequency). For this reason, these oscillation, hopefully, will not adversely affect the confinement of hot ions in the projected thermonuclear systems on the basis of mirror traps.

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