Comparison of Conditional Average Using Threshold and Template Methods for Quasi-Periodic Phenomena in Plasmas

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The comparison of conditional averaging using so-called threshold and template methods is made in their application on quasi-periodic burst of I-mode in ASDEX Upgrade (AUG) I-mode plasma. For obtaining conditionally averaged waveforms, three different manners to determine the 'clocks', using raw signal, envelope of the signal and numerical filtering, are applied to each method. The comparison demonstrates the excellency of the template method to extract typical characteristics of target phenomena in its uniqueness to select the ensemble waveforms and its good convergence property.

Keywords: conditional averaging, template method, threshold method, magnetic burst, intermittence, ASDEX Upgrade, I-mode plasma

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1. Introduction

A number of phenomena characterized with quasi-periodic bursts are known in magnetic confinement plasmas. The examples include sawtooth oscillations [1], edge-localized mode (ELM) in the high confinement mode (H-mode) [2], energetic particle driven modes such as fishbone instabilities [3] and so on. In order to understand and clarify the mechanisms and the transport processes of these kinds of phenomena, it is often necessary to obtain the typical spatio-temporal evolution of the bursts after eliminating the noise of the individual events. For the purpose, the conditional averaging has been widely applied [4–6].

One of the common questions about applying the conditional average for quasi-periodic bursts is how the temporal windows of the ensembles are chosen for the average or how the "clocks", which mean the times for the individual events to occur, are determined. One of the commonly used methods is that the clocks can be chosen as the time when the reference signal has a local peak, which goes beyond a threshold value. In this paper, we call this technique the threshold method. The other technique developed recently is called the template method [7, 8], in which the clocks are determined as the times when the cross-correlation between the template and the signal is sufficiently high, in other words, when the signal resembles to some degree of the standard pattern defined as 'template'. This article presents the results of conditional averaging applied on the intermittent bursts observed during the I-mode plasma of ASDEX Upgrade (AUG) [9–11], and shows the excellent features of the template methods by the comparison to the threshold methods.

2. The Intermittent Bursts in ASDEX Upgrade I-mode Plasma

The I-mode is an improved energy confinement regime, in which heat transport in the pedestal is inhibited to some degree, while particle transport remains at low confinement mode (L-mode levels). The I-mode is characterized by a quasi-periodic phenomenon [12–14], called the Weakly Coherent Mode (WCM), which resides in a plasma edge and is a typical fluctuation during the I-mode. The I-mode is obtained when the ion grad B drift points

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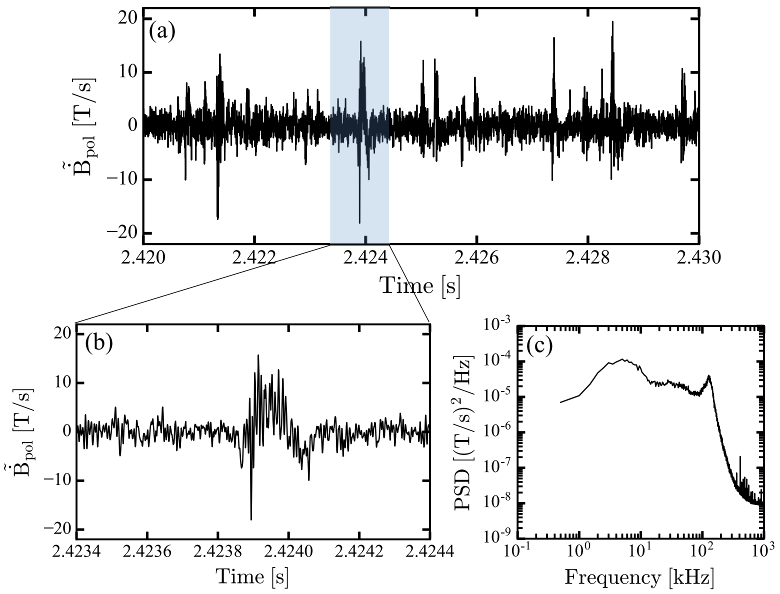

Fig. 1 (a) Raw signal of $\dot{B}_{pol}$ fluctuations measured by Mirnov coil. (b) Enlarged view of the burst. (c) Power spectrum of the $\dot{B}_{pol}$ fluctuations.

away from the active x-point, which is an unfavorable magnetic geometry to access the H-mode power threshold. During the I-mode, intermittent bursts are observed in the Doppler reflectometry, bolometry and Mirnov coils. Figure 1 (a) shows the raw signal of the Mirnov coils during the I-mode in discharge #29744, which is reported in Ref. [9, 10]. The $\dot{B}_{pol}$ fluctuation increases abruptly and intermittently. The enlarged view of the burst within the blue-hatched region is shown in Fig. 1 (b), which indicates the non-monotonic increase of the burst. In Fig. 1 (c), the power spectrum of the $\dot{B}_{pol}$ fluctuation indicates two characteristics. One is the broad frequency spectrum range, which is less than 90 kHz, and the other is high frequency components, which are around 130 kHz.

3. Comparison of Conditional Averages Using Threshold and Template Methods

3.1 Conditional average

The conditional average is used to extract a typical temporal evolution of statistical phenomena. The definition of the conditional average is as follows. Assuming a temporally sequential signal $x(t)$, the conditionally averaged signal $X(\tau)$ is obtained as, $X(\tau) = \frac{1}{N}\sum_{i=1}^{N} x(t_i + \tau)$, where $-T < \tau \leq T$ and T is a specific time width, t_i is the i-th number of the clock and N is the total ensemble number. In the following studies, we use the $\dot{B}_{pol}$ fluctuations signal, which has a sampling frequency of 2 MHz, and the width of temporal window, T, is set to be 500 µs. In the conditional average, the problem is how to determine the temporal windows, or clocks of the ensemble data.

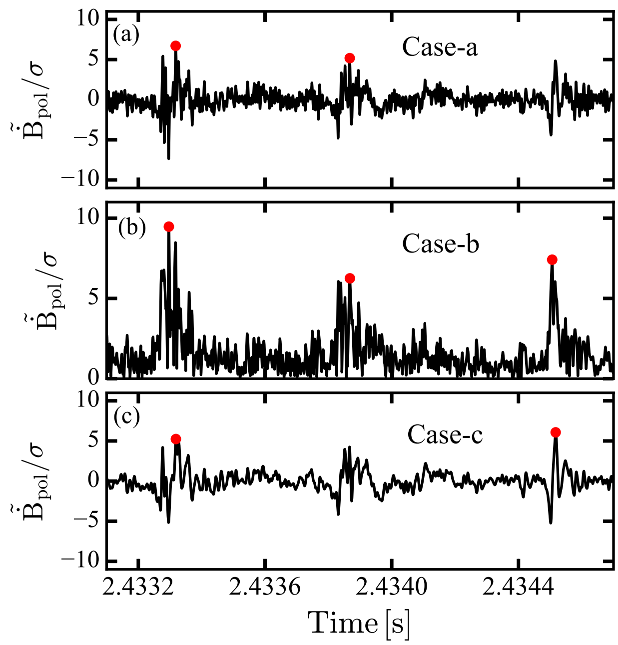

Fig. 2 The threshold method is performed for 3 different proceeded signals of $\dot{B}_{pol}$ bursts. (a) Raw signal, (b) envelope of the raw signal, and (c) low-pass 90 kHz signal are used and named as Case-a, Case-b and Case-c, respectively. Threshold values are defined as 5σ. The red points indicate the clocks for each case.

3.2 Threshold method

The threshold method is one of the techniques to determine the clocks. Here, we show the results of the conditional averaging using threshold method in three different manners of data processing. Figures 2 (a), (b) and (c) show raw signal, envelope of raw signal and low-pass 90 kHz

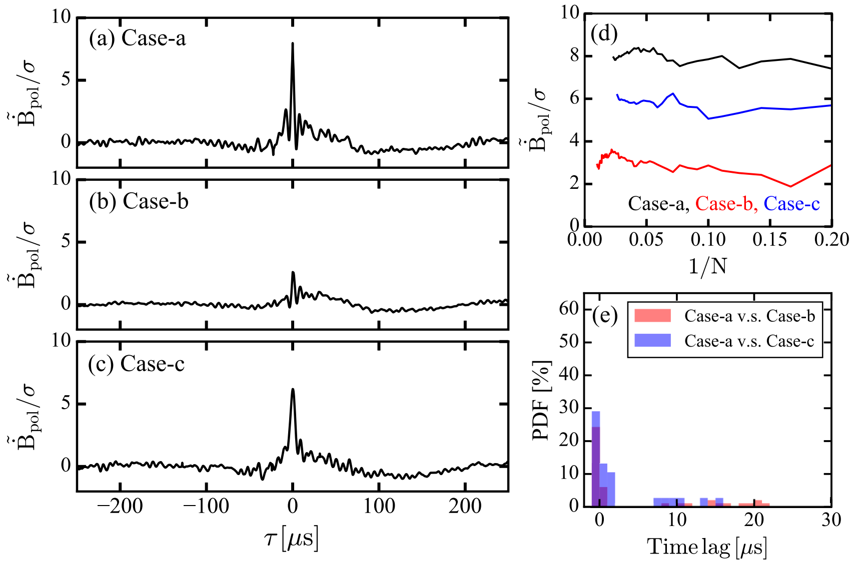

Fig. 3 Threshold method is performed. (a), (b) and (c) show the conditionally averaged waveforms of the bursts for the Case-a, Case-b and Case-c, respectively. (d) Convergence of the amplitude of the bursts at $\tau = 0\,\mu$s. N indicates the ensemble number. (e) PDF of the clocks difference between Case-a and Case-b/Case-c.

filtered signal of $\dot{B}_{\mathrm{pol}}$ fluctuations, which is normalized by their standard deviation σ. Here, the envelope is estimated from the Hilbert transform [15]. The processed signals in Figs. 2 (a), (b) and (c) are named as Case-a, Case-b and Case-c, respectively. In these three cases, all the threshold values are set as 5σ, to determine the clocks for Case-a, Case-b and Case-c, respectively. The red points in the Fig. 2 show the clocks, which are different from each other. Note we only choose maximum points above the threshold value as the clocks in temporal windows, thus the clock is uniquely determine in a temporal window. The conditional averages using these clocks are carried out for these three cases, and the results are shown in Figs. 3 (a)-(c). It is clear that these waveforms are different from each other, in terms their amplitude, phase and the start of their changes.

The convergence property of the three cases is studied. The amplitudes at $\tau = 0\,\mu$s against the ensemble number, as is shown in Fig. 3 (d), show that the amplitudes converge to the different values. The total ensemble number N for Case-a, Case-b and Case-c are 43, 99 and 38, respectively. The time differences of the clocks between three cases are shown in Fig. 3 (e). The probability density function (PDF) of the time lags between three cases clearly demonstrate that the finite difference is produced against the data process of the conditional averages.

3.3 Template method

The rough procedure of the conditional average using the template method is explained in the following. First, an initial template waveform $X_{i=0}(\tau)$, which is similar to the target bursts, is prepared in any manner. For example, the waveform obtained from the threshold method, Case-a, is used as initial template represented as black line in Fig. 4 (c). Then, cross correlation $C_i(t)$ between a signal $x(t)$ (Fig. 4 (a)) and the initial template $X_{i=0}(\tau)$ is calculated as, $C_i(t) = \overline{x(t+\tau)X_i(\tau)}/\sqrt{\overline{x(t+\tau)^2}}\sqrt{\overline{X_i(\tau)^2}}$, where $\overline{f}$ indicates the temporal average over the length of the temporal window T, as $\overline{f(\tau)} = \frac{1}{T}\int_{-T/2}^{T/2} f(\tau)d\tau$.

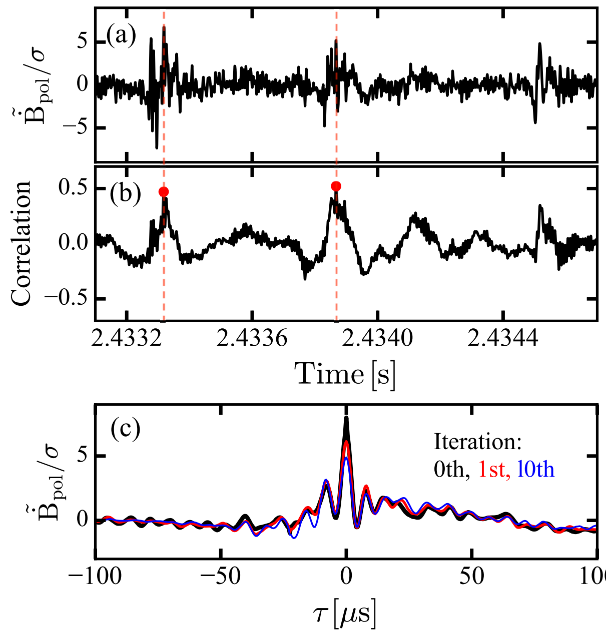

Fig. 4 The procedure of the template method is shown. (a) Raw signal of $\dot{B}_{\mathrm{pol}}$ and (b) cross correlation between raw signal and initial template where the red point indicates the clocks. (c) initial/0th iterated template (black), 1st iterated template (red) and converged, 10th iterated template (blue).

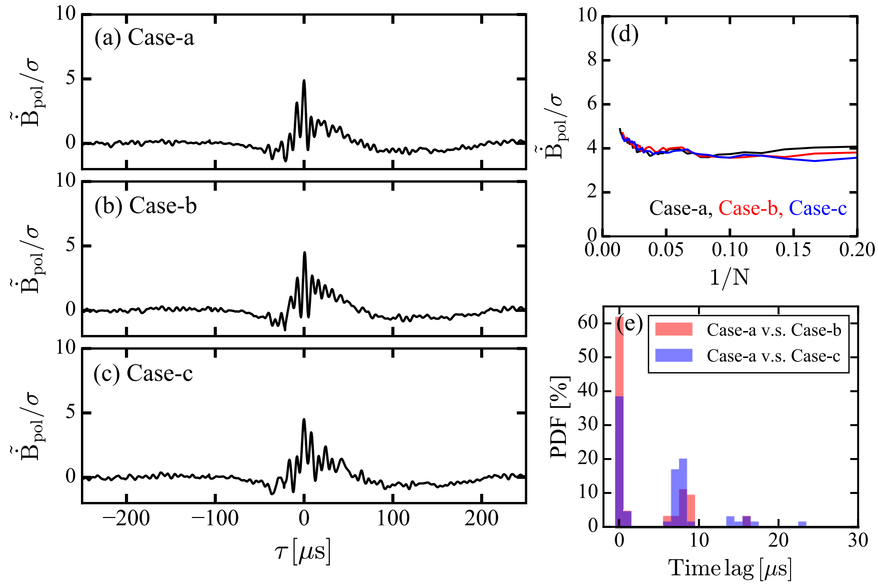

Fig. 5 Template method is performed. (a), (b) and (c) show the conditionally averaged waveforms of the bursts for the Case-a, Case-b and Case-c, respectively. (d) Convergence of the amplitude of the bursts at $\tau = 0\,\mu$s. (e) PDF of the clocks difference between Case-a and Case-b/Case-c.

Figure 4 (b) shows the cross correlation $C_{i=0}(t)$, which shows several local peaks indicated by the red points. Here, the clocks are selected the peak of the reference, where the cross-correlation should be above 0.45. Using the obtained clocks, the conditional average is carried out to give a new template waveform. Using the new template, the iteration is fulfilled until the template converges. In this case, the convergence is obtained after 10 times iteration, as shown in blue line in Fig. 4 (c). In order to investigate the dependence of the initial template, two other cases of the conditional averages are calculated using different initial templates corresponding to Case-b and Case-c in the threshold method in Figs. 3 (b) and 3(c).

Figure 5 show the results of these conditional averages, and demonstrate that the results are almost the same in spite of usage of the different initial templates. Similarly to the threshold method, Fig. 5 (d) indicates the convergence of the amplitudes at $\tau = 0$ μs against the ensemble number for the three cases. The total ensemble number N for Case-a, Case-a and Case-c are 73, 63 and 65, respectively. In the template method, every case converges to the almost same values. Besides, the time lags between the cases are almost around 0 μs, as shown in the PDF at Fig. 5 (e). It is noted that in Figs. (a)-(c), the template method provides the increase and decrease of high-frequency fluctuation components accompany with the bursts, which is consistent with the previous work [11]. Template method can conserve even high frequency components clearly.

4. Discussion and Summary

The comparison between the results of conditional averages using threshold and template methods is made in the case of the quasi-periodic phenomenon observed in I-mode in ASDEX Upgrade. The comparison shows the excellent properties of the template method that the obtained waveforms are independent of the choice of initial template waveforms. The excellent point is associated with the fact that the template method determines the clock and period of the phenomena from the whole properties of the waveforms, *e.g.*, shapes, height, length, and so on. The property guarantees that typical features can be extracted more automatically in quasi-periodic phenomena. On the other hand, here the three cases using the threshold method shows different waveforms, since the method only determines the clocks from a local property of the waveforms, *e.g.*, maximum values of local peaks. This suggests that the method could be useful to extract particular features with focus on a specific aspect of the target phenomena.

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