# Extension of the Ingenious Electrostatic Model to Electromagnetic Model for Large-Scale Plasma Simulation with Self-Consistent Electron Dynamics

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An ingenious model for large-scale electromagnetic (EM) plasma simulations is proposed. By introducing a dielectric tensor $\ddot{\varepsilon}$ with enlarged permittivity elements $\varepsilon_* \gg \varepsilon_0$ to Poisson equation, $\nabla \cdot (\ddot{\varepsilon}\nabla\phi) = -\rho$ ($\varepsilon_0$ is the permittivity of free space, $\phi$ is electrostatic potential and $\rho$ is charge density), the Debye length is artificially elongated and the large-scale system can be numerically treated even for including the self-consistent electron dynamics [T. Takizuka *et al.*, Plasma Fusion Res. **13**, 1203088 (2018)]. In cylindrical coordinates ($R$, $\theta$, $Z$) for three-dimensional tokamak simulations, a toroidal element $\varepsilon_{\theta\theta}$ is chosen much larger than poloidal elements $\varepsilon_{RR} = \varepsilon_{ZZ} = \varepsilon_*$, and a toroidal mesh size $\Delta_\theta$ can be set much larger than poloidal mesh sizes $\Delta_{R,Z}$. Resultantly the total mesh number becomes reasonably small and computation cost can be reduced. EM responses are also simulated using a modified Darwin model for Ampere's law, $\nabla^2 \boldsymbol{A} = -\mu_0(\boldsymbol{J} - \ddot{\varepsilon}\partial\nabla\phi/\partial t)$ ($\boldsymbol{A}$ is magnetic vector potential, $\boldsymbol{J}$ is current density, and $\mu_0$ is the permeability of free space), where light-speed EM waves are neglected. This modification is consistent with the charge-density continuity.



Keywords: electromagnetic simulation, global plasma, electron dynamics, Poisson equation, Darwin model

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Numerical simulation of global plasma dynamics is a powerful tool for the fusion research and development, i.e., understanding the underlying physics of experimental observations in existing machines, verifying the theory and modeling, and predicting the plasma performance for future fusion devices. We recently proposed an ingenious model for large-scale plasma simulations with self-consistent electron dynamics [1]. In the present paper, we aim to extend this electrostatic (ES) ingenious model to an electromagnetic (EM) ingenious model.

Majorities of global plasma simulations, e.g., MHD simulations or divertor simulations, have assumed the quasi-neutral condition. The electron dynamics, however, plays an important role in fusion plasmas especially for the case of interacting with the peripheral walls through SOL-divertor region. The ingenious model [1] was then developed for large-scale plasma simulations by including self-consistent electron dynamics within a reasonable computation cost. In the model, a dielectric tensor $\ddot{\varepsilon}$ with enlarged permittivity elements, $\varepsilon_* \gg \varepsilon_0$, is introduced to the Poisson equation,

$$\nabla \cdot (\ddot{\varepsilon}\nabla\phi) = -\rho, \tag{1}$$

where $\varepsilon_0$ is the permittivity of free space, $\phi$ is electrostatic potential and $\rho$ is charge density. The Debye length $\lambda_{D*} = (\varepsilon_* T_e/e^2 n_e)^{1/2}$ is artificially elongated, the mesh size $\Delta \sim \mathrm{O}(\lambda_{D*})$ is set reasonably large, and the large-scale system can be numerically treated. The plasma frequency $\omega_{p*} = (e^2 n_e/\varepsilon_* m_e)^{1/2}$ becomes small simultaneously, and the time step $\Delta t \sim \mathrm{O}(1/\omega_{p*})$ can be chosen reasonably large.

In cylindrical coordinates ($R$, $\theta$, $Z$) for three-dimensional (3D) tokamak simulations, a toroidal element $\varepsilon_{\theta\theta} = \alpha^2\varepsilon_*$ ($\alpha \gg 1$) is chosen much larger than poloidal elements $\varepsilon_{RR} = \varepsilon_{ZZ} = \varepsilon_*$. The toroidal mesh size $\Delta_\theta$ can be set $\alpha$ times larger than poloidal mesh sizes $\Delta_R$ and $\Delta_Z$. Resultantly the total mesh number becomes reasonably small even for 3D system and computation cost can be satisfactorily reduced [1].

Although the proposed ingenious model is applicable to the two-fluid (ion and electron fluids) modeling using the Poisson equation, we here concentrate its application to the particle-in-cell (PIC) method, which can describe fully the kinetic effect playing a significant role in the edge plasma [2]. In a PIC simulation with lowered $\omega_{p*}$ by the ingenious model, another high-frequency dynamics of electron, gyro motion, is removed by adopting guiding-center (GC) equations, while the ion motion with its gyration is fully followed.

The ES modeling above described is now extended to an EM modeling. EM response becomes important when a plasma pressure increases in a magnetic confinement system. In addition to the ES field, $\boldsymbol{E}_s = -\nabla\phi$, deter-

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mined by the Poisson equation, $\nabla^2\phi = -\rho/\varepsilon_0$, the interplay between magnetic field $\boldsymbol{B}$ and inductive electric field $\boldsymbol{E}_i$ has to be solved simultaneously with Faraday's law, $\nabla\times\boldsymbol{E}_i = -\partial\boldsymbol{B}/\partial t$. Magnetic field response is also coupled through Ampere's law, $\nabla\times\boldsymbol{B} = \mu_0\boldsymbol{J} + \mu_0\varepsilon_0\partial\boldsymbol{E}/\partial t$, where $\boldsymbol{E} = \boldsymbol{E}_s + \boldsymbol{E}_i$, $\boldsymbol{J}$ is current density and $\mu_0$ is the permeability of free space. When time evolution equations for $\boldsymbol{B}$ and $\boldsymbol{E}$, Faraday's law and Ampere's law without Poisson equation (number of equations are 6 in the 3D system), are used in principle for the EM PIC simulation [3], light-speed EM waves arise naturally in the system. This method, therefore, requires usually very small $\Delta t < \Delta/c$ ($c = (\mu_0\varepsilon_0)^{-1/2}$ is the speed of light in free space).

Introducing magnetic vector potential $\boldsymbol{A}$ ($\boldsymbol{B} = \nabla\times\boldsymbol{A}$), the condition of $\nabla\cdot\boldsymbol{B} = 0$ holds automatically, and $\boldsymbol{E}_i = -\partial\boldsymbol{A}/\partial t$. The Ampere's law is then rewritten as $\nabla^2\boldsymbol{A} - (1/c^2)\partial^2\boldsymbol{A}/\partial t^2 = -\mu_0\,(\boldsymbol{J}+\varepsilon_0\partial\boldsymbol{E}_s/\partial t)$, where Coulomb gage $\nabla\cdot\boldsymbol{A} = 0$ is applied. We see explicitly the light-speed wave propagation, $\nabla^2\boldsymbol{A} - (1/c^2)\,\partial^2\boldsymbol{A}/\partial t^2 = 0$. To eliminate the light-speed EM waves for the low-frequency EM plasma simulation, this Ampere's law is reduced to an equation called Darwin model, $\nabla^2\boldsymbol{A} = -\mu_0(\boldsymbol{J} + \varepsilon_0\partial\boldsymbol{E}_s/\partial t)$ [4]. In a PIC simulation based on this scheme, elliptic equations for the potentials, $\phi$ and $\boldsymbol{A}$, Poisson equation and Darwin-model Ampere's law (4 equations in the 3D system), are solved instead of time evolution equations for $\boldsymbol{B}$ and $\boldsymbol{E}$. Note the Poisson solver for $\phi$ (see e.g. [5]) can be applicable for $\boldsymbol{A}$ taking account of the condition, $\nabla\cdot\boldsymbol{A} = 0$.

In order to be consistent with the ingenious model using $\ddot{\varepsilon}$ in the Poisson equation, Eq. (1), the above Darwin model is modified to the form

$$\nabla^2\boldsymbol{A} = -\mu_0\left(\boldsymbol{J} + \ddot{\varepsilon}\frac{\partial\boldsymbol{E}_s}{\partial t}\right). \tag{2}$$

The divergence of the r.h.s. is kept zero, because of the charge-density continuity, $\nabla\cdot(\boldsymbol{J}+\ddot{\varepsilon}\partial\boldsymbol{E}_s/\partial t) = \nabla\cdot\boldsymbol{J}+\partial\rho/\partial t = 0$. The Coulomb gage condition of $\nabla\cdot\boldsymbol{A} = 0$ is then always satisfied.

In the ES PIC simulation, particle motions and Poisson equation are self-consistently coupled in the numerical accuracy of second-order time evolution, for example by using a leap-frog method [2]. Position $\boldsymbol{r}_p$ of a particle $p$ is given at an initial time $t_0$, and field quantities $\rho(\boldsymbol{r}, t_0)$ and $\boldsymbol{E}_s(\boldsymbol{r}, t_0)$ are calculated. The particle velocity $\boldsymbol{v}_p$ is set at a half-time-step earlier time $t_0 - \Delta t/2$, and is numerically accelerated during $\Delta t$ by a Lorentz force $\boldsymbol{F}$ at the time $t_0$; $\boldsymbol{v}_p(t_0 + \Delta t/2) = \boldsymbol{v}_p(t_0 - \Delta t/2) + \boldsymbol{F}(t_0)\Delta t/m$, where $\boldsymbol{F}(t) = q\{\boldsymbol{E}_s(\boldsymbol{r}_p(t), t)+\boldsymbol{v}_p(t)\times\boldsymbol{B}(\boldsymbol{r}_p(t), t)\}$, $q$ is charge, and $m$ is mass. Afterwards $\boldsymbol{r}_p$ is numerically moved from an initial time $t_0$ to a next time step $t_1 = t_0+\Delta t$ with its velocity $\boldsymbol{v}_p$ at the centered time $t_c = t_0+\Delta t/2$; $\boldsymbol{r}_p(t_1) = \boldsymbol{r}_p(t_0)+\boldsymbol{v}_p(t_c)\Delta t$. Field quantities $\rho(\boldsymbol{r}, t_1)$ and $\boldsymbol{E}_s(\boldsymbol{r}, t_1)$ are then recalculated. As for the electron GC equations, they are advanced, for example by a predictor-corrector method ($\boldsymbol{v}_p$ is also set at an initial time $t_0$). To use this method, the Poisson equation is solved twice at the predictor step $\boldsymbol{E}_s(\boldsymbol{r}, t_0)$ and at the corrector step $\boldsymbol{E}_s(\boldsymbol{r}, t_c)$, respectively [2].

When we program an EM PIC code applying the Darwin model, we have to examine several techniques. A candidate of the time sequence is the following predictor-corrector method.

$$\begin{aligned}
&\text{Initial}: \boldsymbol{r}_p(t_0), \boldsymbol{v}_p(t_0),\\
&\quad \text{Eq. (1)}: \rho(t_0) => \phi(t_0),\\
&\quad \text{Eq. (2)}: \boldsymbol{J}(t_0), \partial\boldsymbol{E}_s(t_0)/\partial t => \boldsymbol{A}(t_0), \partial\boldsymbol{A}(t_0)/\partial t.\\
&\text{Predictor}: \boldsymbol{r}_p(t_c) = \boldsymbol{r}_p(t_0) + \boldsymbol{v}_p(t_0)\Delta t/2,\\
&\qquad\qquad \boldsymbol{v}_p(t_c) = \boldsymbol{v}_p(t_0) + \boldsymbol{F}(t_0)\Delta t/2m,\\
&\quad \text{Eq. (1)}: \rho(t_c) => \phi(t_c),\\
&\quad \text{Eq. (2)}: \boldsymbol{J}(t_c), \partial\boldsymbol{E}_s(t_c)/\partial t => \boldsymbol{A}(t_c), \partial\boldsymbol{A}(t_c)/\partial t.\\
&\text{Corrector}: \boldsymbol{r}_p(t_1) = \boldsymbol{r}_p(t_0) + \boldsymbol{v}_p(t_c)\Delta t,\\
&\qquad\qquad \boldsymbol{v}_p(t_1) = \boldsymbol{v}_p(t_0) + \boldsymbol{F}(t_c)\Delta t/m,\\
&\quad \text{Eq. (1)}: \rho(t_1) => \phi(t_1),\\
&\quad \text{Eq. (2)}: \boldsymbol{J}(t_1), \partial\boldsymbol{E}_s(t_1)/\partial t => \boldsymbol{A}(t_1), \partial\boldsymbol{A}(t_1)/\partial t.
\end{aligned}$$

Here the electron diamagnetic current as a field quantity should be added to $\boldsymbol{J}$. Major difficulty compared to the ES PIC code is the calculation of time derivatives, $\partial\boldsymbol{E}_s/\partial t$ for Eq. (2) and $\boldsymbol{E}_i = -\partial\boldsymbol{A}/\partial t$ for equation of motions. Simple backward difference, $\partial\boldsymbol{A}(t_c)/\partial t \approx 2\{\boldsymbol{A}(t_c) - \boldsymbol{A}(t_0)\}/\Delta t$, degrades the numerical accuracy in the time evolution to the first order. When an iteration scheme is adopted to keep the second-order accuracy, the computation time is increased consequently.

If a global simulation code based on the above EM ingenious model is further developed in addition to the ingenious ES simulation code [1], it will be useful to study unresolved issues for tokamak edge plasmas affected by magnetic fluctuations, such as "density limit", "L-H transition", "QH-mode", "ELM dynamics", "transport under RMP", "SOL heat-flux width" etc.


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