

Effects of Hydrogen Negative Ions on Plasma with Magnetic Field Decreasing toward a Wall^{*)}

Azusa FUKANO, Ryoko TATSUMI¹⁾ and Akiyoshi HATAYAMA¹⁾

Monozukuri Department, Tokyo Metropolitan College of Industrial Technology, Higashi-Oi, Shinagawa, Tokyo 140-0011, Japan

¹⁾*Faculty of Science and Technology, Keio University, Hiyoshi, Kohoku-ku, Yokohama, Kanagawa 223-8522, Japan*

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Effects of H^- ion on distributions of electric potential and plasma density near the wall with the magnetic field decreasing toward the wall are investigated analytically. In the analysis, the magnetic field is assumed to be perpendicular to the wall, and the problem is treated as one-dimensional model. The plasma-sheath equation is derived analytically and the potential distribution near the wall is obtained by solving the plasma-sheath equation. The plasma density distributions are obtained from the potential distribution. It is shown that the effect of the production amount of H^- ion on plasma density distributions is large. The effect of the magnetic field profile and the ion temperature on the distributions of electric potential and particle density are also shown.

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1. Introduction

The detached divertor plasma is characterized by the reduction of heat load and ion influx from the core plasma of high temperature and high density toward the divertor plate through the volume recombination process. The molecular activated recombination (MAR) is predicted theoretically to enhance the recombination rate [1–3]. In this process, a hydrogen negative ion (H^-) produced by the reaction of a vibrationally excited hydrogen molecule and an electron contributes the charge exchange recombination process between the hydrogen positive ion (H^+) and the H^- ion. Thus, because the recombination rate of MAR is related to H^- ion, knowing a density distribution of the H^- ion near a wall is important. The distribution of the plasma density near the wall is related to the sheath potential distribution. For unmagnetized plasma, Emmert *et al.* investigated formation of the potential considering both the plasma and the sheath regions self-consistently by using a plasma-sheath equation [4]. Sato *et al.* extended the method of Emmert *et al.* to a case of magnetized plasma with the magnetic field decreasing toward the wall such as the divertor plasma [5], however the effect of the H^- ion has not been considered.

In this paper, we will study the distributions of the electric potential and the plasma density near the wall with the magnetic field decreasing toward the wall, where H^- ions are considered. The plasma-sheath equation is derived analytically and the distributions of the electric potential and the plasma density are obtained. Effects of the H^-

ion on distributions of the electric potential and the plasma density near the wall are shown. Effects of the magnetic field profile and the ion temperature on the distributions are also shown.

2. Derivation of Plasma-Sheath Equation

2.1 Analytical model and basic equations

The geometry of analytical model is shown in Fig. 1. In the analysis, walls on both sides are considered in order to maintain a conservation of particles. The problem is treated as one-dimensional model in z -direction. The electric potential $\phi(z)$ and the magnetic field $B(z)$ are assumed to be symmetric about $z = 0$ and decreases monotonically toward the walls, and $\phi(z)$ is zero and $B(z)$ is B_0 at $z = 0$. Plasma is assumed to consist of H^+ ions, H^- ions, and electrons. It is also assumed that the magnetic field is perpendicular to the walls and an effect of the magnetic presheath is ignored. Total energies E of the H^+ ion and E_- of the

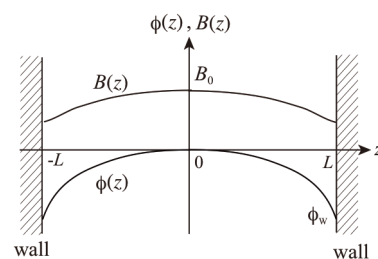


Fig. 1 Geometry of the analysis model.

author's e-mail: fukano@metro-cit.ac.jp

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H⁻ ion in the z-direction are

$$E = \frac{1}{2}M(v_{\perp}^2 + v_{\parallel}^2) + q\phi(z), \quad (1)$$

$$E_- = \frac{1}{2}M_-(v_{\perp-}^2 + v_{\parallel-}^2) - q\phi(z), \quad (2)$$

where M and M_- are the ion masses, v_{\perp} , $v_{\perp-}$ and v_{\parallel} , $v_{\parallel-}$ are the velocities perpendicular and parallel to the magnetic field, q and $-q$ are the charges of the H⁺ ion and the H⁻ ion, respectively. The subscript $-$ denotes value belonging to the H⁻ ion throughout this paper. The magnetic moments are given by

$$\mu = (1/2)Mv_{\perp}^2/B(z), \quad (3)$$

$$\mu_- = (1/2)M_-v_{\perp-}^2/B(z). \quad (4)$$

The kinetic equations for the H⁺ ion and the H⁻ ion in the phase space (z, E, μ) and (z, E_-, μ_-) are described by

$$\sigma v_{\parallel}(z, E, \mu) \frac{\partial f(z, E, \mu, \sigma)}{\partial z} = S(z, E, \mu), \quad (5)$$

$$\sigma v_{\parallel-}(z, E_-, \mu_-) \frac{\partial f_-(z, E_-, \mu_-, \sigma)}{\partial z} = S_-(z, E_-, \mu_-), \quad (6)$$

where $\sigma = \pm 1$ is the direction of the particle motion, $f(z, E, \mu, \sigma)$ and $f_-(z, E_-, \mu_-, \sigma)$ are the distribution functions, and $S(z, E, \mu)$ and $S_-(z, E_-, \mu_-)$ are the source functions. We assume a symmetry about $z = 0$ for $S(z, E, \mu)$ and $S_-(z, E_-, \mu_-)$. Furthermore, we assume that particles are not reflected at the wall, then the boundary conditions of the distribution functions are $f(-L, E, \mu, +1) = f(L, E, \mu, -1) = 0$ and $f_-(L, E_-, \mu_-, +1) = f_-(L, E_-, \mu_-, -1) = 0$.

2.2 Plasma sheath equation

From Eqs. (1) - (4), the parallel velocities of the H⁺ ion and the H⁻ ion are given by $v_{\parallel} = [(2/M)\{E - \mu B(z) - q\phi(z)\}]^{1/2}$ and $v_{\parallel-} = [(2/M_-)\{E_- - \mu_- B(z) + q\phi(z)\}]^{1/2}$. The energy space of the particle is divided to some regions, which is based on the condition that v_{\parallel} and $v_{\parallel-}$ must be real number, that is, $E - \mu B(z) - q\phi(z) \geq 0$ and $E_- - \mu_- B(z) + q\phi(z) \geq 0$ for the H⁺ ion and the H⁻ ion, respectively. The particle motion depends on its energy. The distribution functions $f(z, E, \mu, \sigma)$ and $f_-(z, E_-, \mu_-, \sigma)$ for $\sigma = \pm 1$ are obtained by integrating Eqs. (5) and (6) for particle trajectory with the boundary conditions. The sum of the distribution functions about $\sigma = \pm 1$ for each energy region become

$$\begin{aligned} & \sum_{\sigma} f(z, E, \mu, \sigma) \\ &= \begin{cases} 2 \int_0^L \frac{S(z', E, \mu)}{v_{\parallel}(z', E, \mu)} dz', (\mu B_0 < E < \infty), \\ 2 \int_{z_t}^L \frac{S(z', E, \mu)}{v_{\parallel}(z', E, \mu)} dz', (E_{\min} < E < \mu B_0), \end{cases} \quad (7) \\ & \sum_{\sigma} f_-(z, E_-, \mu_-, \sigma) \end{aligned}$$

$$= \begin{cases} 2 \int_0^L \frac{S_-(z', E_-, \mu_-)}{v_{\parallel-}(z', E_-, \mu_-)} dz', (\mu_- B(\pm L) - q\phi(\pm L) < E_- < \infty), \\ 2 \int_0^{z_{t-}} \frac{S_-(z', E_-, \mu_-)}{v_{\parallel-}(z', E_-, \mu_-)} dz', (E_{\min-} < E_- < \mu_- B(\pm L) - q\phi(\pm L)), \end{cases} \quad (8)$$

where z' and z'_- are the generation positions, z_t , and z_{t-} are the turning points of the H⁺ ion and the H⁻ ion, respectively, and $E_{\min} = \mu B(z) + q\phi(z)$ and $E_{\min-} = \mu_- B(z) - q\phi(z)$. As the source function, we use the expression same as the Emmert *et al.* [4] and Sato *et al.* [5]

$$S(z, E, \mu) = S_0 h(z) \frac{M^2 v_{\parallel}(z, E, \mu)}{4\pi(kT_i)^2} \times \exp\left\{-\frac{E - q\phi(z)}{kT_i}\right\}, \quad (9)$$

$$S_-(z, E_-, \mu_-) = S_{0-} h_-(z) \frac{M_-^2 v_{\parallel-}(z, E_-, \mu_-)}{4\pi(kT_{i-})^2} \times \exp\left\{-\frac{E_- + q\phi(z)}{kT_{i-}}\right\}, \quad (10)$$

where k is the Boltzmann's constant, T_i and T_{i-} are the temperatures, $h(z)$ and $h_-(z)$ are the source strengths that their average about z are normalized to 1, and S_0 and S_{0-} are the average source strengths of the H⁺ ion and the H⁻ ion, respectively. The density $n_i(z)$ of the H⁺ ion and the density $n_{i-}(z)$ of the H⁻ ion are obtained by integrating the Eqs. (7) and (8) over the $E - \mu$ and $E_- - \mu_-$ spaces as [5]

$$n_i(z) = \frac{2\pi B(z)}{M^2} \sum_{\sigma} \int dE \int d\mu \frac{f(z, E, \mu, \sigma)}{v_{\parallel}(z, E, \mu)}, \quad (11)$$

$$n_{i-}(z) = \frac{2\pi B(z)}{M_-^2} \sum_{\sigma} \int dE_- \int d\mu_- \frac{f_-(z, E_-, \mu_-, \sigma)}{v_{\parallel-}(z, E_-, \mu_-)}. \quad (12)$$

By substituting Eqs. (7) and (8) into Eqs. (11) and (12) respectively, and interchanging the order of integrations of them, the ion densities become

$$\begin{aligned} n_i(z) &= \frac{4\pi B(z)}{M^2} \int_0^L dz' \int_{E_s}^{\infty} dE \\ &\quad \times \int_0^{(E-E_s)/B_s} d\mu \frac{1}{v_{\parallel}(z, E, \mu)} \frac{S(z', E, \mu)}{v_{\parallel}(z', E, \mu)}, \quad (13) \\ n_{i-}(z) &= \frac{4\pi B(z)}{M_-^2} \left(\int_0^L dz' \int_{-E_p B_0/(B_0-B_p)}^{\infty} dE_- \right. \\ &\quad \times \int_0^{(E_-+q\phi(z))/B(z)} d\mu_- \frac{1}{v_{\parallel-}(z, E_-, \mu_-)} \frac{S_-(z', E_-, \mu_-)}{v_{\parallel-}(z', E_-, \mu_-)} \\ &\quad \left. + \int_0^L dz' \int_{-E_p}^{-E_p B_0/(B_0-B_p)} dE_- \right. \\ &\quad \left. \times \int_0^{(E_-+E_p)/B_p} d\mu_- \frac{1}{v_{\parallel-}(z, E_-, \mu_-)} \frac{S_-(z', E_-, \mu_-)}{v_{\parallel-}(z', E_-, \mu_-)} \right), \quad (14) \end{aligned}$$

where $E_p = q\phi(z)$, $B_p = B(z)$, $E_s = q\phi(z')$, $B_s = B(z')$ for $z' < z$, and $E_p = q\phi(z')$, $B_p = B(z')$, $E_s = q\phi(z)$, $B_s = B(z)$

for $z' > z$. Where, we considered a case that the decrease rate of the magnetic field toward the wall is smaller than that of the potential. By substituting Eqs. (9) and (10) into Eqs. (13) and (14), respectively and integrating them for μ , μ_- and E , E_- , we obtain

$$n_i(z) = S_0 \left(\frac{\pi M}{2kT_i} \right) \int_0^L dz' I(z, z') h(z'), \quad (15)$$

$$n_{i-}(z) = S_{0-} \left(\frac{\pi M_-}{2kT_{i-}} \right) \int_0^L dz'_- I_-(z, z'_-) h_-(z'_-), \quad (16)$$

where

$$I(z, z') = \begin{cases} \exp \left\{ \frac{q\phi(z') - q\phi(z)}{kT_i} \right\} \operatorname{erfc} \left[\left\{ \frac{q\phi(z') - q\phi(z)}{kT_i} \right\}^{1/2} \right] \\ - \left\{ \frac{B(z') - B(z)}{B(z')} \right\}^{1/2} \times \exp \left\{ \frac{B(z') - q\phi(z') - q\phi(z)}{kT_i} \right\} \\ \times \operatorname{erfc} \left[\left\{ \frac{B(z') - q\phi(z') - q\phi(z)}{kT_i} \right\}^{1/2} \right], \\ q\phi(z') > q\phi(z), \\ \exp \left\{ \frac{q\phi(z') - q\phi(z)}{kT_i} \right\}, q\phi(z') < q\phi(z), \end{cases} \quad (17)$$

$$I_-(z, z'_-) = \begin{cases} \exp \left\{ \frac{q\phi(z) - q\phi(z'_-)}{kT_{i-}} \right\}, q\phi(z'_-) > q\phi(z), \\ \exp \left\{ \frac{q\phi(z) - q\phi(z'_-)}{kT_{i-}} \right\} \operatorname{erfc} \left[\left\{ \frac{q\phi(z) - q\phi(z'_-)}{kT_{i-}} \right\}^{1/2} \right] \\ + \frac{2}{\sqrt{\pi}} \left\{ \frac{q\phi(z)(B_0 - B(z'_-)) - q\phi(z'_-)(B_0 - B(z))}{kT_{i-}(B_0 - B(z'_-))} \right\}^{1/2} \\ \times \exp \left\{ \frac{q\phi(z'_-) B(z'_-)}{kT_{i-}(B_0 - B(z'_-))} \right\} + \left\{ \frac{B(z) - B(z'_-)}{B(z'_-)} \right\}^{1/2} \\ \times \exp \left\{ \frac{(q\phi(z) - q\phi(z'_-)) B(z'_-)}{kT_{i-}(B(z'_-) - B(z))} \right\} \\ \times \left[\operatorname{erfi} \left[\left\{ -\frac{(q\phi(z) - q\phi(z'_-)) B(z'_-)}{kT_{i-}(B(z'_-) - B(z))} \right\}^{1/2} \right] \right. \\ \left. - \operatorname{erfi} \left[\left\{ -\frac{(q\phi(z)(B_0 - B(z'_-)) - q\phi(z'_-)(B_0 - B(z))) B(z'_-)}{kT_{i-}(B(z'_-) - B(z))(B_0 - B(z'_-))} \right\}^{1/2} \right] \right], \\ q\phi(z'_-) < q\phi(z), \end{cases} \quad (18)$$

where $\operatorname{erfc}(x)$ is the complementary error function and $\operatorname{erfi}(x)$ is the imaginary error function. As the electron density n_e , we use a Boltzmann distribution $n_e(z) = n_0 \exp[e\phi(z)/kT_e]$ for simplicity, where n_0 is the electron density at $z = 0$, $-e$ is the electron charge, and T_e is the electron temperature. Substituting Eqs. (15), (16) and the electron density into Poisson's equation, the plasma-sheath equation is derived

$$\begin{aligned} \frac{d^2\phi}{dz^2} &= \frac{n_0 e}{\epsilon_0} \exp \left(\frac{e\phi(z)}{kT_e} \right) \\ &- \frac{S_0 q}{\epsilon_0} \left(\frac{\pi M}{2kT_i} \right) \int_0^L dz' I(z, z') h(z') \\ &+ \frac{S_{0-} q}{\epsilon_0} \left(\frac{\pi M_-}{2kT_{i-}} \right) \int_0^L dz'_- I_-(z, z'_-) h_-(z'_-). \end{aligned} \quad (19)$$

The average source strengths S_0 and S_{0-} are derived from the equilibrium of the fluxes of the plasma particles at the wall. We consider $j_{ew} + j_{iw} + j_{i-w} = 0$, where j_{ew} , j_{iw} , and j_{i-w} are the current densities of the electron, the

H^+ ion, and the H^- ion at the wall, respectively, and given by $j_{ew} = -en_0[kT_e/(2\pi m_e)]^{1/2} \exp[e\phi_w/(kT_e)]$ and $j_{iw} = qS_0 L$ from $j_e = -(1/4)en_e(z)\langle v_e \rangle$ and $\nabla \cdot j_i = qS(z)$, where $\langle v_e \rangle = \{8kT_e/(\pi m_e)\}^{1/2}$ is the electron average velocity, m_e is the electron mass, ϕ_w is the wall potential. Furthermore, we define a rate of production amount of the H^- ions to the H^+ ions to be $\beta = S_{0-}/S_0$. The average source strengths S_0 and S_{0-} are

$$S_0 = \frac{en_0}{qL(1-\beta)} \left(\frac{kT_e}{2\pi m_e} \right)^{1/2} \exp \left(\frac{e\phi_w}{kT_e} \right), \quad (20)$$

$$S_{0-} = \frac{en_0\beta}{qL(1-\beta)} \left(\frac{kT_e}{2\pi m_e} \right)^{1/2} \exp \left(\frac{e\phi_w}{kT_e} \right). \quad (21)$$

3. Numerical Solutions

Since Eq.(19) cannot be solved analytically, it is solved numerically. We introduce the normalized variables such as $\eta = (q/kT_e)(\phi_w - \phi)$, $R = B/B_0$, $s = z/L$, $\tau = T_e/T_i$, $\tau_- = T_e/T_{i-}$, $Z = q/e$, where R is the mirror ratio and $Z = 1$ for the hydrogen plasma. We assume that the ions are generated uniformly, that is, $h(z) = h_-(z) = 1$ for simplicity. The boundary conditions are $dh/ds|_{s=0} = 0$ and $\eta(s = 1) = 0$. We assume the mirror ratio similar to the expression used by Sato *et al.* [5]

$$R(\eta) = \exp[-\alpha\{\eta - e\phi_w/(kT_e)\}], \quad (22)$$

where α is a positive constant and indicates a degree of decrease of the magnetic field toward the wall. The profile of the normalized electric potential $\Phi(s) = -\eta$ for various values of the production amount of the H^- ion to the H^+ ion is shown in Fig. 2, where $\lambda_D/L = 5 \times 10^{-2}$ and λ_D is the Debye length. We will use the value of $\lambda_D/L = 5 \times 10^{-2}$ in all results of this paper. As the value of $\beta = S_{0-}/S_0$ becomes large, the sheath width becomes large and the sheath potential decreases. This seems that because the large value of β means that the amount of the electrons is small, the electrons that reach to the wall decrease and consequently the drop of the sheath potential decreases. The density distributions of the plasma particles are derived from Eqs. (15), (16) and $n_e(z) = n_0 \exp[e\phi(z)/kT_e]$.

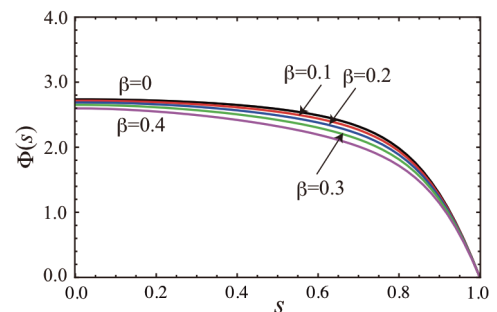


Fig. 2 Profile of the normalized electric potential for various values of $\beta = S_{0-}/S_0$, with $\tau = \tau_- = 1$, $\alpha = 0.4$.

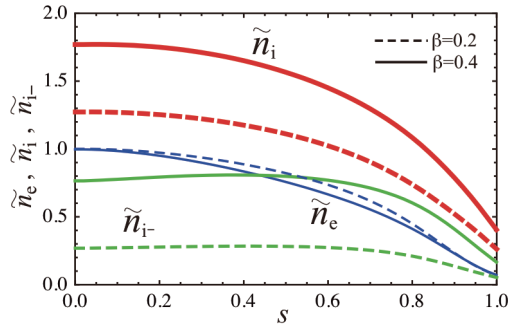


Fig. 3 Profile of the normalized plasma densities for cases of $\beta = S_{0-}/S_0 = 0.2$ and 0.4 , with $\tau = \tau_- = 1$, $\alpha = 0.4$.

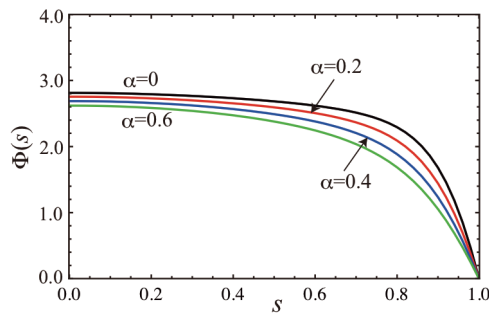


Fig. 4 Profile of the normalized electric potential for various values of α , with $\tau = \tau_- = 1$, $\alpha = 0.2$.

The profile of the plasma densities normalized by the electron density at $s = 0$ for cases of $\beta = 0.2$ and 0.4 are shown in Fig. 3, where thick line, middle thick line, and thin line express the profiles of the H^+ ion, the H^- ion, and the electron, respectively. As the production amount of the H^- ion to the H^+ ion becomes large, the densities of the H^+ ion and the H^- ion become large. It is found that the rate of the H^- ion density to the H^+ ion density near the wall is larger than the production rate of the H^- ion to the H^+ ion. For example, $n_{i-}/n_i = 0.56$ at $s = 0.8$ for the case of $\beta = 0.4$. The profile of $\Phi(s)$ for various values of α is shown in Fig. 4. As the decrease of the magnetic field becomes large, the sheath width becomes large and the sheath potential decreases. This seems because the electrons are easy to move along the magnetic field toward the wall more than the ions. The profile of the normalized particle densities for cases of $\alpha = 0$ and 0.6 are shown in Fig. 5. It is shown that for the case of decreasing magnetic field the densities of the H^+ ion and the electron become small and the H^- ion density becomes large near the wall compared with the case of uniform magnetic field. The profile of $\Phi(s)$ for various values of $\tau = T_e/T_i$ and $\tau_- = T_e/T_{i-}$ is shown in Fig. 6, where $\tau = \tau_-$. The small value of τ corresponds to the large ion temperature. As the ion temperature becomes large, the potential drop becomes small. The profile of the normalized particle densities for cases of $\tau = 0.5$ and 2 are shown in Fig. 7. As the ion temperature becomes large, the H^- ion density near the wall becomes large. It

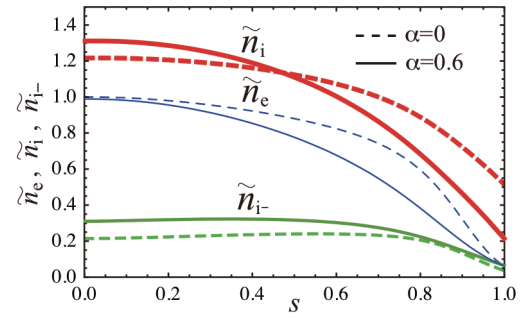


Fig. 5 Profile of the normalized plasma densities for cases of $\alpha = 0$ and 0.6 , with $\tau = \tau_- = 1$, $\beta = 0.2$.

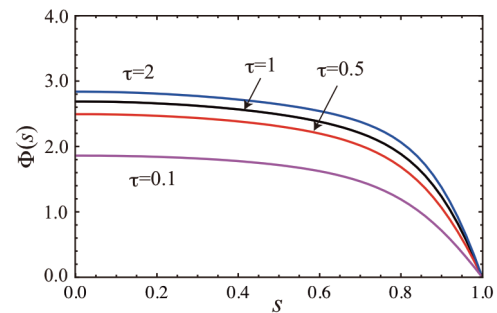


Fig. 6 Profile of the normalized electric potential for various values of $\tau = \tau_-$, with $\alpha = 0.4$, $\beta = 0.2$.

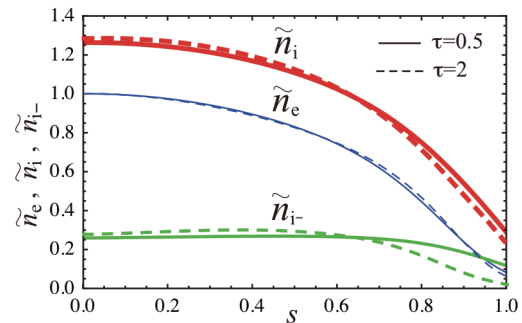


Fig. 7 Profile of the normalized plasma densities for cases of $\tau = \tau_- = 0.5$ and 2 , with $\alpha = 0.4$, $\beta = 0.2$.

seems that H^- ions with large energy are easy to move toward the wall over the potential gradient. On the whole, it is found that the quasi-neutrality is kept in internal plasma and n_i is larger than $n_{i-} + n_e$ near the wall.

4. Conclusions

The distributions of the electric potential and the plasma density near the wall for the plasma that consists of the H^+ ion, the H^- ion, and the electron with the magnetic field decreasing toward the wall are studied analytically. The plasma-sheath equation is derived theoretically and solved numerically. As a result, for the cases of large production rate of the H^- ion and large ion temperature,

the H^- ions are much produced near the wall and it is expected that the plasma recombination rate is enhanced through MAR.

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