Thermal Motion of Charged Particles in Confined Ensemble under Constant Electromagnetic Field

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Thermal motion of charged particles in the field of electrostatic trap under an influence of constant magnetic field is investigated analytically and numerically. For the first time the conditions for energy balance of these particles in the systems with the spatially non-uniform thermal sources are proposed. The numerical simulations were carried out for the ensembles consisting of one to thousands of particles in a wide range of parameters of the analyzed systems. Comparisons of their spectral characteristics are presented. We found that the shape of the spectral density distributions in these systems is practically independent of the number of particles in the analyzed ensembles, and their characteristic frequencies can be obtained by an analytical solution of motion equations for a single charged particle.

Keywords: low temperature plasma, magnetic field, computer simulation, Langevin equation, electrostatic trap

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1. Introduction

Considerable interest in the studies of charged particle dynamics in external electromagnetic fields is defined by development of effective power installations for Controlled Thermonuclear Fusion (CTF) [1–3], and also of methods for plasma reprocessing of Spent Nuclear Fuel (SNF) and Radioactive Wastes (RAW) [4–9]. Spent nuclear fuel (SNF), occasionally called used nuclear fuel, is nuclear fuel that has been irradiated in a nuclear reactor (usually at a nuclear power plant). A reprocessing of SNF and RAW needed for nuclear fuel cycle closure for the purpose of more complete involvement of the resources is actual problem for nuclear industry [4–9].

An influence of the thermal motions of charged particles on their dynamics in the external electric, and magnetic fields is of special interest too [10–15]. Thermal motions have a significant influence on the dynamics of interacting particles in natural systems, as well as in the biological and polymer colloid suspensions, in plasmas of combustion products, in the Earth atmosphere and other media [16,17]. Nevertheless, the simple analytical approaches are developed for two cases only: for the non-interacting grains and for a single charged particle in a field of the trap. The analysis of these problems does not allow to investigate an influence of the number, N, of interacting particles ($N > 1$) on the characteristics of their motion. For this purpose a numerical simulation is commonly used.

The trajectories and mean square displacements of a single Brownian grain in an electrostatic trap under the influence of magnetic field were studied in Refs. [11–13]. Experimental and numerical analyses of the dynamics of an ensemble of interacting Brownian dust particles in the electric fields which are formed in the gas discharge plasma were presented in Refs. [18, 19]. An influence of the stochastic (thermal) motion in the clouds of charged particles on their dynamics in the constant electromagnetic fields was recently studied in Refs. [14, 15].

Note also the possibility of non-uniform distributions of the stochastic kinetic energy on degrees of freedom (for example, due to inhomogeneous parameters of the analyzed systems) for ions/electrons of a plasma or charged dust grains in gas discharges [10, 20–22]. The redistribution of the stochastic kinetic energy for systems of charged particles with such non-uniform thermal sources in the absence of magnetic fields was studied in detail in Refs. [20] and [21]. (Here and below "the thermal sources" are the sources of the stochastic energy of particles with the velocities corresponding to the Maxwell function.)

In this paper, thermal motion of charged particles in the field of electrostatic trap under an influence of constant magnetic field is investigated analytically and numerically. For glow gas discharges without magnetic field, $B = 0$, the concentration of positive ions can excess the electron concentration in the center of the discharge chambers [10]. This leads to the formation of effective traps for negatively charged particles (e.g., for dust particles) [18, 19].

According to the Poisson equation, in case of violation of the plasma electro-neutrality (due to the predominance of concentration of its positive or negative component), the arising electric field restricts the motion of particles of the

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opposite sign [18, 19, 24]. Thus, the confinement fields are formed, which are a trap for these particles.

In the plasma with the magnetic field, $B \neq 0$, the situation in the center of the gas discharge chamber may be differed (due to the "magnetization" of the plasma electrons), i.e., there can be conditions for the retention of positively charged particles. Thus, the presence of electrostatic trap for positive ions and/or for any positively charged particles in installations for the separation of SNF components may occur due to the "magnetization" of electrons on the axes of the discharge chambers, which are typically used for these purposes [7, 8].

In the Sec. 2 of this paper the results of analytical studies of dynamics of single charged particles in the field of the electrostatic trap under the influence of magnetic fields are presented. Analysis of the frequency spectrum of their oscillations is performed. In the Sec. 3 the conditions for an energy balance of the systems in the presence of spatially non-uniform thermal sources are studied.

In the Sec. 4 the results of numerical simulations for systems consisting of one ($N = 1$) to $N \sim 1000$ charged particles are described for a wide range of parameters of these systems. Calculations were performed for ions with the charge number $Z = 1$ and atomic masses of $M_1 = 50$ a.m.u. and $M_2 = 150$ a.m.u. that simulated the fission products of uranium [8, 9]. The friction coefficients of these ions, ν, due to their collisions with gas neutrals ranged from $\sim 8000\,\mathrm{s}^{-1}$ to $\sim 80000\,\mathrm{s}^{-1}$, which correspond to the buffer gas pressure (like argon and helium) $\sim$ 1 mTorr and $\sim$ 10 mTorr, respectively. It should be noted that these types of gases with similar pressures can be used for plasma separation facilities of SNF and RAW components [7, 8].

2. Dynamics of Charged Particles in Electromagnetic Field

For the analysis of the dynamics of particles in the constant electromagnetic fields we consider the motion equations (Langevin equations) for a single charged particle with mass M and charge Q in the external electric field $\mathbf{E} = [E_x; E_y; E_z]$ of electrostatic trap and in the magnetic field with induction $B = B_y$ (which is directed along the **y**-axis) under an action of random force $\mathbf{F_b} = [F_{bx}; F_{by}; F_{bz}]$ that is the source of the stochastic (thermal) energy:

$$\mathrm{d}V_y/\mathrm{d}t = -\nu V_y - Q\alpha_y y/M + F_{by}/M, \tag{1a}$$

$$\mathrm{d}V_x/\mathrm{d}t = -\nu V_x - Q\alpha_x x/M + QBV_z/M + F_{bx}/M, \tag{1b}$$

$$\mathrm{d}V_z/\mathrm{d}t = -\nu V_z - Q\alpha_z z/M - QBV_x/M + F_{bz}/M. \tag{1c}$$

Here y, x, z are displacements of the particle from its equilibrium position; $V_y = \mathrm{d}y/\mathrm{d}t$, $V_x = \mathrm{d}x/\mathrm{d}t$, $V_z = \mathrm{d}z/\mathrm{d}t$ are the components of the particle velocity; α_y, α_x, α_z are the components of gradients of the external electric field for different degrees of freedom, $B = B_y$ is the value of magnetic field induction, and ν is the friction coefficient (reverse deceleration time) for the charged particles due to their collisions with neutrals of surrounding gas.

It is obvious that the motion of a charged particle along the magnetic field (see Eq. (1a)) is stable for the case $\nu > 0$ and $Q\alpha_y > 0$. The characteristic equation for the system of Eqs. (1b), (1c) has the form

$$\lambda^4 + 2\nu\lambda^3 + (\nu^2 + {\omega_B}^2 + Q\{\alpha_x + \alpha_z\}/M)\lambda^2 + \\ + \nu Q(\alpha_z + \alpha_x)\lambda/M + Q^2\alpha_z\alpha_x/M^2 = 0. \tag{2}$$

Here $\omega_B = QB/M$ is the cyclotron frequency.

A particle motion in the plane orthogonal to the magnetic field will be stabile if the Routh-Hurwitz criteria are satisfied. In the case of $\alpha_x = \alpha_z \equiv \alpha$, a solution of the system of Eqs. (1b) and (1c) is always stable for $\nu > 0$ and $Q\alpha > 0$.

The roots of the characteristic Eq. (2) for $\alpha_x = \alpha_z \equiv \alpha$ can be written as

$$\lambda_{1,2} = -\Psi_1 \pm i\Omega_1, \tag{3}$$

$$\lambda_{3,4} = -\Psi_2 \pm i\Omega_2, \tag{4}$$

where $\Psi_1 = \nu(1 + D_1/\sqrt{2})/2$, $\Psi_2 = \nu(1 - D_1/\sqrt{2})/2$, $\Omega_1 = (\omega_B + \nu D_2/\sqrt{2})/2$, $\Omega_2 = (\omega_B - \nu D_2/\sqrt{2})/2$,

$$D_1 = \Big[\Big((1 - (\omega_B/\nu)^2 - 4(\omega_t/\nu)^2)^2 + 4(\omega_B/\nu)^2\Big)^{1/2} + \\ +1 - (\omega_B/\nu)^2 - 4(\omega_t/\nu)^2\Big]^{1/2}, \tag{5a}$$

$$D_2 = \Big[\Big((1 - (\omega_B/\nu)^2 - 4(\omega_t/\nu)^2)^2 + 4(\omega_B/\nu)^2\Big)^{1/2} - \\ -1 + (\omega_B/\nu)^2 + 4(\omega_t/\nu)^2\Big]^{1/2}. \tag{5b}$$

Here $\omega_t = (Q\alpha/M)^{1/2}$ is the characteristic frequency of the electric trap.

The imaginary parts of the roots of Eqs. (3), (4) are responsible for the characteristic frequencies of particle oscillation (Ω_1, Ω_2) in the direction transverse to the magnetic field.

In the case of $\omega_B = 0$ the roots of Eq. (2) for $\alpha_x = \alpha_z \equiv \alpha$ have the well-known form:

$$\lambda_{1,2} = -\nu/2 \pm i\Omega_\mathrm{t}, \tag{6}$$

where $\Omega_\mathrm{t} = ({\omega_t}^2 - \nu^2/4)^{1/2}$.

Note that the system of Eqs. (1a) - (1c) can be used to analyze the motion of the center of mass of any finite ensemble of charged particles with pairwise interparticle interactions and also for weakly non-ideal systems with the coupling parameter $\Gamma = Q^2 n^{-1/3}/T \ll 1$, where n is the concentration of charged particles and T is their kinetic temperature.

3. Energy Balance Condition

We consider the energy balance conditions in the system of Eqs. (1b) - (1c), taking into account that for the case of stochastic motion of particles: $\langle F_{bx}\rangle = \langle F_{bz}\rangle \equiv 0$,

$\langle xF_{bx}\rangle = \langle zF_{bz}\rangle \equiv 0$, $\langle xV_x\rangle = \langle zV_z\rangle \equiv 0$, $\langle xV_z\rangle + \langle zV_y\rangle = 0$, $\langle V_xF_{bx}\rangle = \nu T_x^{\mathrm{o}}$ and $\langle V_zF_{bz}\rangle = \nu T_z^{\mathrm{o}}$, where T_x^{o}, T_z^{o} are the temperatures of thermal sources for the respective degrees of freedom [16, 17]. (Here and below, the angle brackets $\langle\ \rangle$ denote time averaging at $t \to \infty$).

Then, we rewrite the system of Eqs. (1b) - (1c), using the method of correlations [16, 17, 23]:

$$0.5\mathrm{d}(T_x + T_z)/\mathrm{d}t = -\nu(\delta T_x + \delta T_z), \quad (7a)$$

$$0.5\mathrm{d}(T_x - T_z)/\mathrm{d}t = -\nu(\delta T_x - \delta T_z) - 2\omega_B\langle V_xV_z\rangle M, \quad (7b)$$

$$M\mathrm{d}\langle V_xV_z\rangle/\mathrm{d}t = -2\nu\langle V_xV_z\rangle M - \omega_B(T_x - T_z), \quad (7c)$$

where $T_x = M\langle V_x{}^2\rangle$, $T_z = M\langle V_z{}^2\rangle$ are the doubled values of the equilibrium stochastic kinetic energy for the respective degrees of freedom in the plane orthogonal to the direction of the magnetic field $\boldsymbol{B}$; and $\delta T_x = T_x^{\mathrm{o}} - T_x$, $\delta T_z = T_z^{\mathrm{o}} - T_z$.

Procedure of the transformation of Eqs. (1b), (1c) into the system of Eqs. (7a) - (7c) with a use of the correlators, presented in beginning of this Section, was detailed in a set of the works [16, 17, 23]. So, we multiply the Eq. (1b) by the value of V_x, and we multiply the Eq. (1c) by V_z, respectively. Then we average these equations using the above-mentioned correlators. The sum of the transformed equations (i.e. multiplied and averaged Eqs. (1b) - (1c)) is the Eq. (7a); and their difference is the Eq. (7b). Then we multiply the Eq. (1b) by V_z and we multiply the Eq. (1c) by the value of V_x; we average these equations using the above-mentioned correlators. The sum of these transformed equations will give, respectively, the Eq. (7c).

Thus, by solving the system of Eqs. (7a) - (7c), the equation of energy balance for the system of Eqs. (1b) - (1c) with an additional stochastic energy, $T_x^{\mathrm{o}} \neq T_z^{\mathrm{o}}$, arising due to any physical processes, can be represented as:

$$\delta T_x \equiv -\delta T_z = 0.5\omega_B{}^2(T_x^{\mathrm{o}} - T_z^{\mathrm{o}})/(\nu^2 + \omega_B{}^2). \quad (8)$$

Notice that Eq. (8) was obtained by a simple solving the system of Eqs. (7a) - (7c) for the case of equilibrium systems, when the values of T_x, T_z, $\langle V_xV_z\rangle$ are independent on time; i.e. the left parts of Eqs. (7a) - (7c) containing derivatives of these values are equal to 0.

It is readily seen that $\delta T_x \equiv -\delta T_z = 0$ in the case $\omega_B = 0$ and/or $\delta T_x \equiv -\delta T_z \to 0$ for $\nu \gg \omega_B$. In the opposite case ($\nu \ll \omega_B$) the additional stochastic energies will be uniformly redistributed between the corresponding degrees of freedom.

4. Results of Numerical Calculations and Their Discussion

The numerical simulation was performed for the systems consisting of from one ($N = 1$) to $N = 1000$ charged particles in isotropic electrostatic trap ($\alpha_x = \alpha_z = \alpha_y \equiv \alpha$) under the influence of a constant magnetic field, $B = B_y$ by three-dimensional Langevin molecular dynamic method. This method based on the solution of the system of $3N$-differential equations of motion which included the forces of pair inter-particle interaction, external electrical and magnetic forces and the random force $\mathbf{F_b}$ that is the source of the stochastic (thermal) motion of particles [16, 17, 24]. The latter takes into account processes leading to the established equilibrium kinetic temperature T of particles that characterizes kinetic energy of their stochastic (thermal) motion according the fluctuation-dissipation theorem [25, 26]. The simulation technique is detailed in Ref. [24].

For correct simulations the integration step was varied from $\Delta t \cong (40\max[\omega_B;\ \omega_t;\ \nu])^{-1}$ to $\Delta t \cong (100\max[\omega_B;\ \omega_p;\ \nu])^{-1}$ depending on the initial conditions. The computation time t_c after the establishment of equilibrium in the simulated systems was from $\sim 10^3/\min[\omega_B; \omega_t; \nu]$ to $\sim 10^4/\min[\omega_B; \omega_t; \nu]$.

The calculations were performed for ions of atomic masses $M_1 = 50$ a.m.u. and $M_2 = 150$ a.m.u., with the charge number $Z = 1$ and at the room temperature ($T \cong 0.025$ eV) and at the temperature $T \cong 0.077$ eV. In the latter case, we took into account the possible heating of the buffer gas [21]. The friction coefficient for ions, ν, was varied from $\sim 8000\ \mathrm{s}^{-1}$ to $80000\ \mathrm{s}^{-1}$. The magnetic induction, $B = B_y$, was in the range from 200 G to 2000 G; the value of gradients of external electric field, α, was changed from $\sim 0.1\ \mathrm{V/cm^2}$ to $\sim 1000\ \mathrm{V/cm^2}$. The average ion concentration, n, near the trap center for $N > 100$ was changed from $\sim 10^4\ \mathrm{cm^{-3}}$ to $\sim 10^8\ \mathrm{cm^{-3}}$ depending on the value of electric field gradient, α, and on the number of particles, N, in the simulated ensembles. Estimation of the coupling parameter, Γ, for the simulated systems yields: $\Gamma = Q^2n^{-1/3}/T \ll 1$.

In all cases the simulated systems were stable. The temperatures of particles did not differ from the preset points, and the velocity distribution functions corresponded to the Maxwell functions. The mean square displacements of particles at all degrees of freedom were almost equal: $\langle x^2\rangle \cong \langle y^2\rangle \cong \langle z^2\rangle \approx T/(M\omega_t{}^2)$.

Trajectories of individual particles of the ensembles in the plane orthogonal to the magnetic field $\boldsymbol{B}$ and in the plane parallel to $\boldsymbol{B}$, were some different. So, the motions of charged particles in the direction of magnetic field were close to the oscillating motions with the frequency close to ω_{t} for $2\omega_{\mathrm{t}} \gg \nu$. While in the plane orthogonal to $\boldsymbol{B}$, the marked rotational motions were observed. A similar result was obtained earlier for the case of a single particle [11, 12].

The trajectories of an arbitrary selected ion (i.e. an ion chosen randomly from a simulated system consisting of N – particles (ions)) in the ensemble of $N = 500$ ions with $M = M_1$, $\Omega_{\mathrm{t}}/\nu \cong 18.27$, $\omega_B/\nu \cong 28.35$ and $\nu = 8000\ \mathrm{s}^{-1}$ are presented in Figs. 1 (a), 1 (b) in $[\mathbf{x}, \mathbf{y}]$ and $[\mathbf{x}, \mathbf{z}]$ planes, respectively. The displacements of an arbitrary selected ion as function of $\Omega_{\mathrm{t}}t$ in $\mathbf{y}$ and $\mathbf{x}$ directions, as well as the results of spectral analysis of its displacements are shown in Figs. 2 (a), 2 (b) and 2 (c), respectively. Illustration of spectral analysis for one ion with $M = M_2$ and for the mass center of the ensemble of $N = 1000$ ions with $M = M_2$,

$\Omega_t/\nu \cong 6.14$, $\omega_B/\nu \cong 15.07$, $\nu = 8000\,\text{s}^{-1}$ are presented in Figs. 3 (a), 3 (b).

When the value of $B = B_y$ changes, the value of $\omega_B = QB/M$ (the cyclotron frequency) changes too. The results of numerical simulation for different frequencies ω_B due to the change in the magnetic field, $B = B_y$, are shown in Figs. 2, 3 (see figure captions).

It is easy to see that the frequency spectra of the arbitrary selected ions and of the ensemble mass center correspond well to the harmonics Ω_t and Ω_1, Ω_2, which were obtained by solving the system of Eqs. (1a) - (1c) for the case of a single particle in the electric trap, see Table 1.

The energy balance of charged particles in external constant electromagnetic field is studied. The results of simulations for the systems with non-uniform thermal sources of energy have shown that in all analyzed cases the condition of balance (see Eq. (8)) is valid.

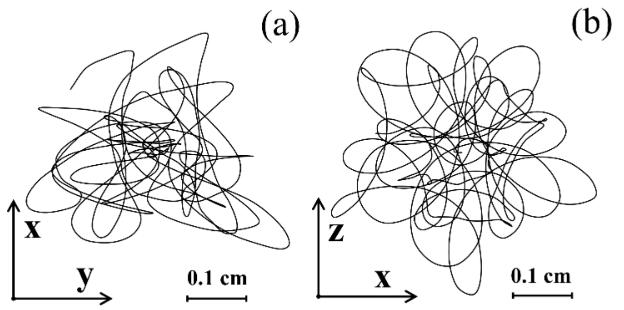

Fig. 1 Trajectories of an arbitrary selected ion in the ensemble of N = 500 ions with M = M_1 in the plane [**x**, **y**] (a) and in the plane [**x**, **z**] (b) over the time $\sim 1/\omega_B$ for $\Omega_t/\nu \cong 18.27$, $\omega_B/\nu \cong 28.35$ and $\nu = 8000\,\text{s}^{-1}$.

With an increase of the value of $B = B_y$, the value of the cyclotron frequency ($\omega_B = QB/M$) increases too. The ratio of $\delta T/\Delta T$, obtained by numerical simulation for different values of $B = B_y$, as function of ω_B/ν (where $\delta T = |\delta T_x| \equiv |\delta T_z|$, $\Delta T = |T_x{}^o - T_z{}^o|$) for a single particle ($N = 1$) and for ensemble of $N = 1000$ particles are presented in Fig. 4 for $\nu = 8000\,\text{s}^{-1}$. Thus, under certain conditions (with an increase of $B = B_y$, for $\omega_B \gg \nu$, see Sec. 3 and Fig. 4), the application of a constant magnetic field can be used as a method of equalizing the stochastic kinetic energy in the plane perpendicular to the vector $\boldsymbol{B}$.

The results of solution for a single charged particle is presented in Fig. 5 for $\omega_t/\nu \cong 18.27$, $\omega_B/\nu \cong 28.35$, $\nu = 8000\,\text{s}^{-1}$ with $T_x{}^o \equiv T_y{}^o \cong 0.077$ eV and $T_z{}^o \cong 0.025$ eV. The temperature of the sources ($T_x{}^o$, $T_y{}^o$, $T_z{}^o$) was set by a random force $\mathbf{F_b}$ [24] (see the beginning of the Sec. 2).

In the last case, the equilibrium velocity distributions of the particle in all degrees of freedom are the Maxwell functions. In the direction of $\boldsymbol{B}$, the velocities of particles corresponding to the Maxwell function is equal to the preset temperature of the thermal source ($T_y = T_y{}^o \cong 0.077$ eV), while in the transverse directions (across $\boldsymbol{B}$) the Maxwell functions are characterized by the following temperatures: $T_x \cong T_z \cong 0.051$ eV.

In conclusion of this section, we note that a solution of Eqs. (1a) - (1c) at the given friction coefficient, ν, and temperatures depends only on the relative values of ω_t/ν and ω_B/ν. Thus, the results obtained in this section are valid for particles of any mass and charges, for example,

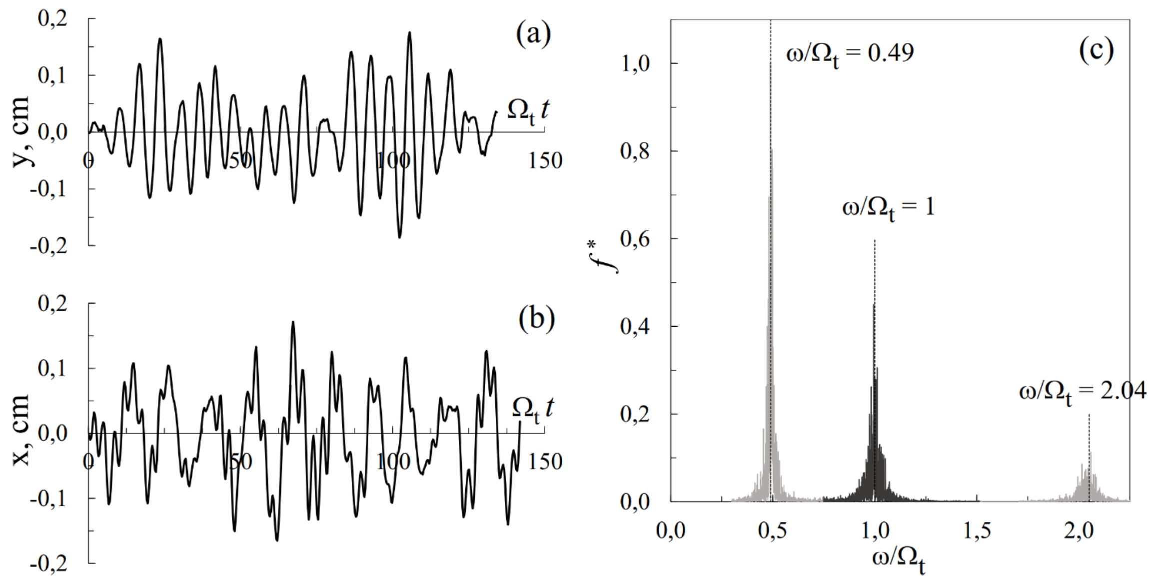

Fig. 2 The displacements of an arbitrary selected ion in the ensemble of $N = 500$ ions with $M = M_1$ as a function of $\Omega_t t$ in the **y** direction (a) and in **x** direction (b), as well as the results of spectral analysis of its displacements (c) for $\Omega_t/\nu \cong 18.27$, $\omega_B/\nu \cong 28.35$, $\nu = 8000\,\text{s}^{-1}$. Here $f_x{}^*$ (light gray line) and $f_y{}^*$ (dark gray line) are the normalized spectral density of the selected ion in the x and y directions, respectively.

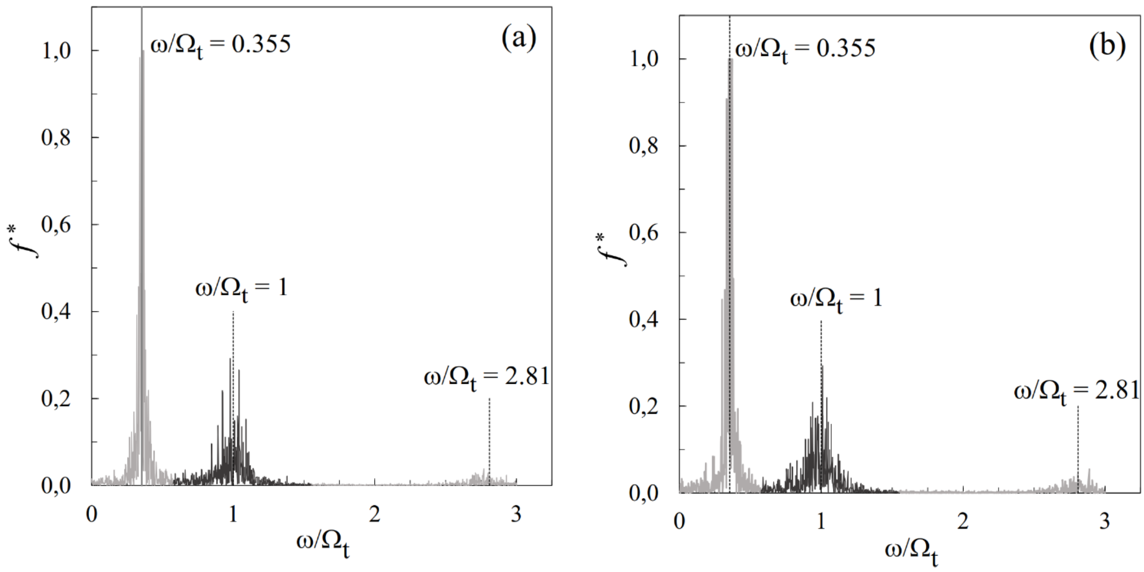

Fig. 3 The normalized spectral density of an arbitrary selected ion (a) and of the mass center (b) of the ensemble consisting of $N = 1000$ ions with $M = M_2$, $\Omega_t/\nu \cong 6.14$, $\omega_B/\nu \cong 15.07$ and $\nu = 8000\,\mathrm{s}^{-1}$. Light gray and dark gray lines correspond to the spectral density $f_x{}^*$, $f_y{}^*$ in the **x** and **y** directions.

Table 1 Parameters Ψ_1, Ψ_2, Ω_1, Ω_2 of the roots of Eq. (2) for the solution of Eqs. (1b), (1c) with $\nu = 8000\,\mathrm{s}^{-1}$ and the given values of Ω_t/ν, ω_B/ν.

Ω_t/ν	ω_B/ν	Ψ_1/Ω_t	Ψ_2/Ω_t	Ω_1/Ω_t	Ω_2/Ω_t
18.27	28.35	0.044	0.011	2.042	0.489
6.14	15.07	0.069	0.012	2.81	0.355

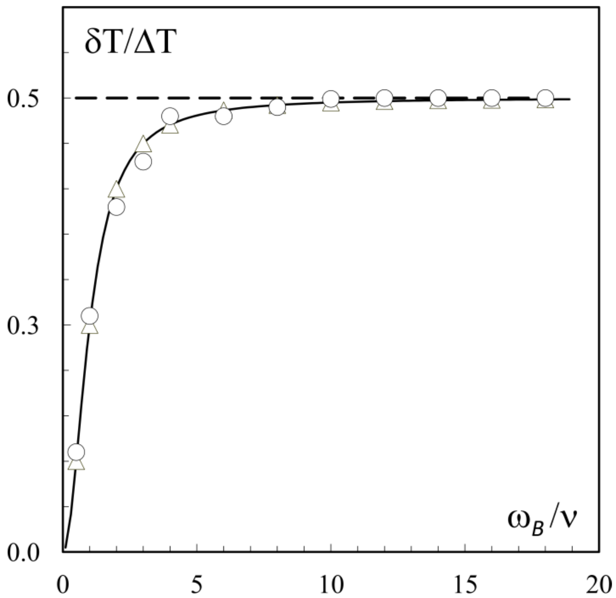

Fig. 4 The ratio of δT/ΔT as function of ω_B/ν, where $\delta T = |\delta T_x| \equiv |\delta T_z|$ and $\Delta T = |T_x - T_z|$. Solid line is Eq. (8); the symbols are the numerical simulation results for: ○ – $N = 1$, Δ – $N = 1000$.

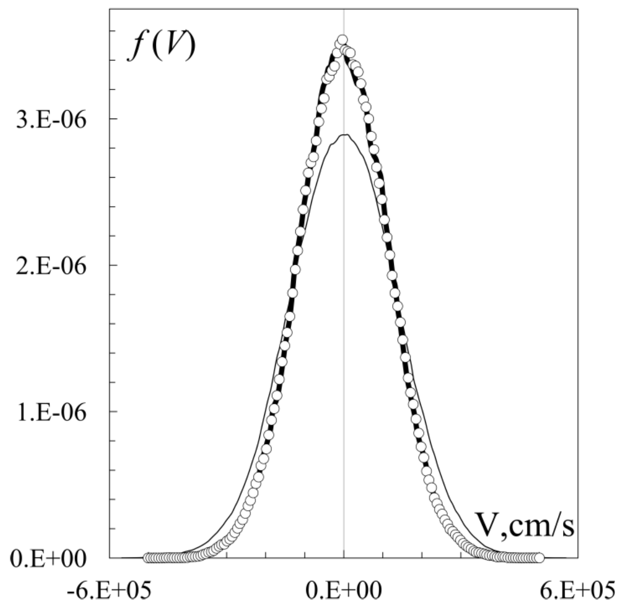

Fig. 5 The velocity distribution function $f(V)$ for a single charged particle in the **y** (thin line), **x** (bold line) and **z** (symbols) directions for the following conditions of the simulation: $\omega_t/\nu \cong 5.78$, $\omega_B/\nu \cong 28.35$, $\nu = 8000\,\mathrm{s}^{-1}$, $T_x{}^o \equiv T_y{}^o \cong 0.077\,\mathrm{eV}$ and $T_z{}^o \cong 0.025\,\mathrm{eV}$. After the establishment of equilibrium in the simulated system: $T_x \cong T_z \cong 0.051\,\mathrm{eV}$, $T_y \cong 0.077\,\mathrm{eV}$.

for the case of weakly non-ideal dusty plasma, etc. [27,28]. (Dusty plasma is an ionized gas with charged particles of micron-sized substance. This plasma is widespread in nature and is formed in a number of technological processes; it is weakly non-ideal when the coupling parameter of system $\Gamma < 1$ [24,27,28].)

5. Conclusions

We performed analytical and numerical studies of the dynamics of confined ensembles of charged particles in the constant electromagnetic fields. The roots of the characteristic equation that allow analyzing the frequency spectrum in these systems were presented. For the first time the conditions for energy balance of these systems with the spatially non-uniform thermal sources were proposed.

Numerical simulation was carried out for systems consisting of from one to thousands of charged particles. Trajectories, velocity distribution of these particles and spectral density in analyzed systems were obtained. Conditions of the energy balance were studied. The spectral characteristics of the simulated systems were compared. We found that the shape of the spectral density distribution practically does not depend on the number of particles in the analyzed ensembles, and their characteristic frequencies can be obtained by an analytical solution of the motion equations for a single charged particle. Numerical simulations of the systems with non-uniform thermal sources have shown that the proposed balance condition is valid for all considered cases.

In conclusion, the obtained results can be used for an estimation of optimal operating parameters of power installations for the effective separation of SNF. In addition, the presented results can be useful for qualitative analysis of the dynamics of weakly non-ideal dusty plasma with grains of any mass and charges in a constant electromagnetic field.

Acknowledgments

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