# Measuring Faraday Effect in Z-Cut Crystal Quartz at Wavelength of 118.8 μm for Polarimeter

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Faraday rotation angle in Z-cut crystal quartz at a wavelength of 118.8 μm was measured for the first time. Z-cut crystal quartz is used in vacuum windows of polarimeters for plasma diagnostics that use far-infrared light. ITER poloidal polarimeter uses double vacuum windows of 10-mm Z-cut crystal quartz discs in compliance with nuclear safety. According to the Becquerel model, thick Z-cut crystal quartz may lead to non-negligible Faraday rotation that must be compensated for to measure the pure polarization change attributable to the plasma. For the first time, this paper measures Faraday rotation at the wavelength of 118.8 μm by means of the rotating linear polarizer method. The Becquerel model is not applicable at the wavelength of 118.8 μm and Faraday rotation resulting from the Z-cut quartz vacuum window is negligible even with a magnetic field of 0.3 T and a 10-mm thick vacuum window.



Keywords: Faraday rotation, Faraday effect, Verdet constant, ITER, polarimeter, Z-cut crystal, quartz, Becquerel model

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## 1. Introduction

Polarimeters are used to measure the birefringence of materials, which provide various kinds of information such as sugar content, mechanical stress distribution or rock provenance. Polarimeters are also important diagnostics in magnetic confinement fusion experiments to measure electron density [1] and magnetic fields [2]. They measure Faraday rotation that is a rotation of the polarization plane of light (or electromagnetic wave) caused by the circular birefringence of plasma attributable to electron density and magnetic fields.

Polarimeters measure not only Faraday rotation in the plasma but also Faraday rotation in vacuum windows. This paper focuses on Faraday rotation in vacuum windows because it could be error source of polarimeters for future fusion reactors. The following paragraphs describe and compare Faraday rotation in plasma and vacuum windows. The authors use parameters of ITER (international thermonuclear experimental reactor) as an example of a future reactor.

Polarimeters have been widely used in fusion experiments: (Tokamak-type devices) TEXTOR [3], JT-60 [4], JET [5], TFTR [6], DIII-D [7], ToreSupra [8], C-Mod [9], NSTX [10], EAST [11], J-TEXT [12], ASDEX-U [13], (Stellarator-type devices) LHD [1], W7-AS [14], CHS [15], H-1 [16], and (Reversed-Field-Pinch-type devices) MST [17], RFX [18], RTP [19]. Probing laser beams pass through plasma, and polarimeters measure the change of the polarization state of the laser light owing to the Faraday effect (i.e. Faraday rotation). The Faraday rotation $\Delta\psi_{\mathrm{p}}$ is given by the equation below;[20]

$$\Delta\psi_{\mathrm{p}} = C\lambda^2 \int_{z_0}^{z_1} n_{\mathrm{e}}(z) B_{//}(z) dz, \tag{1}$$

where $n_{\mathrm{e}}$ is electron density, $B_{//}$ is magnetic field parallel to the probing laser beam, $z$ is the coordinate along the probing laser beam path, and $C = 2.6312 \times 10^{-13}$ rad/T. Here, $\Delta\psi$ denotes the change of the orientation angle, $\psi$, of the polarization ellipse and the subscript p denotes the change of rotation angle attributable to plasma. Far-infrared (FIR) lasers (or THz laser) such as 118.8 μm ($CH_3OH$ laser), 195 μm (DCN laser), 337 μm (HCN laser), and 432.5 μm (HCOOH laser) are typically used to measure the magnetic fields [3,5,6,8,9,11–13,17,18]. Vacuum windows are often made of Z-cut crystal quartz to transmit these FIR lights.

ITER also uses polarimeters, called the ITER poloidal polarimeter, to measure magnetic fields [21]. Probing laser beams are reflected by corner-cube retro-reflectors mounted on the first wall to the diagnostics room along the same path from which the probing laser beams are transmitted. That is, Faraday rotation measured by the polarimeter is twice that of the value given by Eq. (1). Substituting the values relevant to the conditions of ITER poloidal polarimeter ($\lambda = 118.8$ μm, $B_{//} = 1$ T, $z_1 - z_0 = 1$ m, $n_{\mathrm{e}} = 10^{20}$ m$^{-3}$), we get a Faraday rotation of $\Delta\psi_{\mathrm{p}} =$

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42.6°. Here, $z_1 - z_0$ is not the size of an ITER plasma, but the length of path where the magnetic field is to the order of 1 T.

The Faraday rotation in the vacuum window, $\Delta\psi_w$, is given by $\Delta\psi_w = VB_{//}d$, where $V$ is the Verdet constant of the material and $d$ is the thickness of the material. No available references of measuring the Verdet constant of Z-cut crystal quartz in the FIR region (the THz region) exist, but there is a theoretical model exists based on a classical electron theory called the Becquerel equation [22] given by

$$V = -\gamma \frac{e}{2m_e c} \lambda \frac{dn}{d\lambda}, \tag{2}$$

where $n$ is the refractive index of the material, $m_e$ is the mass of the electron, $e$ is the elementary charge, $c$ is light speed, and $\gamma$ is a factor called the magneto-optic anomaly. Reference [23] compares the Verdet constant of the Becquerel equation and experimental results using crystalline quartz, and indicates good agreement between the theory and the experiments for the wavelength below 2 µm. Let the authors estimate Faraday rotation in Z-cut crystal quartz at a wavelength of 118.8 µm assuming that the Becquerel model is still applicable in the FIR region. Substituting $n = 2.13$ and $dn/d\lambda = -261\ \mathrm{m}^{-1}$ at the wavelength of 118.8 µm [24] to the Becquerel equation, we get $V = 9.10$ rad/Tm. Here, $\gamma = 1$ is assumed. According to Ref. [23], $\gamma$ is 0.7 - 0.8 when the wevelength is 0.2 - 2 µm. Since the data at the FIR region is not available, $\gamma$ is assessed at 1 on the safe side (i.e. in the manner of having large impact on the polarimeter). The vacuum window of ITER [25] is thick because of the nuclear safety. One vacuum window consists of two 10-mm Z-cut crystal quartz discs. In the case of the ITER poloidal polarimeter, the highest magnetic field at any vacuum window location is estimated to be 0.8 T. Finally, considering that the probing laser beams pass through the vacuum window twice, the Becquerel model estimates Faraday rotation as $\Delta\psi_w = 17°$, which is large compared with the Faraday rotation resulting from the plasma, $\Delta\psi_p$. It should be noted that $\Delta\psi_w$ will be changed during plasma discharge in tokamak. Thus, if the $\Delta\psi_w$ is not negligible, in-situ technique compensating for $\Delta\psi_w$ needs to be developed. The purpose of this study is to clarify whether this estimation based on the Becquerel equation is applicable at the wavelength of 118.8 µm.

Faraday rotation in vacuum windows is a universal concern and not limited to ITER. However, the ITER poloidal polarimeter faces this issue for the first time because the experimental conditions for ITER are different from those for existing polarimeters. For instance, in the case of poloidal polarimeter of the ASDEX Upgrade, the thickness of the Z-cut crystal quartz and magnetic field at the output vacuum window are 5 mm and 70 mT, respectively. Even if the Becquerel model was applicable ($\lambda = 195$ µm, $n = 2.15$, $dn/d\lambda = -49.1\ \mathrm{m}^{-1}$ $\gamma = 1$ and $V = 2.8$ rad/Tm), the Faraday rotation would be less than 0.11°, which is less than the measurement accuracy of the polarimeter. The high magnetic field and the thick vacuum window in the case of ITER poloidal polarimeter are the cause of the problem, and this problem is applicable to future fusion reactors such as DEMO [26] because future fusion reactors uses higher magnetic field than ITER and nuclear safety will require thick vacuum windows for them. This paper provides important information for designing polarimeters of the future reactors.

## 2. Experimental Setups

The polarization state of a FIR laser beam passing through a Z-cut crystal quartz inside a ring-shaped permanent magnet was measured. The 118.8-µm wavelength FIR laser used was an alcohol laser pumped by a $CO_2$ laser developed by Chubu University and the National Institute for Fusion Science [27]. The power stability of the FIR laser in this experiment is ± 1%/day. The FIR power was 270 mW in this experiment. The size of Z-cut crystal quartz was 10×10×10 (mm). The ring-shaped permanent magnet was made of Neodymium and the outer diameter, the inner diameter and the height were 38.6 mm, 19.1 mm and 30.9 mm, respectively. The catalog specific for remanence of the magnet was 1.25 - 1.29 T. From these values, the spatial distribution of the magnetic field was calculated by the method of Ref. [28], shown in Fig. 1. The magnetic field is uniform and 0.27 T at the area where the crystal quartz is located. Using an old gauss meter, the magnetic field measured was 0.32 T. It is difficult to say which value is more accurate, but it is reasonable to suppose that the magnetic field applied to the Z-cut crystal quartz was approximately 0.3 T. According to the Becquerel equation, the Faraday rotation would be 1.6° in this experimental setup.

Figure 2 shows a photograph of the experimental in-

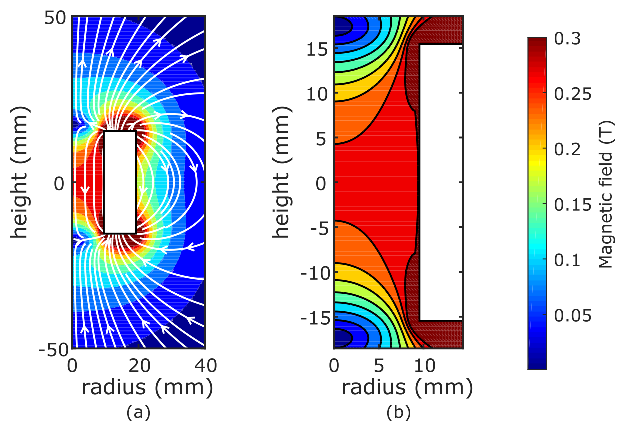


Fig. 1 Spatial distribution of magnetic field of the ring-shaped permanent magnet. Figures (a) and (b) show the wide area around the magnet and the narrow area inside the magnet, respectively. White rectangles are the cross section of the magnet. Solid lines in figure (a) indicate the magnetic field lines.

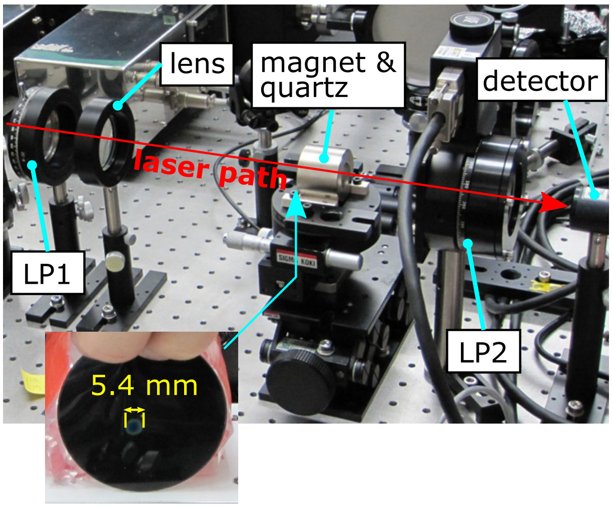


Fig. 2 Photograph of experimental setup. LP stands for a linear polarizer. LP1 was fixed and LP2 was rotated to measure the polarization state. The beam size at the entrance of the magnet was shown in a temperature sensitive liquid crystal sheet.

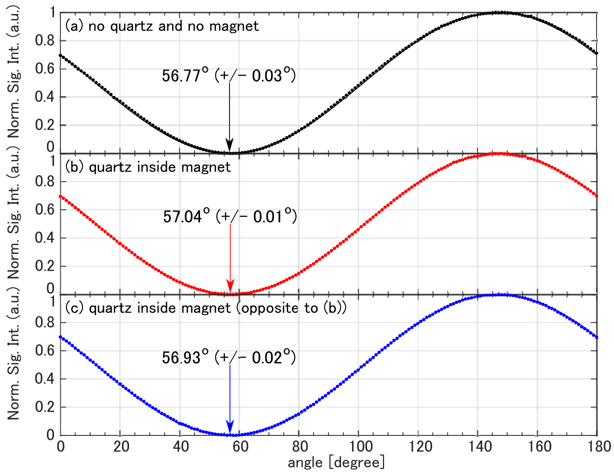


Fig. 3 Measurement results of the rotating linear polarizer method. The laser beam passed through; (a) no quartz and no magnet, (b) the quartz inside the magnet, and (c) the quartz inside the magnet of which the magnetic field direction was opposite to (b). The horizontal and vertical axes are rotation angle of the linear polarizer and normalized detector signal, respectively. The solid lines are fitting results. The values inside plots are extinction angle.

struments. The lens focused the laser light and the beam size inside the magnet was 5.4 mm, smaller than the 10-mm Z-cut crystal quartz. A linear polarizer in front of a pyroelectric sensor was rotated to measure the polarization state. The laser light was chopped by a mechanical chopper having a frequency of 34.5 Hz.

## 3. Results

The polarization state was measured by using the rotating linear polarizer method. The linear polarizer rotated by 180° every one degree; the detector signal was measured per degree. The detector signal is given by $A\sin\{2\pi(x-B)/180\}+C$, where $x$ is a rotation angle of the linear polarizer (unit in degree). Parameters $A$, $B$, and $C$ were determined by fitting the function to the measurement data. Change of parameter $B$ (i.e. phase shift) corresponds to polarization change due to the experimental conditions. Hereafter, the phase shift will be discussed by using the extinction angle (i.e. polarizer angle at the minimum detector signal intensity). The authors repeated the extinction angle measurement and concluded that the precision was less than 0.05° and low enough compared with the estimated Faraday rotation angle of 1.6°.

Figure 3 shows the measurement results of the rotating linear polarizer method, where the different colors indicate the various different experimental setups. The laser beam passed through; (black) no quartz and no magnet, (red) the quartz inside the magnet, and (blue) the quartz and the magnet of which the magnetic field direction was opposite to "red". The extinction angles were 56.77° in the case of "no quartz/magnet (black)", 57.03° in the case of "quartz inside magnet (red)", and 56.93° in the case of "quartz inside opposite magnet (blue)". The accuracy of the extinction angle was less than 0.03° (the precision was less than 0.05° as mentioned above). The difference between the extinction angles having opposite magnetic fields was 0.1° and much smaller than the Becquerel equation's estimation of 3.2°(= 2 × 1.6). The extinction angle of "no quartz/magnet (black)" was not between the extinction angles having opposite magnetic fields because the laser light beam path was not perfectly parallel to the optical axis of the crystal quartz. From these results, we can reasonably conclude that Faraday rotation does not occur in the Z-cut crystal quartz or is negligible.

## 4. Discussion

The Becquerel equation is based on the classical electron theory, which ignores the effect of the motion of the nuclei. The nuclear masses are so heavy that they cannot follow the field in the high-frequency region such as visible light. On the other hand, if the frequency is low, the motion of the nuclei is not negligible. This is also why Cauchy's formula for refractive index and dispersion is not applicable at long wavelengths such as those of FIR light [29]. The assumption of the Becquerel equation (i.e. the classical electron theory) does not hold at the wavelength of 118 μm.

## 5. Conclusions

Faraday rotation in 10-mm thick Z-cut crystal quartz at a wavelength of 118.8 μm was measured under ±0.3 T magnetic field by using the rotating linear polarization method. According to the Becquerel model, the Faraday rotation would be 1.6° in the experimental setup. The difference between the extinction angle having the opposite magnetic field corresponded to twice the Faraday rotation angle (i.e. 3.2°), but the measured extinction angle difference was 0.1° having accuracy of 0.02° and precision of 0.05°. We can conclude that Faraday rotation does not occur at the 118.8-μm wavelength. Even if Faraday rotation did occur, its effect is smaller than the polarization change caused by the unparallel between the laser light beam and the optical axis of the crystal quartz.

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[1] T. Akiyama, E. Sato, T. Nozawa, S. Tsuji-Iio, R. Shimada, H. Murayama, K. Nakayama, S. Okajima, K. Tanaka, K. Watanabe, T. Tokuzawa and K. Kawahata, Rev. Sci. Instrum. **72**, 1073 (2001).
[2] R. Imazawa, Y. Kawano and Y. Kusama, Nucl. Fusion **51**, 113022 (2011).
[3] H. Soltwisch, Rev. Sci. Instrum. **57**, 1939 (1986).
[4] Y. Kawano, S. Chiba and A. Inoue, Rev. Sci. Instrum. **72**, 1068 (2001).
[5] A. Boboc, M. Gelfusa, A. Murari and P. Gaudio, Rev. Sci. Instrum. **81**, 10D538 (2010).
[6] C.H. Ma, D.P. Hutchinson, K.L.V. Sluis, D.K. Mansfield, H. Park and L.C. Johnson, Rev. Sci. Instrum. **57**, 1994 (1986).
[7] M.A.V. Zeeland, R.L. Boivin, T.N. Carlstrom and T.M. Deterly, Rev. Sci. Instrum. **79**, 10E719 (2008).
[8] C. Gil, D. Elbeze, A. Beraud, B. Echard, J. Patterlini, J. Philip, L. Toulouse, M. Lipa and A. Litnovsky, Fusion Eng. Des. **82**, 1238 (2007). Proceedings of the 24th Symposium on Fusion Technology.
[9] W.F. Bergerson, P. Xu, J.H. Irby, D.L. Brower, W.X. Ding and E.S. Marmar, Rev. Sci. Instrum. **83**, 10E316 (2012).
[10] H.K. Park, C.W. Domier, W.R. Geck and N.C.L. Jr., Rev. Sci. Instrum. **70**, 710 (1999).
[11] Z.Y. Zou, H.Q. Liu, Y.X. Jie, W.X. Ding, D.L. Brower, Z.X. Wang, J.S. Shen, Z.H. An, Y. Yang, L. Zeng, X.C. Wei, G.S. Li, X. Zhu and T. Lan, Rev. Sci. Instrum. **85**, 11D409 (2014).
[12] J. Chen, L. Gao, G. Zhuang, Z.J. Wang and K.W. Gentle, Rev. Sci. Instrum. **81**, 10D502 (2010).
[13] A. Mlynek, L. Casali, O. Ford and H. Eixenberger, Rev. Sci. Instrum. **85**, 11D408 (2014).
[14] C. Fuchs and H.J. Hartfuss, Phys. Rev. Lett. **81**, 1626 (1998).
[15] T. Akiyama, K. Kawahata, Y. Ito, S. Okajima, K. Nakayama, S. Okamura, K. Matsuoka, M. Isobe, S. Nishimura, C. Suzuki, Y. Yoshimura, K. Nagaoka and C. Takahashi, Rev. Sci. Instrum. **77**, 10F118 (2006).
[16] H.J. Gardner and J. Howard, Plasma Phys. Control. Fusion **36**, 245 (1994).
[17] B.H. Deng, D.L. Brower, W.X. Ding, M.D. Wyman, B.E. Chapman and J.S. Sarff, Rev. Sci. Instrum. **77**, 10F108 (2006).
[18] E. Zilli, M. O'Gorman, L. Giudicotti, F. Milani, S.L. Prunty, A. Murari and A. Boboc, Int. J. Infrared Millim. Waves **21**, 1673 (2000).
[19] J.H. Rommers, A.J.H. Donne, F.A. Karelse and J. Howard, Rev. Sci. Instrum. **68**, 1217 (1997).
[20] S.E. Segre, Plasma Phys. Control. Fusion **32**, 1249 (1990).
[21] R. Imazawa, Y. Kawano and K. Itami, Proc. 26th Fusion Enegy Conference, (Kyoto, Japan), FIP/P4-5 (2016).
[22] R. Serber, Phys. Rev. **41**, 489 (1932).
[23] S. Ramaseshan, Proceedings of Indian Academy of Sciences **24**, 426 (1946).
[24] E.E. Russell and E.E. Bell, J. Opt. Soc. Am. **57**, 341 (1967).
[25] P. Maquet, C. Walker, R. Barnsley, L. Bertalot, A. Encheva, C. Pitcher, R. Reichle, G. Vayakis, E. Veshchev, V. Udintsev, M. Walsh, C. Watts, K. Patel, T. Giacomin, S. Hughes, N. Taylor, R. Pearce and K. Okayama, Fusion Eng. Des. **88**, 2641 (2013). Proceedings of the 27th Symposium On Fusion Technology (SOFT-27); Liege, Belgium, September 24-28 (2012).
[26] G. Federici, W. Biel, M. Gilbert, R. Kemp, N. Taylor and R. Wenninger, Nucl. Fusion **57**, 092002 (2017).
[27] S. Okajima, K. Nakayama, H. Tazawa, K. Kawahata, K. Tanaka, T. Tokuzawa, Y. Ito and K. Mizuno, Rev. Sci. Instrum. **72**, 1094 (2001).
[28] F.A. Reich, O. Stahn and W.H. Müller, Continuum Mechanics and Thermodynamics **28**, 1435 (2016).
[29] M. Born and E. Wolf, eds., *Principles of Optics* 7th ed., (Cambridge, UK: Cambridge University Press, 1999).