Cyclotron Electromagnetic Instabilities in a Laboratory Dipole Magnetospheric Plasma with bi-Kappa Distributions

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The transverse dielectric susceptibility elements are derived for electromagnetic cyclotron waves in an axisymmetric laboratory dipole magnetosphere accounting for the cyclotron and bounce resonances of trapped and untrapped particles. A bi-Kappa (or bi-Lorentzian) distribution function is invoked to model the energetic particles with anisotropic temperature. The steady-state two-dimensional (2D) magnetic field is modeled by laboratory dipole approximation for a superconducting ring current of finite radius. Derived for field-aligned circularly-polarized waves the dispersion relations are suitable for analyzing both the whistler instability in the range below the electron-cyclotron frequency, and the proton-cyclotron instability in the range below the ion-cyclotron frequency. The instability growth rates in the 2D laboratory magnetosphere are defined by the contributions of energetic particles to the imaginary part of transverse susceptibility.

Keywords: laboratory dipole magnetosphere, anisotropic temperature, trapped and untrapped particles, bi-Kappa distribution, cyclotron wave

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1. Introduction

Plasma confined in a levitated dipole magnetic field is an alternative to the controlled thermonuclear fusion, suitable to model phenomena in the Earth's magnetosphere. The dipole confinement concept to simulate a magnetosphere in laboratory conditions was originally proposed by Hasegawa [1]. Presently, there are active experiments on the plasma creation, confinement and heating in devices such as Levitated Dipole eXperiment (LDX) [2, 3] and Ring Trap 1 (RT-1) [4–7]. The RT-1 device is a laboratory dipole magnetosphere (LDM) created by a levitated superconducting ring magnet. In RT-1 plasma is produced and heated by a high frequency wave power (8.2 GHz) in the range of the fundamental electron-cyclotron resonance (ECR) [5]. The first successful results on the ion-cyclotron resonance (ICR) heating with a frequency of a few MHz in RT-1 plasma were reported recently in Ref. [6]. Both the ECR and ICR plasma heating methods can produce high-energy electrons and ions, respectively, which imply anisotropic temperatures of the resonant particles, with transverse temperature $T_\perp$ (relative to the confining magnetic field) greater than the parallel temperature $T_\parallel$. These anisotropies are observed in RT-1 studying the particle acceleration in LDM plasma [7].

Anisotropic particles with $T_\perp > T_\parallel$ are sources of free energy to drive electromagnetic instabilities, such as the instability of electromagnetic electron-cyclotron (right-hand polarized) waves, and the instability of ion-cyclotron (left-hand polarized) waves. Kinetic theory of electromagnetic cyclotron wave/instabilities in a uniform magnetic field plasma is well developed, see e.g. Refs. [8–22] and references therein. However, the models of plasmas guided by straight magnetic field lines are not conform with LDM configurations in LDX and RT-1, which are axisymmetric and two-dimensional (2D). In this case the dispersion and stability properties need to be described in the frame of a 2D kinetic wave theory implying a specific dielectric tensor. Moreover, the wave-particle interactions should take into account that in a LDM plasma there are two entirely different groups of the so-called trapped and untrapped particles [23, 24]. In the inner Earth's magnetosphere only trapped particles can exist bouncing along the geomagnetic field lines.

In this paper we derive for the first time the dielectric characteristics and dispersion relations for the field-aligned electromagnetic waves in a LDM plasma with anisotropic bi-Kappa distributed particles [15, 18, 19]. These waves have been characterized in Ref. [24] for an idealized LDM plasma with anisotropic bi-Maxwellian particles, while here we generalize the approach by assuming both the electron and ion (proton) populations well described by bi-Kappa distributions in velocity space. The generalized Kappa power-laws are more appropriate to reproduce the velocity distributions with enhanced high-energy tails, which are frequently reported by the in-situ measurements

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in space plasmas, i.e., the solar wind and planetary magnetospheres [25, 26]. On the other hand, cyclotron instabilities have been extensively investigated, but only in bi-Kappa plasmas embedded in a uniform magnetic field [15–22].

2. 2D LDM Plasma Model with bi-Kappa Distributions

The 2D steady-state magnetic field is modeled by a laboratory dipole approximation [23, 24] accounting for a finite radius of the current-ring creating the magnetic dipole. A 2D axisymmetric LDM plasma, schematically shown in Fig. 1, is described with quasi-toroidal coordinates (r, θ, ϕ) connected with cylindrical coordinates (ρ, ϕ, z): $\rho = a + r\cos\theta$, $z = -r\sin\theta$, $\phi = \phi$. Cylindrical components of an equilibrium magnetic field, $\mathbf{H}_0 = (H_{0\rho}, H_{0\phi}, H_{0z})$ write explicitly as

$$H_{0\rho} = \frac{2Ir\sin\theta}{c(a + r\cos\theta)\sqrt{r^2 + 4a^2 + 4ar\cos\theta}} \times\left[K(k) - \frac{r^2 + 2a^2 + 2ar\cos\theta}{r^2}E(k)\right], \tag{1}$$

$$H_{0\phi} = 0, \tag{2}$$

$$H_{0z} = \frac{2I}{c\sqrt{r^2 + 4a^2 + 4ar\cos\theta}} \times\left[K(k) - \frac{r + 2a\cos\theta}{r}E(k)\right], \tag{3}$$

where a is the current ring radius, I is the ring current, c is the speed of light;

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1 - k\sin^2 x}},$$
$$E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1 - k\sin^2 x}dx,$$
$$k = \frac{4a(a + r\cos\theta)}{r^2 + 4a^2 + 4ar\cos\theta}, \tag{4}$$

are, respectively, the complete elliptic integrals of first and second kind, and their explicit argument.

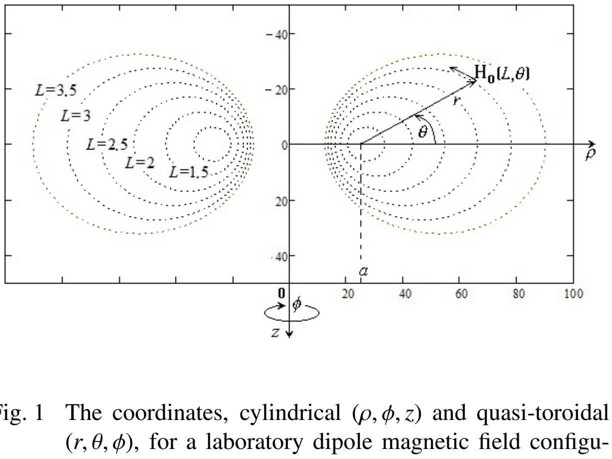

Fig. 1 The coordinates, cylindrical (ρ, ϕ, z) and quasi-toroidal (r, θ, ϕ), for a laboratory dipole magnetic field configuration.

Here we consider a collisionless plasma, where the perturbed distribution functions $f_\alpha(t, \mathbf{r}, \mathbf{v})$ satisfy the Vlasov equation, subscript $\alpha = e, p, i$ denoting different species of particles, e.g., electrons, protons and heavier ions. To simplify the approach, the Vlasov equation is solved using the small magnetization parameters neglecting the drift effects, the finite Larmor radius corrections and the finite orbit widths of untrapped and trapped particles. The Vlasov equation is solved by changing to new variables associated with the conservation integrals for the particle energy $v_\parallel^2 + v_\perp^2 = c_1$ ($v_\parallel$ and $v_\perp$ are the parallel and perpendicular components of the particle velocity relative to $\mathbf{H}_0$), the magnetic moment $v_\perp^2/2H_0 = c_2$ and a stationary magnetic field line $\sqrt{r^2 + 4a^2 + 4ar\cos\theta}\,[(2 - k)K(k) - 2E(k)] = c_3$. Thus, instead of $(v_\parallel, v_\perp, r)$ we introduce the new variables (v, μ, L) as

$$v = \sqrt{v_\parallel^2 + v_\perp^2}, \qquad \mu = \frac{v_\perp^2 H_0(r, 0)}{v^2 H_0(r, \theta)}, \tag{5}$$

$$L = \frac{\pi a}{\sqrt{r^2 + 4a^2 + 4ar\cos\theta}\,[(2 - k)K(k) - 2E(k)]}. \tag{6}$$

The first harmonics of the perturbed distribution

$$f(t, \mathbf{r}, \mathbf{v}) = \sum_s^{\pm 1}\sum_l^{\pm\infty} f_l^s(L, \theta, v, \mu)\exp(-i\omega t + im\phi - il\sigma), \tag{7}$$

satisfy the linearized Vlasov equation, which can be reduced in the zero-order of a magnetization parameter to the set of the first order differential equations

$$\frac{\sqrt{1-\mu b(L, \theta)}}{\delta(L, \theta)}\frac{\partial f_l^s}{\partial\theta} - is\frac{La}{v}\left[\omega - l\Omega_{c0}b(L, \theta)\right] f_l^s = Q_l^s. \tag{8}$$

Here

$$\delta = \frac{0.5cr^2 H_0(r, \theta)(a + r\cos\theta)\sqrt{r^2 + 4a^2 + 4ar\cos\theta}}{ILa\left[(r^2 + 2a^2 + 3ar\cos\theta)E(k) - r(r + a\cos\theta)K(k)\right]}, \tag{9}$$

$$b(L, \theta) = \frac{H_0(r(L, \theta), \theta)}{H_0(r(L, 0), 0)}, \quad \Omega_{c0} = \frac{eH_0(r(L, 0), 0)}{Mc}, \tag{10}$$

$$Q_0^s = \frac{2evLa(1 + \kappa)F\sqrt{1 - \mu b(L, \theta)}E_\parallel}{M\kappa\vartheta_\parallel^2\left\{1 + \frac{v^2}{\kappa\vartheta_\parallel^2}\left[1 - \mu\left(1 - \frac{T_\parallel}{T_\perp}\right)\right]\right\}}, \tag{11}$$

$$Q_{\pm 1}^s = \frac{seLa(1 + \kappa)\sqrt{\mu}F}{M\kappa\vartheta_\parallel^2\left\{1 + \frac{v^2}{\kappa\vartheta_\parallel^2}\left[1 - \mu\left(1 - \frac{T_\parallel}{T_\perp}\right)\right]\right\}} \times\left[\frac{E_{\pm 1}}{\sqrt{b(L, \theta)}}\left(b(L, \theta) + 1 - \frac{T_\parallel}{T_\perp}\right) - i\frac{sv\sqrt{1 - \mu b(L, \theta)}}{\omega La\delta(L, \theta)}\left(1 - \frac{T_\parallel}{T_\perp}\right)\frac{\partial}{\partial\theta}\frac{E_{\pm 1}}{\sqrt{b(L, \theta)}}\right], \tag{12}$$

where $l = 0, \pm 1$ is the order of the cyclotron harmonics, σ is the gyrophase angle in velocity space, $H_0 = \sqrt{H_{0\rho}^2 + H_{0z}^2}$, Ω_{c0} is the minimal cyclotron frequency of plasma particles for the considered magnetic field line (by L-shell parameter, in equatorial plane), $E_{\pm 1} = E_n \pm iE_b$ describe the transverse electric field components with the left-hand and right-hand polarization, where E_n, E_b and $E_\parallel$ are, respectively, the normal, binormal and parallel perturbed electric field components relative to $\mathbf{H}_0$. The steady-state distribution function $F_\alpha(\mathbf{r}, \mathbf{v})$ of plasma particles in Eqs. (11)-(13) is the bi-Kappa distribution

$$F = \frac{N\kappa^{-1.5}}{\pi^{1.5}\vartheta_\parallel \vartheta_\perp^2} \frac{\Gamma(1+\kappa)}{\Gamma(\kappa - 0.5)} \times \left\{1 + \frac{v^2}{\kappa \vartheta_\parallel^2}\left[1 - \mu\left(1 - \frac{T_\parallel}{T_\perp}\right)\right]\right\}^{-(1+\kappa)},$$

$$\vartheta_\parallel^2 = \frac{2\kappa - 3}{\kappa}\frac{T_\parallel}{M}, \qquad \vartheta_\perp^2 = \frac{2\kappa - 3}{\kappa}\frac{T_\perp}{M}, \tag{13}$$

where N is the number density of particles of mass M, charge e, and $T_\parallel$ and $T_\perp \neq T_\parallel$ are their parallel and transverse temperatures, respectively (in energy units). The power-index $\kappa > 3/2$ in F determines the slope of high-energy tails in the velocity spectrum of plasma particles. In the limit of large $\kappa \to \infty$, the bi-Kappa distribution function degenerates into a bi-Maxwellian function. The α-index is omitted in Eqs. (7), (8), (11)-(13). By $s = \pm 1$ we distinguish the particles with positive and negative parallel velocity relative to $\mathbf{H}_0$:

$$v_\parallel = sv\sqrt{1 - \mu b(L, \theta)}. \tag{14}$$

Variable $r(L, \theta)$ in Eqs. (9), (10) is function of $L = L(r, \theta)$ defined in Eq. (6) and radial variable L (normalized) is the conventional label for the magnetic field lines in Fig. 1, corresponding to different magnetic shells.

3. Trapped and Untrapped Particles

Since LDM plasma is a configuration with a single minimum of $\mathbf{H}_0$, we can identify two distinct populations of the so-called trapped (t-) and untrapped (u-) particles. In the phase volume such separation can be done by the μ variable as

$$0 \leq \mu \leq \mu_0, \qquad -\pi \leq \theta \leq \pi, \tag{15}$$

for untrapped particles, where $\mu_0 = 1/b(L, \pi)$ is the inverse mirror ratio of the L-magnetic field line, and

$$\mu_0 \leq \mu \leq 1, \qquad -\theta_t \leq \theta \leq \theta_t, \tag{16}$$

for trapped particles, where $\pm\theta_t(\mu, L)$ are the reflection points (stop points, mirror points) of trapped particles, which are defined by the zeros of parallel velocity:

$$v_\parallel(v, \mu, L, \pm\theta_t) = 0. \tag{17}$$

In our notation the reflection points $\pm\theta_t$ are independent of the particle energy v and depend only on the pitch angle (by the parameter μ) satisfying the equation $1/b(L, \pm\theta_t) = \mu$. As a characteristic of LDM plasma, the population of u-particles is very small at the external magnetic shells-surfaces since $\mu_0 \to 0$ if L-shell parameter increases.

With the solutions of Eq. (8), we can derive the 2D transverse and longitudinal (relative to $\mathbf{H}_0$) current density components, respectively,

$$j_l(L, \theta) = \frac{\pi e}{2} b^{1.5}(L, \theta) \sum_s^{\pm 1} \int_0^\infty v^3 dv \times \left[\int_0^{\mu_0} \frac{f_{l,u}^s \sqrt{\mu} d\mu}{\sqrt{1 - \mu b(L, \theta)}} + \int_{\mu_0}^{1/b(L,\theta)} \frac{f_{l,t}^s \sqrt{\mu} d\mu}{\sqrt{1 - \mu b(L, \theta)}}\right] \quad l = \pm 1, \tag{18}$$

$$j_\parallel(L, \theta) = \pi e b(L, \theta) \sum_s^{\pm 1} s \int_0^\infty v^3 dv \times \left[\int_0^{\mu_0} f_{0,u}^s d\mu + \int_{\mu_0}^{1/b(L,\theta)} f_{0,t}^s d\mu\right], \tag{19}$$

where subscripts u and t correspond to untrapped and trapped particles, respectively. Note that in our notation the normal and binormal current density components are given by $j_n = j_{-1} + j_{+1}$ and $j_b = i(j_{-1} - j_{+1})$, respectively. The expressions for $j_l|_{l=\pm 1}$ are convenient to analyze the cyclotron resonance effects at the fundamental cyclotron frequency of both the ions (if $l = 1$) and electrons (if $l = -1$) in an explicit form. To derive the dispersion relations for circularly-polarized waves in the 2D LDM we should evaluate the 2D transverse current density components in Eq. (18), which are defined by harmonics $f_{\pm 1,u}^s$ and $f_{\pm 1,t}^s$ of the perturbed distribution functions of untrapped and trapped particles. Therefore, the implicit solution of Eq. (8) is obtained for $f_{l,u}^s$ and $f_{l,t}^s$ harmonics with $l = \pm 1$.

To describe the bounce-periodic motion of u- and t-particles along the $\mathbf{H}_0$-field lines, the poloidal angle θ is replaced with the new time-like variable

$$\tau(\theta) = \int_0^\theta \frac{\delta(L, \eta)}{\sqrt{1 - \mu b(L, \eta)}} d\eta, \tag{20}$$

given that the transit-time and bounce-period of untrapped and trapped particles are proportional to $\tau_{b,u} = 2\tau(\pi)$ and $\tau_{b,t} = 4\tau(\theta_t)$, respectively. The distribution functions of u- and t-particles can be defined by the corresponding Fourier series ($\beta = u, t$)

$$f_{l,\beta}^s = \sum_p^{\pm\infty} f_{l,\beta}^{s,p} \times \exp\left[ip\frac{2\pi}{\tau_{b,\beta}}\tau(\theta) - isl\frac{La}{v}\int_0^\theta \frac{\widetilde{\Omega}_{c,\beta}\delta(L, \eta)}{\sqrt{1 - \mu b(L, \eta)}} d\eta\right], \tag{21}$$

where $\widetilde{\Omega}_{c,u} = \Omega_{c0} b(L, \theta) - \overline{\Omega}_{c,u}$ and $\widetilde{\Omega}_{c,t} = \Omega_{c0} b(L, \theta) - \overline{\Omega}_{c,t}$ are the oscillating parts of the cyclotron frequencies, and $\overline{\Omega}_{c,u}$ and $\overline{\Omega}_{c,t}$ are the corresponding bounce-averaged cyclotron frequencies of the u- and t-particles

$$\overline{\Omega}_{c,u} = \frac{2\Omega_{c0}}{\tau_{b,u}} \int_0^{\pi} \frac{b(L,\theta)\delta(L,\theta)}{\sqrt{1-\mu b(L,\theta)}} d\theta, \tag{22}$$

$$\overline{\Omega}_{c,t} = \frac{4\Omega_{c0}}{\tau_{b,t}} \int_0^{\theta_t} \frac{b(L,\theta)\delta(L,\theta)}{\sqrt{1-\mu b(L,\theta)}} d\theta. \tag{22a}$$

The Fourier amplitudes $f_{l,\beta}^{s,p}$ (for $l = 0, \pm 1$ and $\beta = u, t$) are found by the corresponding bounce-averaging

$$f_{l,\beta}^{s,p} = \frac{-iv}{2\pi p v - s\tau_{b,\beta} La(\omega + l\overline{\Omega}_{c,\beta})} \times \int_{-0.5\tau_{b,\beta}}^{0.5\tau_{b,\beta}} Q_l^s \exp\left(-ip\frac{2\pi}{\tau_{b,\beta}}\tau + isl\frac{La}{v}\int_0^{\tau} \widetilde{\Omega}_{c,\beta} d\tau'\right) d\tau. \tag{23}$$

Using solutions (21) we satisfy automatically the boundary conditions for the perturbed distribution functions, namely, (a) the periodicity of the u-particles circulating along the equilibrium magnetic field $\mathbf{H}_0$:

$$f_{l,u}^s(L,\theta) = f_{l,u}^s(L,\theta + 2\pi) \quad \text{or} \quad f_{l,u}^s(L,\tau) = f_{l,u}^s(L,\tau + \tau_{b,u}),$$

and (b) the continuity of the perturbed distribution functions of t-particles at the reflection points:

$$f_{l,t}^s(L,\pm\theta_t) = f_{l,t}^{-s}(L,\pm\theta_t) \quad \text{or} \quad f_{l,t}^s(L,\tau) = f_{l,t}^s(L,\tau + \tau_{b,t}).$$

4. Dielectric Properties Function of κ

The perturbed current density components in Eqs. (18) and (19) are related to $\mathbf{E}$ to derive the dielectric tensor. We use the Fourier expansions of the 2D perturbed electric field and current density components over λ varying along the magnetic field line:

$$\lambda(\theta) = \int_0^{\theta} \delta(L,\eta) d\eta. \tag{24}$$

In this case

$$\frac{j_l(L,\theta)}{\sqrt{b(L,\theta)}} = \sum_n^{\pm\infty} j_l^{(n)}(L) \exp\left(i\pi n \frac{\lambda(\theta)}{\lambda_0}\right), \tag{25}$$

$$\frac{E_l(L,\theta)}{\sqrt{b(L,\theta)}} = \sum_{n'}^{\pm\infty} E_l^{(n')}(L) \exp\left(i\pi n' \frac{\lambda(\theta)}{\lambda_0}\right), \tag{26}$$

where $\lambda_0 = \lambda(\pi)$, so that $La\lambda_0$ is the half-length of the magnetic field line of order L. As a result

$$\frac{4\pi i}{\omega} j_l^{(n)} = \frac{2i}{\omega} \int_{-\pi}^{\pi} \frac{j_l(L,\theta)}{\sqrt{b(L,\theta)}} \exp\left(-i\pi n \frac{\lambda(\theta)}{\lambda_0}\right) d\theta = \sum_{n'}^{\pm\infty} \left(\chi_{l,u}^{n,n'} + \chi_{l,t}^{n,n'}\right) E_l^{(n')}, \qquad l = \pm 1, \tag{27}$$

where $\chi_{l,u}^{n,n'}$ and $\chi_{l,t}^{n,n'}$ are the contributions of u- and t-particles to the transverse susceptibility, respectively. After the s-summation, these dielectric characteristics for the radio frequency waves in the LDM plasma with a bi-Kappa background can be expressed as

$$\chi_{l,u}^{n,n'} = \frac{\omega_p^2 LaT_{\|}(1+\kappa)\Gamma(1+\kappa)}{8\omega\pi^{1.5}\lambda_0 T_{\perp}\sqrt{\kappa}\vartheta_{\|}\Gamma(\kappa-0.5)} \times \sum_{p=-\infty}^{\infty} \int_0^{\mu_0} \mu d\mu \int_{-\infty}^{\infty} \frac{A_{l,p}^n(u,\mu) B_{l,p}^{n'}(u,\mu)}{\left\{1+u^2\left[1-\mu\left(1-\frac{T_{\|}}{T_{\perp}}\right)\right]\right\}^{2+\kappa}} \frac{u^4 du}{pu - Z_{l,u}}, \tag{28}$$

$$\chi_{l,t}^{n,n'} = \frac{\omega_p^2 LaT_{\|}(1+\kappa)\Gamma(1+\kappa)}{8\omega\pi^{1.5}\lambda_0 T_{\perp}\sqrt{\kappa}\vartheta_{\|}\Gamma(\kappa-0.5)} \times \sum_{p=-\infty}^{\infty} \int_{\mu_0}^{1} \mu d\mu \int_{-\infty}^{\infty} \frac{C_{l,p}^n(u,\mu) D_{l,p}^{n'}(u,\mu)}{\left\{1+u^2\left[1-\mu\left(1-\frac{T_{\|}}{T_{\perp}}\right)\right]\right\}^{2+\kappa}} \frac{u^4 du}{pu - Z_{l,t}}. \tag{29}$$

Here we have introduced the following definitions

$$A_{l,p}^n(u,\mu) = \int_{-\pi}^{\pi} \cos\left[\Psi_{l,p}^n(u,\mu,\theta)\right] \frac{b(L,\theta)\delta(L,\theta)}{\sqrt{1-\mu b(L,\theta)}} d\theta, \tag{30}$$

$$B_{l,p}^n(u,\mu) = \int_{-\pi}^{\pi} \left[b(L,\theta) - 1 + \frac{T_{\|}}{T_{\perp}} + \frac{\pi n v_{T\|} u}{\omega La\lambda_0}\left(1 - \frac{T_{\|}}{T_{\perp}}\right) \times \sqrt{1-\mu b(L,\theta)}\right] \cos\left[\Psi_{l,p}^n(u,\mu,\theta)\right] \frac{\delta(L,\theta) d\theta}{\sqrt{1-\mu b(L,\theta)}}, \tag{31}$$

$$C_{l,p}^n(u,\mu) = \int_{-\theta_t}^{\theta_t} \cos\left[\Phi_{l,p}^n(u,\mu,\theta)\right] \frac{b(L,\theta)\delta(L,\theta)}{\sqrt{1-\mu b(L,\theta)}} d\theta, \tag{32}$$

$$\begin{aligned} D_{l,p}^n(u,\mu) = &\int_{-\theta_t}^{\theta_t} \left[b(L,\theta) - 1 + \frac{T_{\|}}{T_{\perp}} + \frac{\pi n\sqrt{\kappa}\vartheta_{\|} u}{\omega La\lambda_0}\left(1 - \frac{T_{\|}}{T_{\perp}}\right) \times \sqrt{1-\mu b(L,\theta)}\right] \cos\left[\Phi_{l,p}^n(u,\mu,\theta)\right] \frac{\delta(L,\theta) d\theta}{\sqrt{1-\mu b(L,\theta)}} \\ &+ (-1)^p \int_{-\theta_t}^{\theta_t} \left[b(L,\theta) - 1 + \frac{T_{\|}}{T_{\perp}} + \frac{\pi n\sqrt{\kappa}\vartheta_{\|} u}{\omega La\lambda_0}\left(1 - \frac{T_{\|}}{T_{\perp}}\right) \times \sqrt{1-\mu b(L,\theta)}\right] \cos\left[\Phi_{l,-p}^n(-u,\mu,\theta)\right] \frac{\delta(L,\theta) d\theta}{\sqrt{1-\mu b(L,\theta)}}, \end{aligned} \tag{33}$$

$$\Psi_{l,p}^n(u,\mu,\theta) = n\pi \frac{\lambda(\theta)}{\lambda_0} - \left(p\frac{2\pi}{\tau_{b,u}} + \frac{lLa}{u\sqrt{\kappa}\vartheta_{\|}}\overline{\Omega}_{c,u}\right)\tau(\theta) + \frac{lLa}{u\sqrt{\kappa}\vartheta_{\|}}\Omega_{c0}\int_0^{\theta} \frac{b(L,\eta)\delta(L,\eta)}{\sqrt{1-\mu b(L,\eta)}} d\eta, \tag{34}$$

$$\Phi_{l,p}^n(u,\mu,\theta) = n\pi \frac{\lambda(\theta)}{\lambda_0} - \left(p\frac{2\pi}{\tau_{b,t}} + \frac{lLa}{u\sqrt{\kappa}\vartheta_{\|}}\overline{\Omega}_{c,t}\right)\tau(\theta) + \frac{lLa}{u\sqrt{\kappa}\vartheta_{\|}}\Omega_{c0}\int_0^{\theta} \frac{b(L,\eta)\delta(L,\eta) d\eta}{\sqrt{1-\mu b(L,\eta)}}, \tag{35}$$

$$Z_{l,u} = \frac{La\tau_{b,u}}{2\pi\sqrt{\kappa}\vartheta_{\|}}\left(\omega - l\overline{\Omega}_{c,u}\right), \quad Z_{l,t} = \frac{La\tau_{b,t}}{2\pi\sqrt{\kappa}\vartheta_{\|}}\left(\omega - l\overline{\Omega}_{c,t}\right),$$
$$u = \frac{v}{\sqrt{\kappa}\vartheta_{\|}}, \quad \omega_p^2 = \frac{4\pi Ne^2}{M}. \tag{36}$$

Equations (28)-(36) describe the contribution of u- and t-particles to the transverse susceptibility. Note the explicit dependence of the power-index κ, which quantifies the presence of suprathermal populations. To obtain complete expressions of the susceptibility it is necessary to carry out the summation over all species α of plasma particles. For isotropic temperatures $T_{\parallel} = T_{\perp} = T$, from Eqs. (28)-(33) we can obtain the susceptibility elements for a plasma with isotropic Kappa distribution functions.

5. Dispersion Relations for Cyclotron Waves

From an analogy with the cyclotron waves in a uniform magnetic field (with straight lines), we can assume that the n-th harmonic of the electric field gives the main contribution to the n-th harmonic of the current density (*single-mode approximation*). In this case, for the field-aligned electromagnetic cyclotron waves (when $m = 0$, $\partial/\partial L = 0$, $E_{\parallel} = 0$, $H_{\parallel} = 0$), from the Maxwell's equations, excluding the $E_l^{(n)}$-harmonics by Eq. (27), we obtain the following dispersion equation

$$\left(\frac{\pi nc}{La\lambda_0\omega}\right)^2 = 1 + 2\sum_{\alpha}^{e,i_1,i_2,\ldots} \chi_{l,\alpha}^{n,n}(L), \quad (37)$$

where α denotes the particle species (electron, proton, or heavier ions). This equation can describe the instability of the right-hand polarized waves (whistler instability) if $l = -1$, and the left-hand polarized waves (proton- or ion-cyclotron instability) if $l = 1$. Note that, in our notation, the parallel wave vector is defined as $k_{\parallel} = n\pi/(La\lambda_0)$, such that $n\pi c/(La\lambda_0\omega)$ is the normalized parallel refractive index. Equation (37) needs to be resolved numerically for the real and imaginary parts of the wave frequency, $\omega = \mathrm{Re}\,\omega + i\,\mathrm{Im}\,\omega$, in order to obtain the instability conditions ($\mathrm{Im}\,\omega > 0$) in the laboratory dipole plasmas with anisotropic temperature.

The growth (damping) rate, i.e., $\mathrm{Im}\,\omega$, of the electromagnetic cyclotron waves is defined by the contribution of the resonant particles to the imaginary part of the transverse susceptibility, $\mathrm{Im}\,\chi_{l,\alpha}^{n,n}$, and can be readily derived from Eqs. (28) and (29), using the Landau residue. In this case

$$\mathrm{Im}\,\chi_{l,\alpha}^{n,n} = \mathrm{Im}\,\chi_{l,u,\alpha}^{n,n} + \mathrm{Im}\,\chi_{l,t,\alpha}^{n,n} = \sum_{p=1}^{\infty}\left(\mathrm{Im}\,\chi_{l,p,u,\alpha}^{n,n} + \mathrm{Im}\,\chi_{l,p,t,\alpha}^{n,n}\right),$$

where

$$\mathrm{Im}\,\chi_{l,p,u,\alpha}^{n,n} = \frac{\omega_p^2 LaT_{\parallel\alpha}(1+\kappa)\Gamma(1+\kappa)}{8\omega\pi^{1.5}\lambda_0 T_{\perp\alpha}\sqrt{\kappa}\vartheta_{\parallel}\Gamma(\kappa-0.5)p^5} \times \int_0^{\mu_0} \frac{A_{l,p}^n\left(\frac{Z_{l,u,\alpha}}{p},\mu\right) B_{l,p}^n\left(\frac{Z_{l,u,\alpha}}{p},\mu\right) Z_{l,u,\alpha}^4}{\left\{1 + \frac{Z_{l,u,\alpha}^2}{p^2}\left[1-\mu\left(1-\frac{T_{\parallel}}{T_{\perp}}\right)\right]\right\}^{2+\kappa}}\mu d\mu, \quad (38)$$

$$\mathrm{Im}\,\chi_{l,p,t,\alpha}^{n,n} = \frac{\omega_p^2 LaT_{\parallel\alpha}(1+\kappa)\Gamma(1+\kappa)}{8\omega\pi^{1.5}\lambda_0 T_{\perp\alpha}\sqrt{\kappa}\vartheta_{\parallel}\Gamma(\kappa-0.5)p^5} \times \int_{\mu_0}^{1} \frac{C_{l,p}^n\left(\frac{Z_{l,t,\alpha}}{p},\mu\right) D_{l,p}^n\left(\frac{Z_{l,t,\alpha}}{p},\mu\right) Z_{l,t,\alpha}^4}{\left\{1 + \frac{Z_{l,t,\alpha}^2}{p^2}\left[1-\mu\left(1-\frac{T_{\parallel}}{T_{\perp}}\right)\right]\right\}^{2+\kappa}}\mu d\mu, \quad (39)$$

are the separate contributions of the bounce resonance terms for, respectively, untrapped and trapped particles having anisotropic bi-Kappa distributions in velocity space. These expressions are markedly different from those obtained in LDM plasmas with bi-Maxwellian distributions [24], and which can be recovered in the limit $\kappa \to \infty$. The electromagnetic ion-cyclotron and electron-cyclotron waves in LDM plasmas with anisotropic temperature are limited to frequency ranges below the minimal cyclotron frequencies of ions, $\mathrm{Re}\,\omega < \Omega_{co,i}$, and electrons, $\mathrm{Re}\,\omega < |\Omega_{co,e}|$, respectively.

6. Conclusion

To conclude, let us first summarize the main results of the paper. The transverse current density components in the 2D LDM plasma with anisotropic bi-Kappa distributions are evaluated by solving the Vlasov equation for perturbed distribution functions of u- and t-particles in the lowest (zero-) order of the magnetization parameter. The new dielectric elements obtained in Eqs. (28) and (29) are expressed by a summation of the bounce-resonant terms including the double integration in velocity space, the resonant denominators, and the corresponding phase coefficients in Eqs. (30)-(33). Due to a 2D magnetic field non-uniformity, the bounce resonance conditions for t- and u-particles in the laboratory magnetospheric plasmas are different from the resonance conditions in a uniform magnetic field. The whole spectrum of the electric field is present in the current density harmonic in Eq. (27), and the left-hand and right-hand polarized waves are coupled.

In Eq. (37) we have provided the dispersion relations for the field-aligned cyclotron waves in LDM with bi-Kappa distributed particles. In the limit of a very large power-index $\kappa \to \infty$ the dielectric characteristics (28) and (29) and dispersion relations (37) reproduce the corresponding expressions obtained in Ref. [24] for bi-Maxwellian distributions. It should be mentioned that for isotropic temperatures ($T_{\parallel} = T_{\perp} = T$) expressions (28)-(36) simplify, and provide the corresponding expressions for the dielectric susceptibility elements in a LDM plasma with isotropic Kappa distributions. In the limit of a very small (negligible) ring-current radius ($a \to 0$), our plasma model reduces to a 2D Earth's magnetosphere with bi-Kappa distributed particles [27].

Similarly to linear cyclotron plasma waves in a uniform magnetic field, the n-th harmonic of the electric field is assumed giving the main contribution to the n-th harmonic of the transverse current density components, and the coupling between the left-hand and right-hand waves is negligibly small. Dispersion Eq. (37) can describe the electromagnetic instabilities of both the electron-cyclotron ($l = -1$) and ion-cyclotron ($l = 1$) modes accounting for the cyclotron, transit-time and bounce resonances. As for a plasma confined in a uniform magnetic field, the growth (or damping) rates of the cyclotron waves in a 2D LDM plasma are defined by the contributions of the resonant trapped and untrapped particles to the imaginary part of the transverse dielectric susceptibility, Eqs. (38) and (39). Present results are of particular interest for understanding the role of cyclotron waves as ECR/ICR heating mechanisms in the laboratory dipole magnetospheric plasmas.

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