Fast Algorithm for Tomographic Reconstruction for Plasma Emission

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Temporal evolution of two dimensional image of plasma emission has been obtained using a reconstruction algorithm called Maximum Likelihood-Expectation Maximization (MLEM), with a tomography system in a linear cylindrical plasma. The calculation of tomography images for the whole plasma duration needs parallel computation due to the iteration processes of MLEM. However, faster and simpler tomography algorithm giving a consistent image with MLEM should be desirable for monitoring the emission profile and quick analysis between shots. This paper proposes such a new algorithm of a modified Tikhonov regularization with introducing a weight matrix, termed here direct matrix reconstruction (DMR).

Keywords: plasma turbulence, turbulence tomography, MLEM, local emission fluctuation

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Tomography is a useful tool to investigate an internal structure without giving any or less perturbation on the target to observe. In plasma research the tomography has been applied to study internal structure of plasmas by measuring its line-integrated emissions [1]. Recently, a prototype tomography system for plasma spontaneous emission has been installed on a linear cylindrical device named PANTA (Plasma Assembly for Nonlinear Turbulence Analysis). The prototype system succeeded in showing dynamic evolution of plasmas, using a reconstruction algorithm termed *Maximum Likelihood-Expectation Maximization* (MLEM) with a sufficient temporal resolution and a good signal-to-noise ratio [2].

Although the MLEM reconstruction provides quite precise and reasonable images, the reconstruction for the whole discharge needs extraordinary long CPU time compared with usual experimental data analysis. Therefore, faster tomography algorithms are desirable for monitoring data for determineing operation scenario during experiment. An algorithm has been developed based on the Tikhonov reguralization [3] termed here direct matrix reconstruction (DMR) method. The article describes the algorithm and the feature of reconstructed image when this method is applied to the experimental data obtained from the tomography system in PANTA.

The local emission can be inferred by minimizing the following function,

$$\hat{\chi}^2_{g,\mathrm{DMR}} = \sum_{i=1}^{M}(g_i - \hat{g}_i)^2 + \gamma \sum_{j=1}^{N}(w_j \varepsilon_j)^2, \quad (1)$$

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where the local emission on the i-th grid and the inferred line-integrated emission are ε_i ($i = 1, \cdots, 121$) and $\hat{g}_i = \sum_k h_{ik}\varepsilon_k$, respectively, and w_j represents the weight for each grid. Here, the area where the j-th line-of-sight intersects with the i-th grid is denoted as h_{ji}. The second term on the right-hand-side is a penalty function to make the obtained solution reasonable. The condition to minimize Eq. (1), $(\partial\hat{\chi}^2_{g,\mathrm{DMR}}/\partial\varepsilon_k) = 0$, gives a set of equations, which are written in a matrix form, as

$$(H + \gamma W)\vec{\varepsilon} = \vec{G}, \quad (2)$$

where H is a matrix whose components are defined as $H_{ij} = \sum h_{ki}h_{kj}$ and $\vec{G}$ is a vector whose components are defined as $G_i = \sum h_{ki}g_k$, and W represents a diagonal matrix composed of the weight, w_j. If we choose an appropriate constant γ for the penalty function, the matrix equation gives an appropriate solution. The solution is obtained as $\vec{\varepsilon} = (H + \gamma W)^{-1}\vec{G}$.

The DMR method is applied on an assumed emission profile for a trial. Figure 1 shows, for comparison, the original image, the MLEM and DMR images, together with the residual errors to the MLEM image and the line-integrated data defined as $\chi^2_{e,\mathrm{MLEM}} = \sum(\varepsilon_{i,\mathrm{MLEM}} - \hat{\varepsilon}_i)^2$ and $\chi^2_g = \sum(g_i - \hat{g}_i)^2$, respectively, as a function of γ, where $\varepsilon_{i,\mathrm{MLEM}}$ represents the emission intensity at i-th grid obtained by MLEM. The residual error, $\chi^2_{e,\mathrm{MLEM}}$, shows a constant minimum values when $10^{-8} < \gamma < 10^{-6}$. The original image is well reproduced in this range of γ, therefore, a proper emission profile can be obtained by choosing appropriate γ and weight w_i.

Next, the method is applied on the experimental data obtained in a linear cylindrical plasma, with 5 cm radius

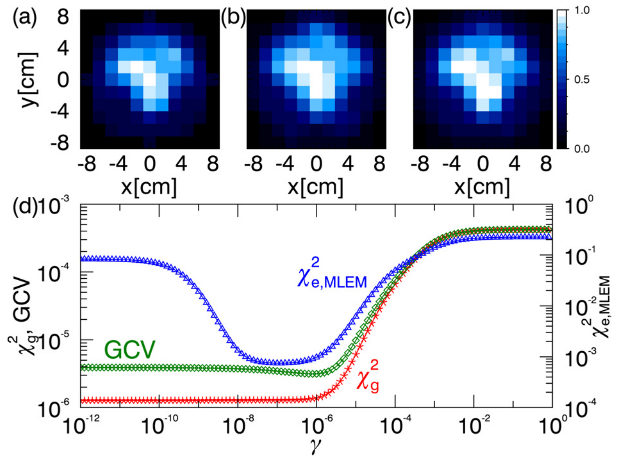

Fig. 1 (a) The original image, (b) MLEM reconstructed image using the line-of-sight data created from the test image and (c) DMR image when $\gamma = 1.0 \times 10^{-6}$. (d) γ dependence of the residual error between DMR image and MLEM image ($\chi^2_{e,\mathrm{MLEM}}$), the residual error between the original line-of-sight signal and that obtained from the DMR image (χ^2_g) and GCV [4]. The GCV is an indicator often used to evaluate the best parameter, γ.

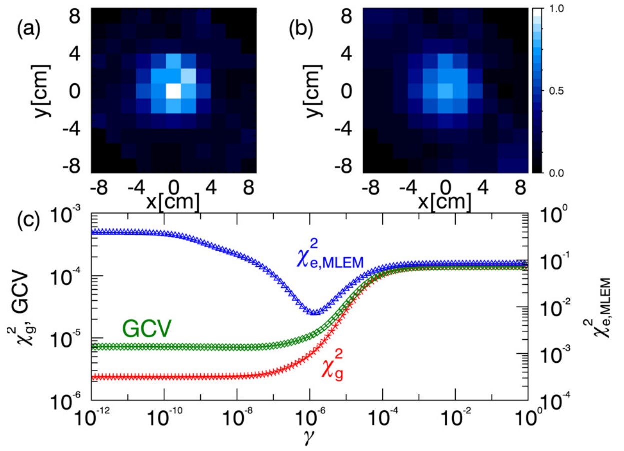

Fig. 2 (a) MLEM imege obtained from the experimental data, (b) the DMR image with $w_i = 1$ when $\gamma = 1.0 \times 10^{-6}$, where $\chi^2_{e,\mathrm{MLEM}}$ takes its minimum. (c) γ dependence of GCV, $\chi^2_{e,\mathrm{MLEM}}$ and χ^2_g.

column and 4 m length, in the PANTA device. Figure 2 (a) shows the plasma emission profile obtained with MLEM method applied to 128 line-integrated emission data covering the plasma cross-section ranging from $L = -8$ to $L = 8$ cm. Figure 2 (c) shows the residual error as a function of γ, where the residual errors are evaluated by taking the summation of the square of difference between MLEM and reconstructed images, assuming that the MLEM image gives a proper emission profile. Here, in the calculation the weight is assumed to be $w_i = 1$ for every grid.

The residual error, $\chi^2_{e,\mathrm{MLEM}}$, shows its minimum when $\gamma = 10^{-6}$, so the reconstructed image with this value should be similar to the MLEM image. Figure 2 (b) shows the best

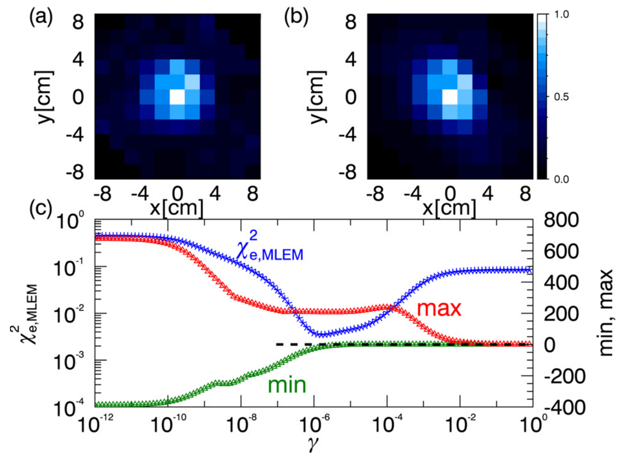

Fig. 3 (a) MLEM imege obtained from the experimental data and (b) the DMR image with $w_i = 0.9r_i^2 + 0.1$ when $\gamma = 4.0 \times 10^{-6}$, where $\chi^2_{e,\mathrm{MLEM}}$ takes its minimum. (c) γ dependence of $\chi^2_{e,\mathrm{MLEM}}$, the minimum and maximum value of the reconstructed image.

DMR image. One can easily find two obvious problems in the image. First, the obtained emission image is too flattened and does not reproduce rather sharp peaks at the center and on the upper-right side. Second, several grid around the peripheral region show significant negative values. In this case, one grid at the edge has emission value of −0.2, while the peak value of the emission profile is about 0.8.

A possible manner to solve the above-mentioned problems is to choose the weigh function, w_i, properly. Here we assume the form of the weight function as $w_i = ar_i^2 + b$, where r_i means the distance from the center of the coordinate or plasma, and a and b are the constants which determine the weight profile. Figure 3 shows a reconstructed image which is quite similar to the MLEM image, where a and b are set to 0.9 and 0.1, respectively. In this case, the constant γ is varied and the residual errors are obtained as a function of γ. When $\gamma = 4 \times 10^{-6}$, $\chi^2_{e,\mathrm{MLEM}}$ shows almost minimum value and the reconstructed image is quite similar to the MLEM image. The mean error of single grid estimated from $\chi^2_{e,\mathrm{MLEM}}$ is less than 0.01, which is quite small compared to the maximum value of the emission profile. Although the reconstructed image has a grid with the emission value of −0.025, the negative value is negligibly small compared to the peak emission value.

The calculation time of DMR is quite short compared to that of MLEM. It takes 0.3 ms and 51 ms for DMR and MLEM, respectively, to calculate a reconstruction image from a set of the line-integrated data once γ and the inverse matrix are determined. Therefore, DMR is 150 times faster than MLEM in the calculation of a reconstruction, although the DMR needs to calculate the inverse matrix once, which takes about 30 ms, in advance for the reconstruction. However, the calculation time of the inverse matrix is negligible since the total number of reconstruction images is 0.6 million; the data is sampled in 1 μs for the whole discharge duration of 600 ms in PANTA.

At present, the whole reconstruction with MLEM is performed for a discharge using parallel processing on 10 power macintosh computers with totally 12-core CPUs (2× 2.4 GHz 6-Core Intel Xeon). In contrast, the reconstruction of the emission profile for whole discharge duration with the DMR needs a similar calculation time with a single computer. Therefore, the DMR method introduced here can be used for the monitoring and quick analysis of the first stage for plasma tomography data.

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