On the Application of Cross Bispectrum and Cross Bicoherence

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Bispectrum and bicoherence analysis is a powerful method to analyze nonlinear interaction of turbulent plasmas. In this review, we explain the difference of the bispectrum and bicoherence analysis and possible research that can be pursued with these methods. Basic concept is explained by using several examples from previous successes brought by bicoherence and bispectrum analysis, such as drift wave-zonal flow interaction. Open questions are discussed that can be challenged by these methods. Problems such as spatial transport of fluctuation energy (i.e. turbulence spreading), momentum transport by the triplet correlation, and so on, are treated. Bispectrum and bicoherence analysis can be utilized to expand the forefront of plasma turbulence research.

1. Introduction

In magnetically confined plasmas, low frequency fluctuation around drift wave frequency plays an important role [1]. Nonlinear interaction of drift waves is an important issue to understand turbulence and structural formation in magnetized hot plasmas, both from theoretical and experimental sides. This is since nonlinear interaction is strong enough to modify the linear dispersion [2]. Nonliear interaction is analyzed by applying bicoherence analysis. Early application is discussed in [3–9]. It has become possible to experimentally quantify the nonlinear interaction, by comparing the observed bisepectrum and nonlinear coupling coefficients [10]. More recently, bispectrum analysis is widely applied in experimental studies, given the importance of linearly stable but nonlinearly excited fluctuations, such as zonal flows [11–17]. As a result, several important observations are reported which identify excitation mechanisms of zonal flows, zonal fields, streamer, and so on [12, 15, 16, 18]. Including these examples, bicoherence analysis is widely used as a powerful method. A comprehensive review is given on the application to the physics of structural formation in plasmas [19, 20]. Nowadays, bicoherence analysis has become a standard method and has been routinely applied. The understanding on the statistical convergence is deepened and the method is important to challenge a wider class of problems. Inroductory explanation is also available [21, 22].

Bicoherence analysis can be classified into two different approaches. One is auto-bicoherence, which analyzes the same physical quantity. The other is cross-coherence, where different physical quantities are combined. Utilizing the both methods can accelerate plasma research. For instance, in the case of zonal flows, we expect that there are differences between the auto-bicoherence and the cross-bicoherence of density and potential fluctuations. The difference can be used to experimentally characterize zonal flows. By extracting the envelope variation of microscopic density fluctuation, we can observe a footprint of zonal flows on the density fluctuation [14]. This is a typical example of the new method, referred as parametric spectroscopy [23].

Recent experiments report observation of, not only scalar fields such as density and potential, but also vector fields such as velocity fluctuation. Non-linear coupling of scalar fields and vector fields attracts attention. In searching the several physical quantities, it is desired to have perspective both on physics issue to address with these quantities and on the required advanced statistical analysis.

In this note, we explain possible research that can be pursued by analyzing cross-bispectrum and/or cross-bicoherence of density fluctuation, potential fluctuation, velocity fluctuation, and so on. Note that this type of approach is often referred to as bispectrum analysis or bicoherence analysis. Here we distinguish these and explain them separately. In cross- and auto-coherence analysis, a normalized quantity is analyzed. This is important to quantify nonlinear coupling by focusing on the combination of mode, for instance. On the other hand, in cross-bispectrum analysis, an unnormalized quantity is used. Cross-bispectrum analysis plays an important role to discuss the absolute value of the intensity of physical quantities, spatial transport flux, and so on. As the both methods have their own merits, by using both cross-coherence and cross-bispectrum analysis, we can experimentally quantify nonlinear coupling and transport flux of physical quantities.

2. Model

2.1 Plasma of interest

We consider a magnetized plasma. For simplicity, a slab geometry is employed, with the magnetic field pointing in the z direction. See Fig. 1, with drift velocity and parallel flow. The local coordinate (x, y) is introduced, where x corresponds to the radial direction r and y corresponds to the poloidal direction θ.

Fluctuations are assumed quasi-electrostatic. Density (n), potential (ϕ), and velocity (V) fluctuations are assumed to be measured. Temperature fluctuation is also an important quantity. However, its observation is limited, and we do not consider it further here.

Under these conditions, there can be drift mode [1] and D'Angelo mode [24–35] as a primary modes, and zonal flows [11] and streamers [18] as a secondary mode.

2.2 Second order correlation

$\langle X, Y \rangle$ ($\langle \ldots \rangle$ is a long time average at the same point) is calculated routinely. Since we consider physical quantities at the same point, we use the temporally Fourier analyzed

$$X(f)Y(f)^*. \tag{1}$$

Combinations are shown in Table 1.

Here the auto-correlation of density, potential, and velocity fluctuations (1,2,10,11,12) are calculated as power spectrum.

Starting from scalar quantities, the ratio of $\langle nn \rangle$ and $\langle \phi\phi \rangle$ is a standard quantity for drift waves. Under a proper normalization, the relation

$$\left\langle \frac{n}{n_0} \frac{n}{n_0} \right\rangle \approx \left\langle \frac{e\phi}{T_e} \frac{e\phi}{T_e} \right\rangle, \tag{2}$$

is a basis for identifying drift waves. Here n_0 is the time averaged density.

Among cross correlations, $\langle n\phi \rangle$ (3) is used to evaluate the phase difference between density and potential. In the case of drift waves,

$$\frac{n}{n_0} = (1 - i\delta)\frac{e\phi}{T_e}, \tag{3}$$

is well known. For instance, for the collisional drift waves [30], the phase shift is given by

$$\delta = \frac{\omega_{*e} - \omega}{k_z^2 D_z}. \tag{4}$$

ω_{*e} is drift frequency, D_z is the electron parallel diffusivity. The relation represents the causal relation of the excitation of drift waves. The finite phase shift also indicates a finite particle flux associated with the relaxation of density profile.

In the case of D'Angelo mode,

$$\frac{n}{n_0} \approx \frac{e\phi}{T_e}. \tag{5}$$

Density is close to adiabatic response, and D'Angelo mode does not yield particle flux at the simplest model. This is plausible since D'Angelo mode is driven by the parallel velocity shear. D'Angelo mode has distinctive feature in parallel velocity fluctuation and relaxes the parallel flow profile.

When drift waves and D'Angelo modes co-exist, the density is given by

$$\frac{n}{n_0} = (1 - i\delta_D + i\delta_V)\frac{e\phi}{T_e}. \tag{6}$$

The phase shift δ_D is the same as that for drift waves. The contribution [30] from D'Angelo modes is given by

$$\delta_V = \frac{c_s k_z \rho_s k_y \langle V_z \rangle' - c_s^2 k_z^2}{k_z^2 D_z \omega}, \tag{7}$$

c_s is ion sound speed and ρ_s is ion sound Lamour radius. While drift waves have the phase relation that indicates the relation of density profile, D'Angelo modes have the phase
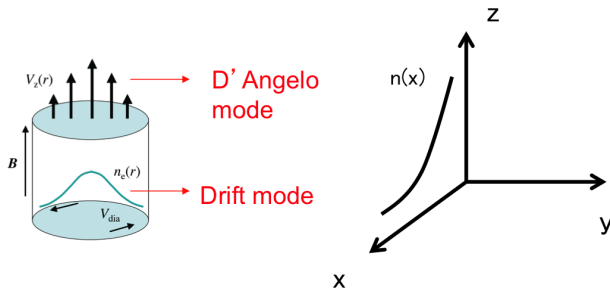

Fig. 1 Magnetized inhomogeneous plasmas with two driving sources. Shown are radial gradients of density (pressure) and flows along the magnetic field. In the local coordinate, z is in the direction of the magnetic field, x is in the direction of inhomogeneity, and y corresponds to the poloidal direction.

Table 1 Combinations of second order correlation.

#	X	Y
1	n	n
2	ϕ	ϕ
3	n	ϕ
4	n	V_x
5	n	V_y
6	n	V_z
7	ϕ	V_x
8	ϕ	V_y
9	ϕ	V_z
10	V_x	V_x
11	V_y	V_y
12	V_z	V_z
13	V_x	V_y
14	V_x	V_z
15	V_y	V_z

relation that peak the density profile (yield up-hill particle flux).

In the case with both vector and scalar quantities,

$$nv_x \quad (4),$$

has been analyzed to discuss turbulent flux. On the other hand, while similar quantities

$$nv_y \quad (5), \quad nv_z \quad (6),$$

correspond to momentum density of fluctuations, they have not been experimentally evaluated. For drift waves, since $n \sim \phi$, 4 is finite with first order correction, and 7 becomes zero.

In the case of the correlation of vector quantities,

$$v_x v_y \quad (13), \quad v_x v_z \quad (14), \quad v_y v_z \quad (15),$$

are often evaluated.

Finally,

$$nn \ (1), \quad \phi\phi \ (2), \quad v_x v_x + v_y v_y + v_z v_z \ (10 + 11 + 12),$$

are related to fluctuation intensity. For simple models of drift waves, fluctuation intensity is given by

$$\left(\frac{n}{n_0}\right)^2 + \left(\frac{V_\perp}{c_s}\right)^2 \approx (1 + \rho_s^2 k_\perp^2)\left(\frac{e\phi}{T_e}\right)^2. \quad (8)$$

In recent studies, fluctuation amplitude obtained by two body correlations are utilized to distinguish characteristic modes in the system. Compared to drift waves, D'Angelo modes have larger parallel velocity fluctuation. This feature can be expressed by using 1, 12 as

$$\left\langle \frac{n}{n_0}\frac{n}{n_0}\right\rangle > \left\langle \frac{V_z}{c_s}\frac{V_z}{c_s}\right\rangle, \quad (9)$$

for drift waves, and

$$\left\langle \frac{n}{n_0}\frac{n}{n_0}\right\rangle \approx \left\langle \frac{V_z}{c_s}\frac{V_z}{c_s}\right\rangle, \quad (10)$$

for D'Angelo modes. Recent data analysis on basic experiments reports that drift waves and D'Angelo modes can co-exist in different radial and frequency domains [31, 35].

3. Bispectrum and Cross Bispectrum
3.1 Third order correlation

Triplet correlation

$\langle X, Y, Z \rangle$ ($\langle \ldots \rangle$ is a long time average at a given point), attracts attention and is analyzed to elucidate nonlinear mechanisms.

In many cases, bispectrum analysis is performed by using quantities Fourier transformed in time

$$X(f1)Y(f2)Z(f1 + f2)^*,$$

and the results are applied to experiments. See Fig. 2 for instance. Nonlinear coupling also needs to satisfy the matching condition for wave number. There are studies using
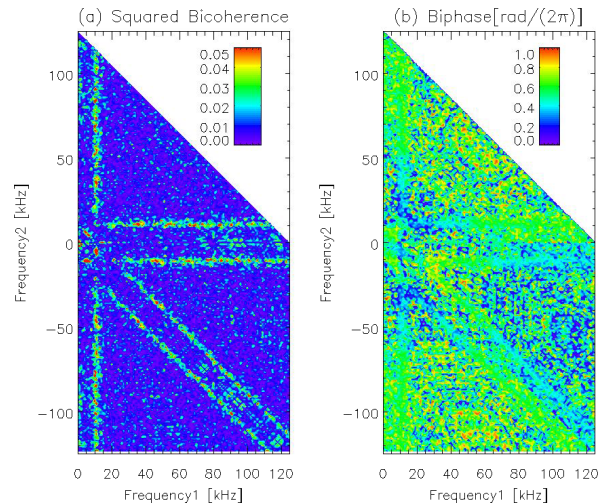

Fig. 2 Examples of bispectral analysis for drift wave-zonal flow turbulence in JFT-2M tokamak. (a) Bicoherence and (b) biphase analyses of floating potential fluctuation. The figures are quoted from Y. Nagashima, *et al.*, PPCF 45, S1 (2006), and their layout is changed. Here the data used is SN98390 and the significance level is 0.005.

data Fourier transformed in space, wave number spectrum [17, 36]. In more advanced analysis, nonlinear correlation is evaluated in frequency domain with the wave number matching as a constraint. Figure 3 shows a typical example of such analysis. With these studies in mind, we discuss the bispectrum analysis in frequency domain in this work.

Fourier analyzing in time, bi-spectrum

$$X(f1)Y(f2)Z(f1 + f2)^*, \quad (11)$$

is applied for the analysis of data obtained from experiments.

Triplet correlations of scalar and vector quantities are shown in Table 2. b1~b4 are bispectrum of scalar quantities, b5~b25 are bispectrum between scalar and vector quantities, b26~b35 are bispectrum of vector quantities.

3.2 Scalar quantity

Bispectrum of scalar quantities has been applied for the study of zonal flows and its utility has been proven [8, 9, 12–14]. For the problem of the excitation of zonal flows by drift waves, cross-bispectrum of both (1) potentials themselves and (2) potential and density is analyzed. By using the difference in the density variations by drift waves and by zonal flows, it has been reported that they have a different nonlinearity and that zonal flows modulate the spectrum of random turbulent fluctuation.

Moreover, the difference in b1, b2, b3, b4, can be used to extract qualitative difference between drift waves and zonal flows (Fig. 4). This is possible since while density variation by zonal flows is small, zonal flows mainly appear as a variation in potential fluctuation. Let the frequency of zonal flow f. In the coupling of density n that
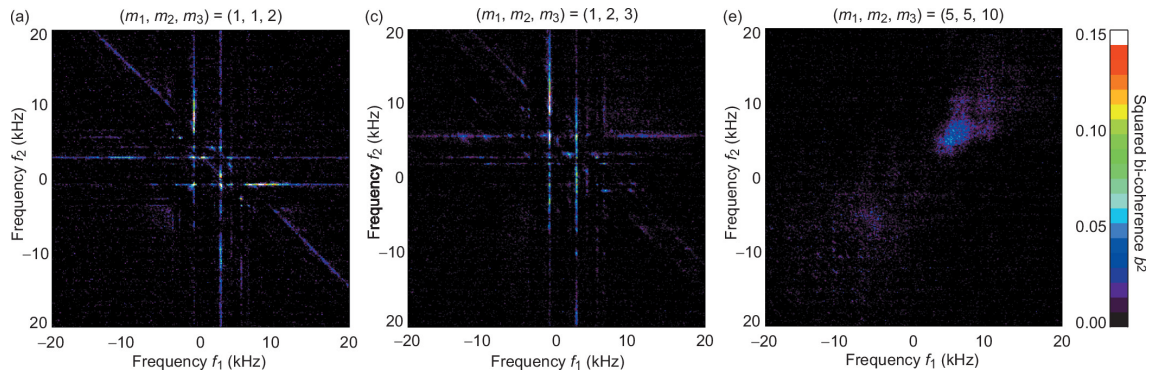
Fig. 3 Example of bicoherence analysis with the mode number matched. Here the 2D fourier specrum in frequency and the poloidal mode number is evaluated from the azimuhtal probe array (64 channels). Bioherence in frequency is evaluated for the data with mode number mathced, $m_3 = m_1 + m_2$. Three different cases are shown, with $(m_1, m_2, m_3) = (1, 1, 2), (1, 2, 3)$, and $(5, 5, 10)$. The figure is quoted from [37].

Table 2 Combinations of triplet correlation.

#	X	Y	Z
b1	n	n	n
b2	ϕ	ϕ	ϕ
b3	n	n	ϕ
b4	n	ϕ	ϕ
b5	n	n	V_x
b6	ϕ	ϕ	V_x
b7	n	ϕ	V_x
b8	n	n	V_y
b9	ϕ	ϕ	V_y
b10	n	ϕ	V_y
b11	n	n	V_z
b12	ϕ	ϕ	V_z
b13	n	ϕ	V_z
b14	n	V_x	V_x
b15	n	V_y	V_y
b16	n	V_z	V_z
b17	n	V_x	V_y
b18	n	V_x	V_z
b19	n	V_y	V_z
b20	ϕ	V_x	V_x
b21	ϕ	V_y	V_y
b22	ϕ	V_z	V_z
b23	ϕ	V_x	V_y
b24	ϕ	V_x	V_z
b25	ϕ	V_y	V_z
b26	V_x	V_x	V_x
b27	V_x	V_y	V_y
b28	V_x	V_z	V_z
b29	V_x	V_x	V_y
b30	V_x	V_x	V_z
b31	V_x	V_y	V_z
b32	V_y	V_y	V_y
b33	V_y	V_z	V_z
b34	V_y	V_y	V_z
b35	V_z	V_z	V_z

gives the frequency f, there is no appreciable peak in bispectrum. On the other hand, in the coupling of potential ϕ that gives the frequency f, there is a clear peak in bispectrum. If we do not use the density variation itself, but extract envelope variation of density variation, we can ob-

serve the nonlinear coupling of drift waves and zonal flows from the combination of {envelope of n, n, n}. This is since zonal flows modulate density fluctuation.

As a caveat, radial electric field by $E \times B$ drift is obtained under the assumption that the temporal variation is slower than the cyclotron motion. However, there can be a case that the polarization drift obtained from the time derivative of electric field fluctuation cannot be neglected. The effect of the polarization drift is introduced in Hasegawa-Mima model, for example. In the case of collisional drift waves, where radial flux is finite, the polarization drift may produce poloidal current, which can be evaluated by the statistical average of density fluctuation and poloidal electric field. This may open a new path for the study of observation of currents in plasmas.

3.3 Cross-bispectrum of scalar and vector fields
3.3.1 Spatial transfer of fluctuation intensity

Fluctuation amplitude can spatially transfer. The behavior is observed in experiments and studied widely. In order to study the process, bispectrum containing vector quantities is important. Cross-bispectrum of scalar and vector quantities plays a key role to study spatial transport.

For drift waves, fluctuation intensity due to density variation is given by

$$nn. \tag{12}$$

Then

$$V_x nn, \tag{13}$$

describes the amount of spatial transfer in the x direction of the intensity of density fluctuation of drift waves. It is the physical quantity treated in turbulence spreading [38, 39]. As depicted in Fig. 5, we consider fluctuation intensity (evaluated by density fluctuation) varying in time. At the same time, V_x varies in time as well. If their variation is in phase, fluctuation intensity can spatially move in the x
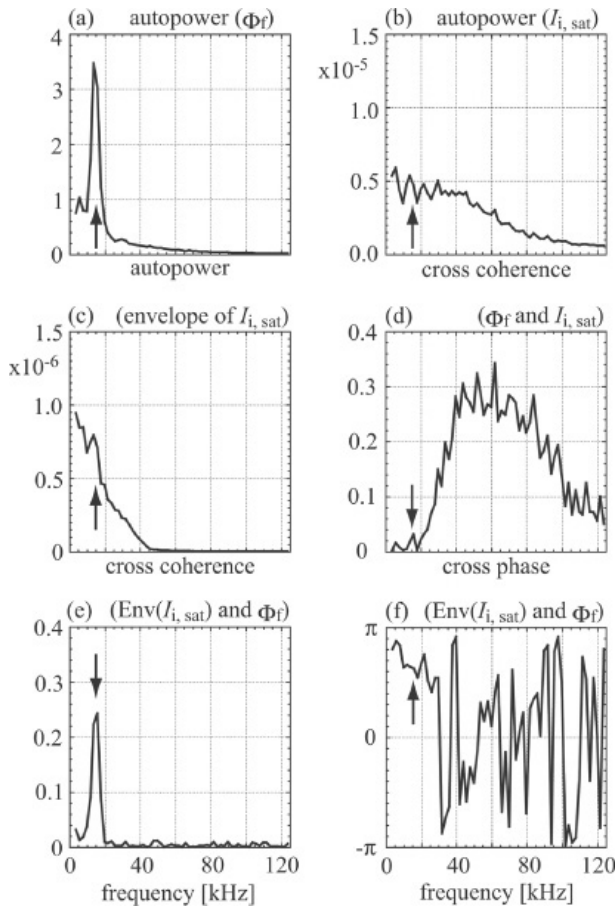
Fig. 4 Analysis of low frequency fluctuation with floating potential and ion saturation current. Combination of floating potential, (a) auto-power of the floating potential, (b) auto-power of the ion saturation current, and (c) auto-power of the envelope of the ion saturation current, (d) squared cross-coherence between the floating potential and ion saturation current, (e) squared cross-coherence between the floating potential and the envelope of the ion saturation current, and (f) cross-phase between the floating potential and the envelope of the ion saturation current. In (e), significant coherence between the envelope and floating potential at zonal flow frequency ($\sim 10\,\mathrm{kHz}$) is observed, indicating the envelope of the ion saturation current has information of zonal flows. The figure is quoted from Ref. [14].

direction on average. In order to evaluate the contribution, we need to evaluate the cross-bispectrum

$$n(f1)n(f2)V_x(f1 + f2)^*. \tag{14}$$

Spatial transport of turbulence intensity (by background turbulence) is considered to be an important element to determine the evolution of fluctuation intensity. For instance, the spatial flux of turbulence intensity is theoretically expressed as $-D\,\mathrm{grad}\,I + VI$ (within the local model. The diffusive term is often considered to be important.). As a result, turbulence intensity evolution is given
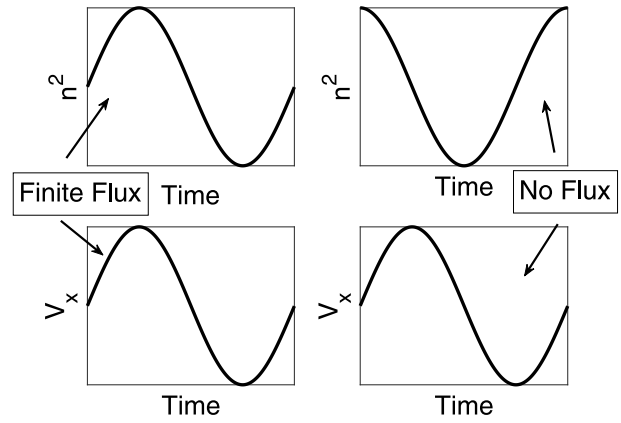

Fig. 5 Spatial flux of fluctuation energy (represented by density fluctuation). Finite spatial flux results when the time evolution of fluctuation energy and radial velocity fluctuation are in phase.

by

$$\frac{\mathrm{d}}{\mathrm{d}t}I = \gamma_\mathrm{L}I - \gamma_\mathrm{NL}I - \gamma_\mathrm{mod}I + D\Delta I. \tag{15}$$

The first term in RHS is linear excitation, the second term is nonlinear stabilization (including the stabilization by the first spatial derivative of DC radial electric field [40]), the third term is due to coupling to zonal flow, and the fourth term is the nonlinear stabilization (excitation) due to spatial transport of turbulence intensity by background turbulence [41]. Leaving aside the validity of the localized model of the flux, within the model, the fourth term cannot be neglected compared to the other three terms. Indeed, theoretical and numerical studies report the propagation of turbulence front and entrainment of stable region [42]. Front propagation is also reported from experiments [43,44]. Experiments on linear machines also report the spatial broadening of fluctuation spectrum from the linearly unstable region with steep density gradient into linearly stable region [35]. Cross-bispectrum can be used to further verify the processes.

3.3.2 Reynolds stress

The importance of Reynolds stress is recently pointed out both by theory and experiment. Typically, the excitation of poloidal flow (excitation of radial electric field) by turbulence is studied by evaluating the second order correlation

$$V_x V_y, \tag{16}$$

as the stress per particle. In the case of the excitation of flows along magnetic field,

$$V_x V_z, \tag{17}$$

is analyzed as the stress par particle. However, Reynolds stress is defined by the triplet correlation,
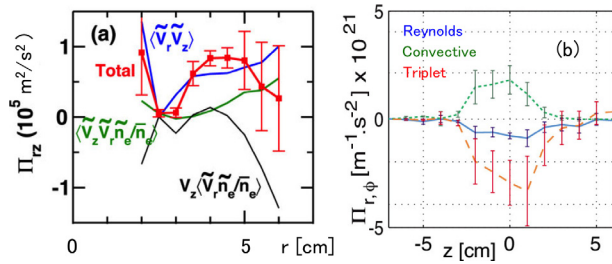
$$n\,V_x\,V_y,$$

Fig. 6 Observation of momentum flux. (a) Result on PANTA [31]. r is the radial direction and z is the direction of the magnetic field. (b) Result on TORPEX [48]. z is the vertical direction and ϕ is the toroidal direction. In both cases, the triplet term (green for (a) and red for (b)) is finite and can be dominant in some cases.

$$n\, V_x\, V_z,$$
$$n\, V_y\, V_z. \tag{18}$$

They should be evaluated by cross-bispectrum [45–50].

First, if we consider the excitation of poloidal flow (radial electric field), the difference between

$$\langle n\, V_x\, V_y \rangle \quad \text{and} \quad n_0\langle V_x\, V_y \rangle, \tag{19}$$

can be studied experimentally [49]. Most experiments focus on the latter quantity and evaluate the phase relation between poloidal flow fluctuation and fluctuation velocity in the radial direction. In actual experiments, density fluctuation is simultaneously obtained in many cases. The triplet correlation can be evaluated.

In the case of the excitation of flows along the magnetic field, the study on the difference of the quantities (Fig. 6)

$$\langle n\, V_x\, V_z \rangle \quad \text{and} \quad n_0\langle V_x\, V_z \rangle, \tag{20}$$

have been started [31, 50]. Recent theoretical study reports that these quantities are key to understand the variety of toroidal rotation of plasmas [45]. In addition to the residual stress evaluated by the two body correlation, the triplet correlation can be considered. The triplet correlation describes the process such as turbulence momentum spreading [28]. The effect of coupling edge-core toroidal flows is pointed out, which awaits experimental validation [50]. Related to this, the Reynolds stress by the triplet correlation is evaluated on the DIII-D tokamak [51].

The simultaneous analysis of multiple Reynolds stresses and fluxes such as particle transport is important for turbulence excited by multiple inhomogeneities (free energy source) such as density gradient and velocity gradient. Recently, the concept of cross-ferroic turbulence has been proposed and the framework of plasma turbulence research is expanded [52]. In condensed matter physics, research on multi-ferroic materials is on-going, and the spontaneous excitation (cross correlation) of macroscopic field, such as ferromagnetism, ferroelectricity, and so on,
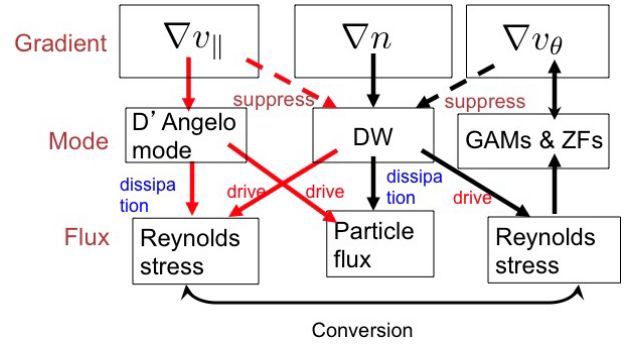

Fig. 7 Structural formation in plasmas with radial gradients of density (pressure) and parallel velocity. In addition to the excitation and suppression of drift waves and zonal flows, D'Angelo modes can drive relaxation and steepening of parallel velocity profile and density profile.

is discussed. The cross-correlation in turbulent plasmas is also important. It can be a key to understand the mechanisms behind generation, competition, and annihilation of macroscopic fields, topological change (change in dissipation and symmetry), and symmetry breaking and conversion. Plasmas with both pressure and velocity gradients, as in Fig. 1, are often observed in laboratory and space plasmas. Fluctuations of interest in these situations are drift waves, D'Angle modes, and so on. Figure 7 depicts the structural formation in plasmas with density gradient and parallel flow gradient. It is well known that zonal flows are excited by drift waves and regulate the primary drift waves [11]. In addition, density profile and parallel velocity profile evolve in time via D'Angelo modes. It is possible to have a relation of one profile and generation of the secondary strucuture in the other. Flows along the magnetic field can be converted into the perpendicular flow, and vice versa. These processes reflect rich structural formation processes in nature. Theoretical studies are in progress to understand the most probable structure, theoretical structure of cross-correlation (variety of nonlinear relations) and production rate of competing dissipateion in the system with multiple unstable modes and structural formations. In addition, experimental studies start revealing the coexistence of multiple fluctuations [31, 32]. Mutual nonlinear coupling is analyzed via cross-coherence analysis.

3.3.3 Reynolds stress (additional remarks)

Another combination

$$n\, V_y\, V_z, \tag{21}$$

and resultant force

$$d\langle n\, V_y\, V_z \rangle/dy \quad \text{and} \quad d\langle n\, V_y\, V_z \rangle/dz, \tag{22}$$

are not studied extensively, since they are assumed to disappear upon flux surface average. However, theory predicts the formation of poloidal shock across L-H transition

[53–57]. Improved future experiment may report the importance of the localized nonlinear structures on the magnetic flux surface, such as poloidal shock. In the process to induce the localized nonlinear structure on the flux surface, forces due to inhomogeneity on the flux surface

$$\mathrm{d}\langle n\, V_y\, V_z\rangle/\mathrm{d}y \quad \text{and} \quad \mathrm{d}\langle n\, V_y\, V_z\rangle/\mathrm{d}z,$$

may be at work. Cross-bispectrum can be important to evaluate these. (Theory points out the importance of, not only flux surface averaged transport fluxes, but also the up-down asymmetry of transport fluxes. For instance, several studies report the mechanism of Stringer's spin-up [56,57], excitation of toroidal flows via the up-down asymmetry of fluctuation [58]. Recent experiments also reveal the inhomogeneity of fluctuation [59,60]. A new cross-bispectrum can be applied in experimental validation of these processes.)

3.3.4 Difference between n and ϕ

b5-b13 contain n and ϕ. Since we approximately have $n \sim \phi$ for drift waves,

b5, b6, b7,
b8, b9, b10,
b11, b12, 13,

are assumed to have similar feature. However, when quasi-modes are excited, they do not have to satisfy the Boltzman relation. Excitation mechanism may be experimentally distinguished by using the multi-point data (spatially) and by evaluating the spatial broadening of linearly excited drift wave and that by turbulence spreading.

3.3.5 Zonal flow generated by vector field

Bicoherence of scalar quantity is very useful to verify the nonlinear coupling of drift waves and zonal flows. On the other hand, recent study indicates that zonal flows can be excited in turbulence driven by vector field, such as D'Angelo mode [33, 34], Fig. 8. In order to verify such coupling, the bispectrum of scalar and vector fields, e.g.

b22,

can be useful. Also, if perpendicular flow (externally excited, not excited as a secondary mode) exists, they can drive Kelvin-Helmholtz instability. In these cases,

b20, b21,

are important. There are several future directions.

3.4 Cross-bispectrum of vector quantities
3.4.1 Spatial transport of fluctuation intensity (1)

When fluctuation energy spatially transfer, not only density fluctuation, but also fluctuation kinetic energy transfers.
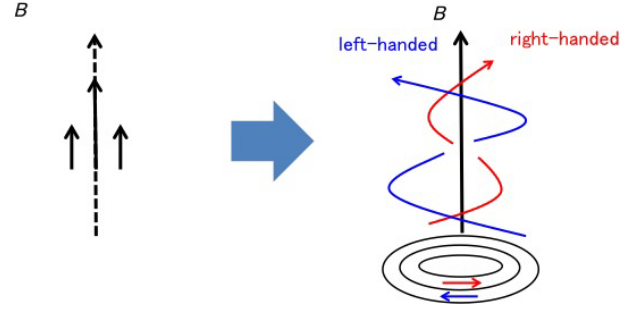

Fig. 8 Development of zonal flows and helical flow pattern by D'Angelo modes [33]. The bicoherence of vector quantity V_z and scalar quantity ϕ, $V_z V_{z\phi}$, can be important for analyzing the process.
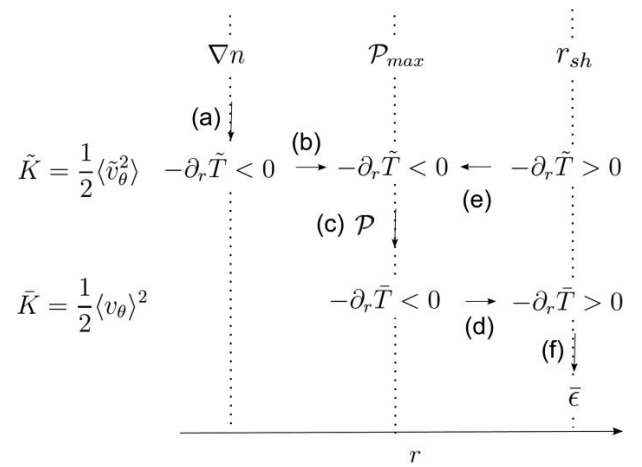

Fig. 9 Energy balance of mean field and turbulent field [61]. The spatial flux of fluctuation energy, $\tilde{T} = \left\langle \tilde{V}_r \tilde{v}_\theta^2 \right\rangle$, can play an important role in fluctuation energy balance.

If we consider the balance as Eq. (15),

$$V_x V_x V_x,$$
$$V_x V_y V_y,$$
$$V_x V_z V_z, \tag{23}$$

describe the spatial transfer in the x direction of fluctuation kinetic energy. Indeed [61], it is pointed out that the spatial flux of fluctuation kinetic energy plays an important role by analyzing the dynamics of the kinetic energy of poloidal flow (Fig. 9).

For drift waves, fluctuation velocities in the x-direction and in the y-direction are comparable. We have

$$V_x V_x \sim V_y V_y. \tag{24}$$

Velocity fluctuation in the z-direction is relatively small,

$$V_x V_x \sim V_y V_y \gg V_z V_z. \tag{25}$$

b28 is less important compared to b26 and b27. Then, spatial transfer of (total) fluctuation energy is evaluated by

b5, b26, b27.

Taking into account the normalization factors, the flux is

$$\left[\left(\frac{n}{n_0} \right)^2 + \left(\frac{V_x}{c_s} \right)^2 + \left(\frac{V_y}{c_s} \right)^2 \right] V_x. \qquad (26)$$

For D'Angelo mode, fluctuation velocity in the z direction cannot be neglected. The flux of fluctuation energy is then

$$\left[\left(\frac{n}{n_0} \right)^2 + \left(\frac{V_x}{c_s} \right)^2 + \left(\frac{V_y}{c_s} \right)^2 + \left(\frac{V_z}{c_s} \right)^2 \right] V_x. \qquad (27)$$

As a consequence of the spatial transfer, even when the region with steep parallel velocity gradient is spatially localized, the region for D'Angelo mode to exist can be broader. Cross-bicoherence analysis can be useful to verify this type of hypothesis.

3.4.2 Spatial transport of fluctuation intensity (2)

We can extend the above discussion to the transfer in the z-direction. For instance,

$$V_z V_x V_x,$$
$$V_z V_y V_y,$$
$$V_z V_z V_z, \qquad (28)$$

describe the spatial transfer of fluctuation kinetic energy along the magnetic field. The process has not been analyzed before. However, if we consider transport in SoL, inhomogeneity along the magnetic field exists, and the structure along the magnetic field needs to be taken into account. The problem of the SoL width becomes increasingly important. Study by auto- and cross-bispectrum is necessary.

In this case, since drift waves typically have

$$V_x > V_z, \qquad (29)$$

spatial transfer is mainly in the radial direction. On the other hand, since D'Angelo modes have

$$V_x \approx V_z, \qquad (30)$$

transfer in the z-direction is important, in addition to that in the x-direction. By comparing the transfer in the radial direction

$$\left[\left(\frac{n}{n_0} \right)^2 + \left(\frac{V_x}{c_s} \right)^2 + \left(\frac{V_y}{c_s} \right)^2 + \left(\frac{V_z}{c_s} \right)^2 \right] V_x, \qquad (31)$$

and the transfer in the z-direction

$$\left[\left(\frac{n}{n_0} \right)^2 + \left(\frac{V_x}{c_s} \right)^2 + \left(\frac{V_y}{c_s} \right)^2 + \left(\frac{V_z}{c_s} \right)^2 \right] V_z, \qquad (32)$$

D'Angelo mode may be identified. It is important to identify relevant mode to understand transport characteristics. Further consideration may be worthwhile.

3.4.3 Others

$$b31 \quad Vx \quad Vy \quad Vz,$$

describe, e.g., spatial transfer of poloidal Reynolds stress

in the magnetic field direction. As discussed above, this may not be neglected in the system with inhomogeneity along the magnetic field.

4. Summary

In this note, we have explained the situation that the bicoherence analysis has become a standard method and that the broader class of problems can be analyzed by the bicoherence analysis in the forefront of experimental data analysis with better understanding of statistical convergence. In recent studies, not only scalar quantities such as density and potential, but also vector quantities such as magnetic fields and velocity are measured, and the nonlinear interaction between the scalar and vector quantities attracts attention. With the situation in mind, we gave a bird's eye view on the studies pursued by analyzing cross-bispectrum and cross-bicoherence. Cross-bispectrum and cross-bicoherence are important and these methods should be utilized.

In the research of mupltiple physical quantities, it is important to have perspective on which problem we are trying to clarify and on what type of advanced statistical analysis is required.

Acknowledgments

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