Distortion of Fast α-Particle Two-Dimensional Velocity Distribution Function due to the Transition of Particle Orbit by Nuclear Elastic Scattering in Magnetic Field Confinement

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The effect of nuclear elastic scattering (NES) on energetic alpha-particles two-dimensional (2D) velocity distribution function in a magnetic fusion device is examined by particle orbit simulation. Energetic alpha-particles exhibit an anisotropic 2D velocity distribution function, because the alpha particle loss reduces the amount of particles moving in a particular direction, and an alpha particle orbit depends on the initial direction of particle. When NES changes the direction of fast alpha particle motion, its orbit can be changed. It is shown that NES causes particle orbit transitions and changes the energetic alpha particles 2D velocity distribution function. We discuss the manner in which the distribution function is changed and how it affects the confinement of fast alpha particles.

Keywords: nuclear elastic scattering, particle orbit, alpha particle, 2D velocity distribution function, ITER

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1. Introduction

In the studies of magnetic-confined fusion plasma, it is important to understand fast-ion behavior from the viewpoint of plasma heating and heat load on the first wall [1]. It is well known that Coulomb and non-Coulombic, i.e., nuclear elastic scattering (NES) [2] contributes to the slowing down of fast ions. NES accelerates the slowing down of fast ions and is caused by a combination of the nuclear force and nuclear Coulomb interference when ions are sufficiently energetic to be close. Neglecting the NES effect leads to an overestimation of the fast ion slowing-down time [3]. While the influence on the fast-ion distribution function, i.e., the decrease in the energetic component in the slowing-down distribution function, has been evaluated previously [4,5]. However, in these analyses, uniform plasmas (i.e., the particle orbit in the magnetic field configuration was not considered) was assumed.

For Coulomb scattering, it is well known that collisions change the particle orbit pattern and affect the particles confinement properties [6]. The Coulomb scattering process is characterized by many small-energy-transfer events. In contrast, NES is a large-angle scattering process, and a large fraction of the fast-ion energy is transferred in a single event. While Coulomb scattering changes the particle orbit pattern restrictively, NES can change the particle orbit pattern more broadly than Coulomb scattering because of the large-angle scattering. Assuming a magnetic mirror device, Kantrowitz and Conn noted that particle loss is increased by NES [7]. In these analyses, the particle orbit was not considered. This is because only the ratio of parallel to vertical velocity component is necessary to evaluate the lost particles in a mirror device. However, in a toroidal device, it is important to consider the particle orbit for the accurate evaluation of the effect of NES on particles' behavior. In this paper, we discuss particle behavior based on the particle velocity, direction, and position, resulting in a 2D velocity distribution function considering velocity and space.

Because particle orbit simulation can be used to calculate orbit transition, particle orbit analysis is useful for understanding the fast-ion behavior in thermonuclear plasmas [8]. Stringer incorporated the Coulomb scattering effect into the particle orbit analysis and indicated that the confinement of bulk ions is stabilized by the scattering [9]. In a previous study, we incorporated the NES effect into the particle orbit analysis, finding that alpha particle loss is reduced [10]. We found that the particle orbit transition was caused by NES, and explained that the reason why the alpha particle loss was reduced..

As a result of the particle orbit transition, the 2D velocity distribution function may also be changed. The purpose of this study is to reveal the effect of nuclear elastic scattering on the fast alpha-particle 2D velocity distribution function caused by the transition by using particle orbit analysis and investigate the effect of the distortion of the 2D velocity distribution function on the plasma.

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2. Analyses Model

The NES effect was incorporated into the ORBIT [11] charged-particle orbit analysis code; this guiding center orbit code was developed at the PPPL.

The probability $p(v)$ that a test particle moving with velocity v undergoes NES during a small time step Δt is

$$p(v) = n_b \sigma_{NES} \Delta t, \tag{1}$$

where n_b is the number density of the background ions, and σ_{NES} is the NES cross section. As the first step, the effect of the thermal motion of the target particles, i.e., deuteron and triton, was neglected. This is because the NES cross section is small in the low energy region (< 1 MeV). Scattering in the center-of-mass system was assumed to be isotropic. The transferred energy in a single scattering event ΔE can be written as

$$\Delta E = \frac{E}{2}(1-\alpha)(1-\cos\phi), \tag{2}$$

E is the kinetic energy of the test particle before scattering and ϕ is the scattering angle in the center-of-mass system. Here, $\alpha = \{(m_t - m_b)/(m_t + m_b)\}^2$, and m_t and m_b are the masses of the test particle and background ion, respectively. In this paper, the NES cross section values are taken from the study by Perkins and Cullen [12].

We assumed an ITER-like plasma and chose the 3.52 MeV alpha-particle as a test particle. Toroidal magnetic field $B_T = 5.3$ T, the radial profile of bulk ions and electrons densities $n_i = n_e = 1.0 \times 10^{14} \times (1 - \psi_{pol,n}^2)^{1.5}$, and the radial profiles of bulk ions and electron temperature $T_i = T_e = 28.0 \times (0.9 \times (1 - \psi_{pol,n}^2) + 0.1)$ are assumed, where $\psi_{pol,n}$ represents the normalized poloidal magnetic flux function ψ_{pol}. Radial profiles of the safety factor and current density used in the present simulations are taken from Ref. [13] and shown in Fig. 1. Magnetic flux surface is shown in Fig. 2. Throughout the calculations, 210,000 test particles and 0.6 sec are assumed. The time correspond to 100000 toroidal transit period of 3.52 MeV alpha-particle (ignored collisions) generated at center of plasma and in magnetic field line direction in condition of this paper.

The 2D velocity distribution function f_{2D} satisfies the following relational expressions.

$$S\,[\text{count}] = \iiint_0^\infty \int_{-\infty}^\infty 2\pi v_\perp f_{2D}(v_\parallel, v_\perp)\, dv_\parallel dv_\perp dV, \tag{3}$$

$$N[\text{m}^{-3}] = \int_0^\infty \int_{-\infty}^\infty 2\pi v_\perp f_{2D}(v_\parallel, v_\perp)\, dv_\parallel dv_\perp, \tag{4}$$

where S and N are the number of alpha particles and the density of alpha particles in plasmas, respectively, V is the volume of plasma, and $v_{//}$ and $v_\perp$ represent the parallel and vertical components of the particle velocity relative to the magnetic axis, respectively.

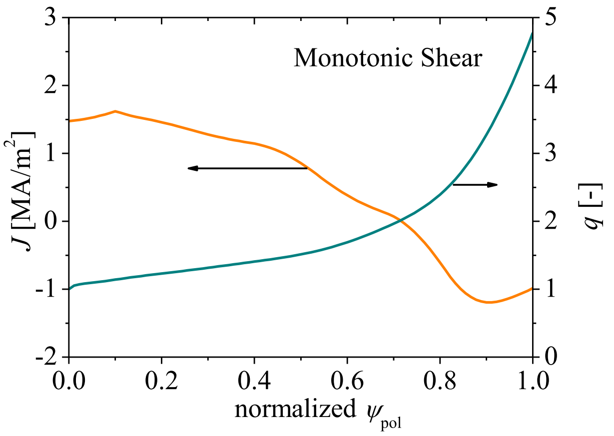

Fig. 1 Radial profiles of safety factor and current density assumed in the present calculations [12].

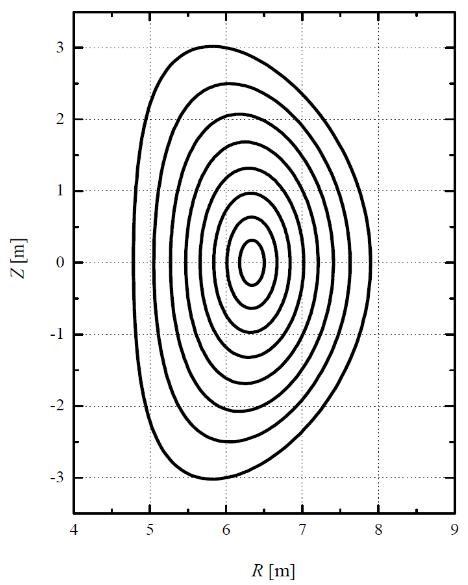

Fig. 2 Magnetic flux surface.

3. Result and Discussion

3.1 NES frequency and effect on particle pitch angle

Figure 3 shows the NES probability as a function of ΔE in $\psi_0 < 0.5$ and $\psi_0 > 0.5$ regions, where ψ_0 is the normalized poloidal magnetic flux function when the particle passes mid-plane at last or second last with lager R. R is coordinate value in major radius direction. All particles pass through the mid-plane twice during the round in the poloidal direction. ψ with larger R is important when we discuss particle confinement. ΔE is the energy transferred in a single scattering event by NES, and $P(\Delta E)$ is the NES fraction for all NES events at each condition, where $\int_0^\infty P(\Delta E) d\Delta E$ = (Number of NES)/(Number of alpha particles in the energy region) is required. The calculation conditions are described in previous section i.e., 2. Analyses Model. The $P(\Delta E)$ of low ΔE is larger than that for high ΔE, because NES with high ΔE occurs only for the

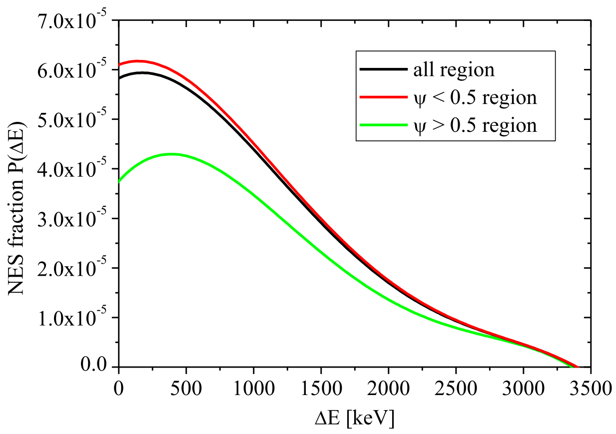

Fig. 3 NES fractions as a function of ΔE in $\psi_0 < 0.5$ and $\psi_0 > 0.5$ regions.

alpha particles with energy higher than ΔE. NES with low ΔE can occur for the alpha particles after slowing down; in contrast the NES with high ΔE occurs only for alpha particles before slowing down. However, NES cross-section of high energy particles is larger than that of low energy particles, and the scattering angle is also large for large ΔE. Therefore, NES with high ΔE is important. Alpha particle losing more than 1 MeV of energy by NES is 42% to all alpha particles occurring NES. NES occurs to about 9% of the alpha particles during their slowing down process to thermal energy. Therefore about 3.8% of alpha particles loss more than 1 MeV of energy by single NES. Typically, energy loss of energetic alpha particles is 1 to 2% of generated energy of alpha particles. Accordingly NES can affect the alpha particle distribution function or confinement. For the $\psi_0 < 0.5$ region, the NES fraction is the largest of the entire studied range. NES probability depends on the bulk ions densities in Eq. (1); the bulk ions densities in the smaller ψ_0 region are larger than those in the lager ψ_0 region in this calculation. In a similar fashion, the NES fraction in the $\psi_0 > 0.5$ region is smaller than that in the entire region. However, the NES in the larger ψ_0 region should not be ignored, because the alpha particles in the lager ψ_0 region are leaky from the plasmas. The reason for this is discussed in detail in the following section.

3.2 Effect of NES on 2D energetic alpha-particles velocity distribution function

Figure 4 shows the 2D velocity distribution function of alpha particles after 0.6 sec (100000 toroidal transit periods) with and without considering NES. Here, v_α represents the velocity of 3.52 MeV alpha particles, so that $v_\parallel/v_\alpha$ and $v_\perp/v_\alpha$ are the normalized parallel and vertical components of the particle velocity. The other calculation conditions are the same as those in Fig. 3. We can see that the alpha particle energy peak is about $v/v_\alpha = 0.6$ and energy range is $v/v_\alpha = 0.6 \pm 0.2$ when NES is ignored. Alpha particle energy converge in a certain range in this calculation because Coulomb scattering is a small angle scattering process and have large cross-section. In contrast, we can see low velocity alpha particles, i.e., $v/v_\alpha < 0.25$, when NES is considered. NES can change fast alpha particles energy to nearly bulk energy in one scattering event. This possibility is shown in Fig. 3, where the distribution functions indicate that NES accelerates the slowing-down of the energetic alpha particles. This effect has already been reported previously as the enhancement of energy deposition to bulk ions by the NES, e.g., in [5–7]. Large changes of alpha particles' energies due to NES are accompanied by large changes in the pitch angle of the alpha particles. Such a large change of the pitch angle can affect the alpha particles' orbits. The effect on the alpha particles orbit is hard to see in Fig. 4, because these distributions are presented over the entire studied range. Therefore, individual regions must be examined separately to discuss the effect of the NES on the particle orbit.

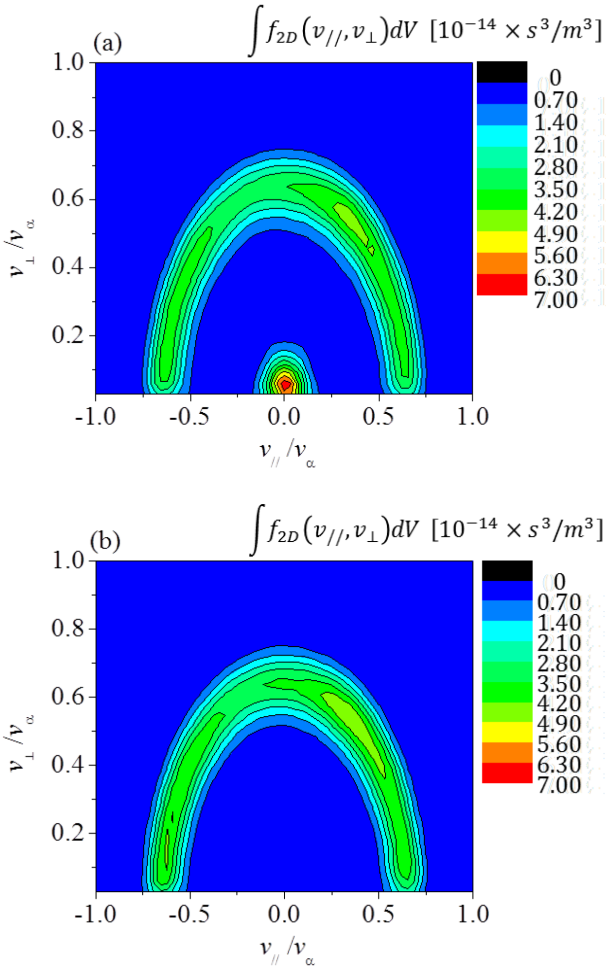

Fig. 4 2D alpha particles distribution function when NES is (a) considered, (b) ignored.

An alpha particle orbit can be divided into two patterns, i.e., trapped and untrapped orbits. The projections of these orbits on the poloidal plane are shown in Fig. 5. Alpha-particles moving on the trapped orbit satisfy the fol-

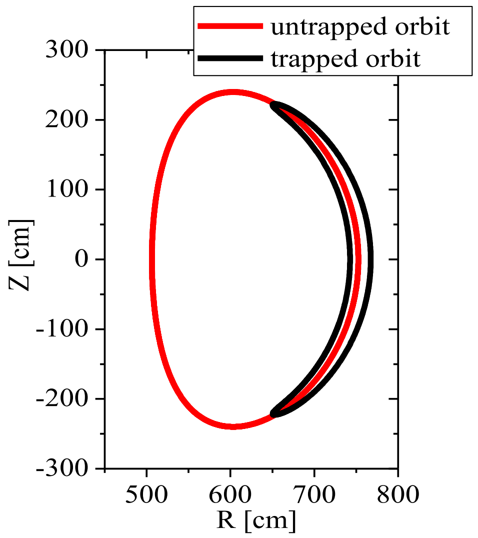

Fig. 5 An example of trapped and untrapped orbit projected on poloidal plane.

lowing requirement [8]:

$$\frac{v_{//0}}{v_{\perp 0}} = \frac{1}{|\tan\theta_0|} < \left(2\frac{r_0}{R_m}\right)^{1/2}, \quad (5)$$

where θ_0 and r_0 are the pitch angle and the distance from the plasma center at the minimum-magnetic-field point, respectively, and R_m is the major radius. This requirement means that the alpha particles in the lager r_0 region or with a small $v_{\|0}/v_{\perp 0}$ are easily trapped. The following paragraph, we discuss these effects with Figs. 6 and 7.

Figure 6 shows a 2D velocity distribution function of untrapped and trapped alpha particles. The other calculation conditions are the same as those in Fig. 3. The 2D distribution function of trapped particles exhibits 2 peaks at $v_\perp/v_\alpha = 0$. One peak is at $v_\|/v_\alpha = 0$, another is at $v_\|/v_\alpha = 0.7$. In contrast, the 2D distribution function of untrapped particles hardly has any elements at near $v_\|/v_\alpha = 0$ and $v_\|/v_\alpha = 0.7$. These results show that distribution functions follow inequality (5). When NES is ignored, the tendencies of the 2D distribution function of trapped and untrapped particles are similar to that NES is considered except for existing $v/v_\alpha < 0.25$ component.

Next, the number of trapped or untrapped particles and trapped orbit fractions are shown in Fig. 7. The numbers of trapped or untrapped particles are normalized by total number of particles. The trapped orbit fraction is defined as (the number of trapped particles at the ψ_0)/(the number of trapped and untrapped particles at the ψ_0). In the lager ψ_0 region, few numbers of trapped and untrapped particles are present as compared with that in the smaller ψ_0 region, because the alpha particle generation rate depends on the temperature and the density of bulk ions. Temperature and density decrease to small values in the lager ψ_0 region and we observe an increased number of trapped particles in the lager ψ_0 region. The trapped orbit rate is about

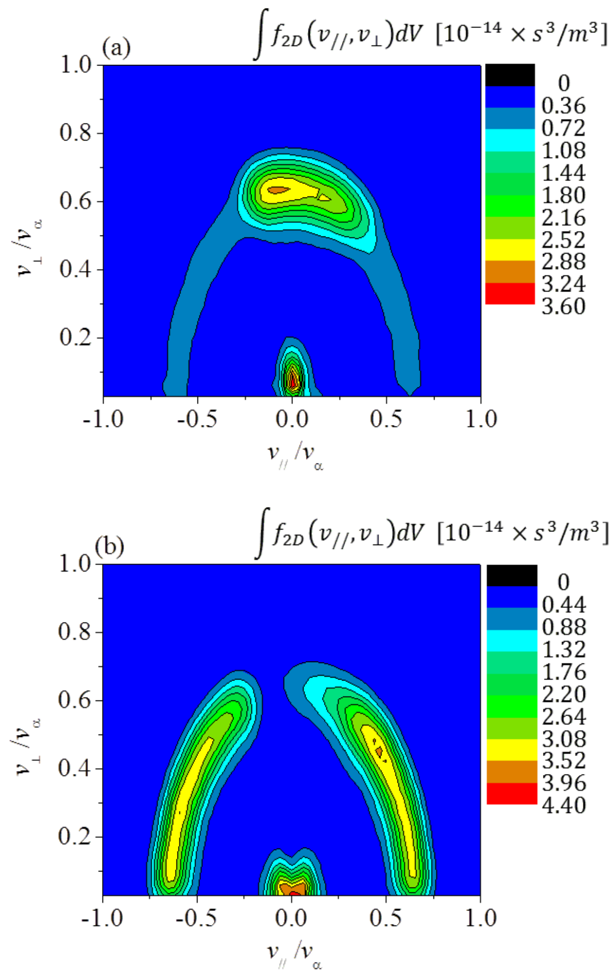

Fig. 6 2D (a) trapped, (b) untrapped alpha particles distribution function when NES is considered.

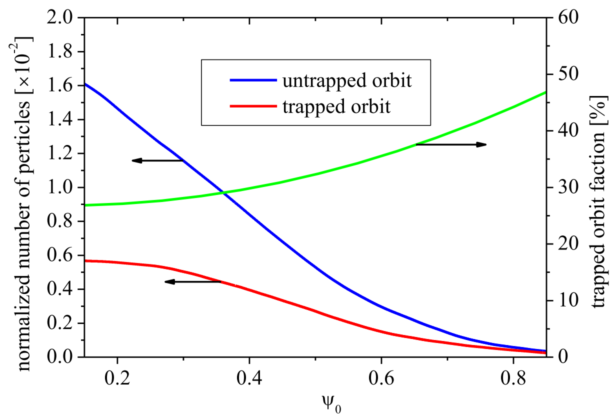

Fig. 7 Normalized number of trapped and untrapped orbit particles as a function of ψ_0 and proportion of trapped orbit particle.

28% at $\psi_0 = 0.15$, while the trapped orbit fraction is about 47% at $\psi_0 = 0.85$. This is because r_0 is large in the larger ψ_0 region so that it is easy to satisfy inequality (5). In addition, the trapped orbit swells outward as shown in Fig. 5.

Next, in Fig. 8, we show the 2D energetic alpha-

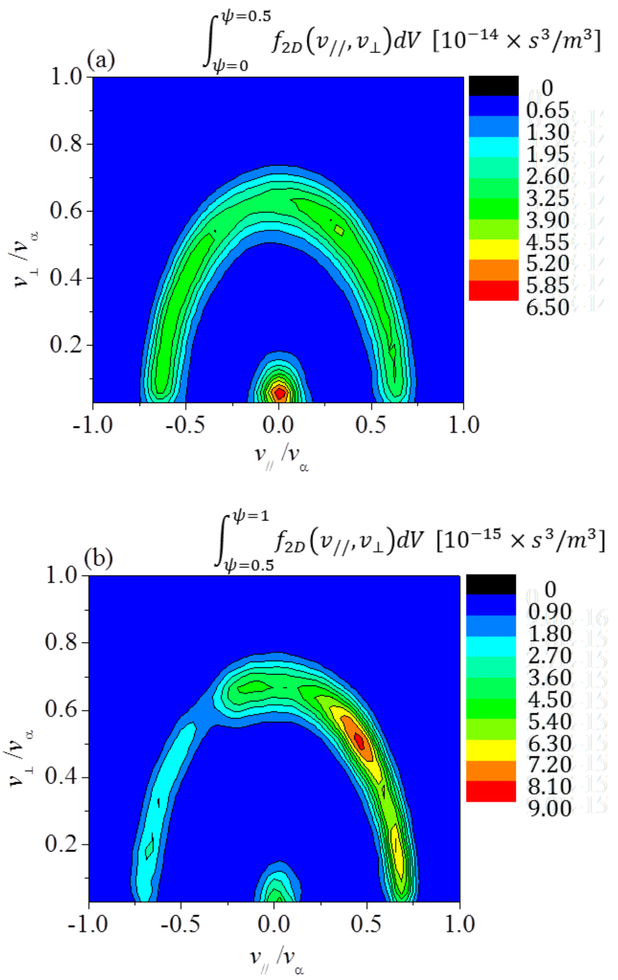

Fig. 8 2D alpha particles distribution function in the region of (a) $\psi_0 < 0.5$, (b) $\psi_0 > 0.5$ when NES is considered.

particles velocity distribution function in $\psi_0 < 0.5$ and $\psi_0 > 0.5$ region when NES is considered. The other calculation conditions are the same as those for Fig. 3. The distribution function in $\psi_0 < 0.5$ region is similar to the results for the entire range in Fig. 4, because alpha particles in $\psi_0 < 0.5$ region comprise a majority of the particles in plasmas. Temperature and density of bulk ions are large in the smaller ψ_0 region, and the generation rates of alpha particles are large for large temperature and density. In contrast, the component of $v_{\|0} < 0$ is obviously small in the $\psi_0 > 0.5$ region. The particle trajectory with $v_{\|0} < 0$ is shifted to the inner (high field) region by curvature and ∇B drift. On the whole, the fraction of particles with $v_{\|0} < 0$ is decreased in the $\psi_0 > 0.5$ region. Considering NES leads to a broadening of the peak near $v_{\|0} = 0$ of the distribution function in the $\psi_0 > 0.5$ region. When NES is ignored, the tendency of the 2D distribution function in $\psi_0 < 0.5$ region is similar to that NES is considered. In contrast, in $\psi_0 > 0.5$ region, NES change the tendency of the 2D distribution function. However, this change is difficult to discuss using the 2D distribution function because this change is not so large. Therefore we discuss this change by NES

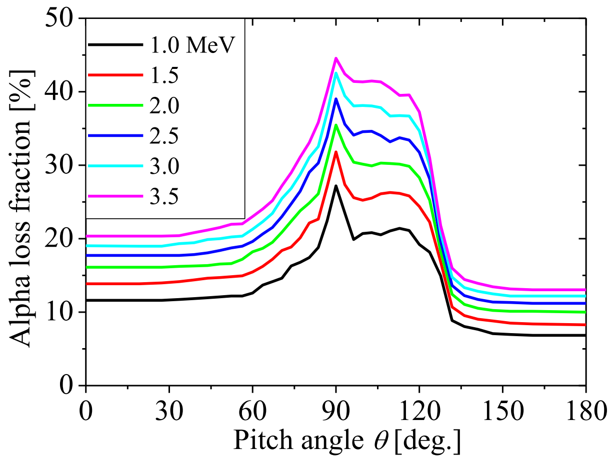

Fig. 9 Fraction of the number of lost particles as a function of pitch angle for each energy obtained using a uniform generation function.

and its effect to plasma confinement using pitch angle distribution function in next section.

3.3 Effect of NES on alpha particle energy loss

The fraction of the particles lost from the plasma is plotted as a function of the pitch angle for each energy in Fig. 9. Alpha loss fraction is defined as (number of lost particles generated at the pitch angle)/(number of particles generated at the pitch angle). In this calculation, the generation distribution of the alpha particles is made uniform and independent of poloidal magnetic flux, poloidal angle, pitch angle. Energy of alpha particles is assumed monoenergetic i.e., 1 MeV, 1.5 MeV, 2 MeV, 2.5 MeV, 3 MeV and 3.5 MeV. The calculation time is taken as 6.0×10^{-4} sec (100 toroidal transit periods). The other calculation conditions are the same as those in Fig. 3. Most of the alpha particles are lost at a pitch angle $\theta = 90°$, because alpha particles are most trapped by toroidal field ripples at $\theta = 90°$ [14]. In the $\theta > 130°$ region, the loss fraction is greatly reduced. The ions are the untrapped orbit particles in the $\theta > 130°$ region, and the ions move toward opposite directions to the magnetic field in $\theta > 90°$ region. The untrapped orbit particles moving toward opposite directions tend to be shifted the inner region in the poloidal plane by curvature and ∇B drift, therefor the untrapped orbit particles in the $\theta > 130°$ region is reduced on the whole. The loss fraction in the $90° < \theta < 120°$ region is larger than that in the $60° < \theta < 90°$ region, because the ions that are bounced back by the magnetic mirror move in the opposite direction to their previous movement direction. After bouncing back, the ions previously moving toward the opposite direction now move in the same direction and the orbit tends to shift outside the region in the poloidal plane.

In the lager ψ_0 region, many alpha particles are trapped and lost in the poloidal plane. For example, the

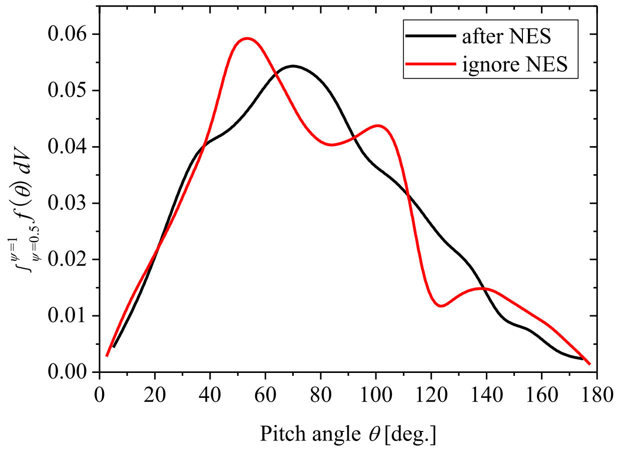

Fig. 10 Alpha particles pitch angle distribution function when NES is considered and when it is ignored.

fraction of alpha particles lost from the trapped orbit and the region of the poloidal plane $\psi_0 > 0.5$ with the exception of first orbit (FO) loss is evaluated as over 90%. The FO loss depends on the position and velocity when alpha particles are generated; therefore, FO loss is hardly affected by collisions and is ignored in this discussion. Figure 10 shows the pitch angle distribution function of alpha particles $f(\theta)$ in the $\psi_0 > 0.5$ region with and without considering NES. Calculation conditions are the same as those in Fig. 7. In the $100° < \theta < 110°$ region, alpha particle distribution is 4% smaller after the occurrence of NES. The decrease in the distribution probability in this region reduces the loss fraction of energetic alpha particles, because loss fraction in this region is large, as seen in Fig. 9. The center of ions moving toward the opposite direction to the magnetic field, i.e., $\theta > 90°$, on untrapped orbit tend to be shifted to the inner region, i.e., $\psi_0 < 0.5$, by curvature and ∇B drift. Because the NES probability does not depend on pitch angle, in the region of $\psi_0 > 0.5$, the number of ions moving in the region of pitch angle where particles can be easily lost tends to be decreased by the NES. On the other hands, the number of alpha particles in the $60° < \theta < 90°$ region increases after NES. However, alpha particles in region $\theta < 80°$ are suddenly not leaky in Fig. 9. Therefore, this increase is largely ineffective. The energy loss of energetic alpha particles from the plasma depends on alpha particle loss fraction and energy of lost alpha particles. In this section, we have discussed that NES reduce alpha particles loss fraction. We described NES accelerate slowing-down of energetic alpha particles in the previous section, i.e., 3.1 and 3.2, therefore NES also reduce energy of lost alpha particles. For example, the energy loss of energetic alpha particles from the plasma reduced by about 6.3% when NES is considered in the same condition as Fig. 4.

4. Concluding Remarks

We calculate 2D alpha particles distribution functions considering NES using particle orbit analysis. For trapped particles or particles in the $\psi_0 > 0.5$ region, 2D velocity distributions functions of alpha particle show strong anisotropy. It has been shown that NES accelerates the slowing-down of energetic alpha particles and changes the distribution functions to isotropic. In particular, NES tend to changes the alpha particles pitch angle distribution in the $\psi_0 > 0.5$ region to isotropic; the alpha particles energy loss is also decreased by 6.3%.

We assumed an ITER-like plasma in this study. Changes in the magnetic confinement may lead to changes in the particle orbit and distribution function. In particular, devices other than the Tokamak device exhibit markedly different orbits of charged particles. Therefore, distribution function and NES effect can be changed, for example, when a helical device is assumed.

While we ignored the effect of thermal motion of bulk ions when we calculated the NES effect with the orbit code, NES actually affects the thermal motion of bulk ions. If the bulk ions distribution function is anisotropic, alpha particle generation distribution function becomes anisotropic. The alpha particle distribution function and confinement may be described by anisotropic generation distribution functions. However, the energies of bulk ions are sufficiently smaller than those of the particles undergoing NES. Therefore, it is most likely unnecessary to consider the thermal motion of bulk ions.

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