**Flux of Parallel Flow Momentum by Parallel Shear Flow Driven Instability**

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The flux of parallel momentum by parallel shear flow driven instability is calculated with the self-consistent mode dispersion. The result indicates that the diffusive component has two characteristic terms: \(v_{D1} \sim e^2/v\gamma_0\) and \(v_{D2} \sim e^2/(k^2 D\parallel)\) where \(e\) is the fluctuation radial velocity, \(\gamma_0\) is the growth rate of the mode, \(k\parallel\) is the parallel wave number, and \(D\parallel\) is the electron diffusivity along the magnetic field. \(v_{D1}\) results when the parallel flow shear is above the threshold, while \(v_{D2}\) is important around the marginal state. Since typically \(v_{D1} \gg v_{D2} \sim D\parallel\), where \(D_n\) is the particle diffusivity, the Prandtl number \((\equiv \nu/D\parallel)\) becomes large when parallel flow shear driven instability occurs. This feature may explain the experimental observation on the difference between profiles of density and toroidal flow in edge and SOL plasmas.

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Flow along the magnetic field (parallel flow) is an important element to understand the behavior of magnetized plasmas. While parallel flows have beneficial effect for fusion plasmas by controlling transport, parallel flow shear itself can drive instability and be a source for fluctuation. The parallel flow shear driven instability was predicted by D’Agnelo\(^{1}\) and the cases of NBI plasmas and SOL plasmas were analyzed\(^{2-4}\). It was shown that the Prandtl number is of order of unity in the case of drift waves\(^{5}\). More recently, the impact of parallel flow driven instability on particle transport and parallel momentum transport is reported from basic experiment\(^{6}\), and analysis has been performed\(^{7}\). However, while the earlier studies reveal relevant contribution in transport fluxes and production rate, an approximate value for the frequency \(\omega \sim \omega_c\), where \(\omega_c\) is the drift wave frequency, was used. Though this is true for an order of magnitude estimate, the fluctuation of interest has a dispersion relation \(\omega = \omega_k\), and this has to be taken into account for more consistent analysis. Moreover, measurements of toroidal plasma flow near edge reported the difference between profiles of density and toroidal flow\(^{8}\). This motivates the theoretical evaluation of turbulent Prandtl number.

In this work, we present an analysis of transport of parallel flows by parallel flow shear driven instability, with the dispersion relation of the mode treated consistently. In order to address this, we use a fluid model\(^{7}\) with the evolution of vorticity, density, and parallel flows:

\[
\frac{d}{dt} \frac{\epsilon_{\text{\parallel}}}{T_e} \frac{\nabla^2 \frac{e\phi}{T_e}}{\epsilon_{\text{\parallel}}} = -D\parallel \nabla^2 \left(\frac{e\phi}{T_e} - \frac{e\phi_{\text{\parallel}}}{n_0}\right),
\]

(1)

\[
\frac{d}{dt} \frac{n_e}{n_0} = -D\parallel \nabla^2 \left(\frac{e\phi}{T_e} - \frac{e\phi_{\text{\parallel}}}{n_0}\right) - \nabla \psi_{\parallel},
\]

(2)

\[
\frac{d}{dt} \psi_{\parallel} = -c_s^2 \nabla^2 \left(\frac{e\phi_{\text{\parallel}}}{n_0}\right).
\]

(3)

Here \(d/dt = \partial_t + (c/B) \times \nabla \psi_{\parallel}\), \(\psi_{\parallel}\) is the electrostatic potential, \(n_i\) is the ion sound Larmor radius, \(D\parallel = e^2/\nu_{\parallel}\) is the parallel diffusivity of electrons, \(\nu_{\parallel}\) is the electron collision frequency, \(n_e\) is the electron density, \(n_0\) is a reference density, \(\nu_{\parallel}\) is the ion fluid velocity, \(c_s\) is the ion sound speed.

Linearization of the model equation yields the dispersion relation as:

\[
\frac{e^2 k^2_{\text{\parallel}}}{ik^2 D\parallel} \omega = -(\omega - \omega_{\text{ce}})\omega + \left(c_s^2 k^2_{\text{\parallel}} - c_i^2 k^2_p - c_s^2 k^2_n\right) \frac{(\omega + ik^2 D\parallel)\omega - c^2_s k^2_{\text{\parallel}}}{(\omega + ik^2 D\parallel)\omega - c^2_s k^2_{\text{\parallel}}},
\]

(4)

Here \(\omega_{\text{ce}} = k_y (\rho_{\perp}/L_n) c_s\), \(k^2_{\text{\parallel}} = k^2_{\perp} + k^2_{\parallel}\), and \(L_n^{-1} = -\langle n'\rangle/n_0\) is the density scale length. Equation (4) is solved perturbatively using the inverse of the adiabaticity parameter \(\omega_{\text{ce}}/(k^2 D\parallel) \ll 1\). For parallel flow shear driven mode, the dispersion relation is:

\[
\omega_{\text{ce}}(0) = \frac{\omega_{\text{ce}}}{2(1 + k^2_{\text{\parallel}}/k^2_p)},
\]

(5)

\[
\gamma_{(0)} = sgn(k_y) \frac{\omega_{\text{ce}}}{2(1 + k^2_{\text{\parallel}}/k^2_p)} \sqrt{D\parallel},
\]

(6)

\[
D\parallel = 4(1 + k^2_{\text{\parallel}}) \left\{ k_{L_n} \left( \frac{c_{\perp}}{L_n} - \frac{k^2_{\parallel} L^2_{\parallel}}{k^2_{\text{\parallel}} + k^2_{\perp}} \right)^{-1}\right\} - 1.
\]

(7)
Basic feature of the instability is obtained from the zeroth order growth rate and is reported in literature [7, 9]. Here note that the onset of the instability requires parallel flow shear needs to be large enough to make $D_e > 0$. We also note that the parallel wave number should not be too large to avoid the stabilizing effect due to acoustic wave coupling. The adiabaticity condition $\omega_c/(k_D^2 D_n) \ll 1$ is guaranteed with the large parallel electron diffusivity. The next order correction is calculated as

$$
\delta \omega_r = \text{sgn}(k_y) \frac{\omega^2_e}{k_D^2 D_n} \left\{ \frac{1}{1 + k_y^2 \rho_s^2} - \frac{3 k_y L_n}{k_D \rho_s c_s / L_n} \right\}.
$$

The adiabaticity condition holds, we have

$$
\delta \gamma = \frac{\omega^2_e}{k_D^2 D_n} \left\{ \frac{1}{1 + k_y^2 \rho_s^2} \right\}.
$$

The flux of parallel momentum is calculated. General expression is obtained by using the quasilinear theory [10]

$$
\frac{\Pi^f}{c^2} = \text{Re} \sum_k \frac{c_k y}{(\omega + \gamma \omega_c) c_k k_y^2} \left| \frac{e \phi}{T_e} \right|^2 \left( \frac{k_y^2}{k_D^2} \right) \frac{1}{k_y^2 \rho_s^2} \left( \frac{k_y^2 L_n}{c_s / L_n} \right) - \frac{1}{k_y^2 \rho_s^2} \left( \frac{k_y^2 L_n}{c_s / L_n} \right).
$$

Using the dispersion relations Eqs. (5), (6), (8) and (9), the momentum flux reduces to

$$
\frac{\Pi^f}{c^2} = - \sum_k \frac{k_y \rho_s c_k y}{c_k k_y} \left( \gamma \omega_c (1 + k_y^2 \rho_s^2) \right) \left| \frac{e \phi}{T_e} \right|^2 \left( \frac{k_y^2}{k_D^2} \right) \frac{1}{k_y^2 \rho_s^2} \left( \frac{k_y^2 L_n}{c_s / L_n} \right) - \frac{1}{k_y^2 \rho_s^2} \left( \frac{k_y^2 L_n}{c_s / L_n} \right)\left( \frac{k_y^2 \rho_s^2}{k_D^2} \right) \frac{1}{k_y^2 \rho_s^2} \left( \frac{k_y^2 L_n}{c_s / L_n} \right).
$$

Note that in the adiabatic limit, only the first term remains finite. When parallel flow shear exceeds the critical value, this term is approximately given as

$$
- \sum_k \frac{k_y \rho_s c_k y}{c_k k_y} \left( \gamma \omega_c (1 + k_y^2 \rho_s^2) \right) \left| \frac{e \phi}{T_e} \right|^2 \left( \frac{k_y^2}{k_D^2} \right) \frac{1}{k_y^2 \rho_s^2} \left( \frac{k_y^2 L_n}{c_s / L_n} \right).
$$

This is diagonal, diffusive flux $-v_{D1}\langle \gamma \omega_c \rangle'$ with the viscosity given by

$$
\nu_{D1} \sim \frac{\tilde{\nu}^2}{\gamma(0)}.
$$

The second term in the momentum flux is also related to the diffusive component, with the viscosity

$$
\nu_{D2} \sim \frac{\tilde{\nu}^2}{k_y^2 D_n}.
$$

There may be a competing effect from the residual stress, the last term in the momentum flux.

The viscosity induced by the parallel flow shear driven instability has two characteristic values, $\nu_{D1}$ and $\nu_{D2}$. The difference in the viscosity may be used to apply parallel flow shear driven instability to experiment. Since typically $k_D^2 D_n \gg \omega_c - \gamma$ holds, we have $\nu_{D1} \gg \nu_{D2}$.

In summary, we presented an analysis of the flux of parallel momentum driven by parallel flow shear driven instability, with the mode dispersion treated consistently. The results indicate that the viscosity on the flow has two typical values. Above the critical velocity shear, the viscosity is larger than the particle diffusivity, so the Prandtl number is larger than one. On the other hand, if the shear is close to the marginal, the Prandtl number is $\sim O(1)$. These features may be important to understand the difference in the profile structures of toroidal flow and density reported from experiments.

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