A Parameter that Denotes Non-Equilibrium Property for Turbulent Plasmas*)

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One of the nonequilibrium features of turbulent plasmas is the mixing of the timescales of macroscopic and microscopic dynamics. We here study the direct influence of heating on the turbulence and turbulent transport, by paying attention to the coupling between source and fluctuations in the phase space dynamics. This coupling causes the immediate influence of external source (like heating) on the turbulence and turbulent transport, and thus introduces the mixing of time scales. A control parameter is introduced to denote the distance from thermodynamical equilibrium.

Keywords: nonequilibrium plasma, turbulence, turbulent transport, phase space

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1. Introduction

Plasmas in nature and laboratories are often far from thermal equilibrium. In employing the terminology 'far', one naively assumes that the 'distance' from thermal equilibrium may be definable. The distance from thermal equilibrium, if it is quantified, is one of the essential parameters that specify the turbulent plasmas.

In the history of study of turbulence, the parameters such as Reynolds number or Rayleigh number have played the central role in describing turbulent viscosity and turbulent heat flux, respectively. These parameters are determined by the competition between the production via spatial inhomogeneity and damping due to microscopic processes, and have provided universal descriptions of turbulence. Similar argument holds for plasmas, and an example of distance was discussed [1], in order to describe the non-equilibrium properties [2, 3]. In plasmas, additional degrees of freedom in dynamics, i.e., the dynamics in velocity space, must also be taken into account.

The recent result on LHD clearly has shown that, under the experimental condition of periodic modulation of heating power, the radial heat flux changes immediately (within experimental time resolution) when the heating power changes in time [4–8]. The change of flux is much faster than those of the mean plasma parameters. The heat flux cannot be expressed in terms of a unique relation of the global plasma parameters, and a hysteresis appears in the gradient-flux relation. The spatial inhomogeneity alone is insufficient to specify the turbulence and turbulent transport. Stimulated by these striking observations, a new thermodynamic force, defined in the parameters of velocity space, was introduced, and its relation to the inhomogeneities in real space was also discussed [9]. This new idea has given an explanation of transport hysteresis [4–8] as was discussed in [10, 11].

The nonequilibrium feature of matter, which is far from thermal equilibrium, is also described by the mixing of the timescales of macroscopic and microscopic dynamics [12]. It is usually assumed that the micro- and macro-timescales are discriminated. The cross-scale dynamics in multi-scale turbulence leads the mixing of timescales [9, 10]. Here we discuss the timescale mixing, which is introduced by the new thermodynamic force in the plasma turbulence. The control parameters are discussed, and are compared to the standard parameter that denotes the deviation of Maxwell distribution. Putting an emphasis on this timescale mixing, we discuss the distance from thermodynamic equilibrium.

2. Model

The kinetic equation in the presence of the source in the phase space is written as

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla+\frac{e_{\rm s}}{m_{\rm s}}(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})\cdot\nabla_v\right)f(\boldsymbol{x},\boldsymbol{v}:t)=S+C, \quad (1)$$

where $\boldsymbol{x}$ and $\boldsymbol{v}$ denote the spatial and velocity coordinates, S is the source in the phase space, C is the collision operator, and suffix s is the particle species. The functional form of source term S, $S[f;\boldsymbol{v},x,t]$, is treated as prescribed in this analysis. The distribution function is separated into mean and perturbation, as $f = f_0 + \tilde{f}$, where the symbol ∼

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indicates fluctuation part. Owing to the fluctuations in f, $S[f; \boldsymbol{v}, x, t]$ contains the component, which is coherent to the fluctuation of interest. The linear contribution to it is expanded to obtain

$$S[f; \boldsymbol{x}, \boldsymbol{v}, t] = \tilde{S}[f_0; \boldsymbol{x}, \boldsymbol{v}, t] + \frac{\delta S[f_0; \boldsymbol{x}, \boldsymbol{v}, t]}{\delta f_0}\tilde{f} + \cdots . \tag{2}$$

Thus the kinetic equation is separated into the fluctuating part and the mean component, for electrostatic perturbations,

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla + \frac{e_s}{m_s}(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})\cdot\nabla_v\right)\tilde{f} = -\frac{e_s}{m_s}\tilde{\boldsymbol{E}}\cdot\nabla_v f_0 + \frac{\delta S[f_0; \boldsymbol{v}, \boldsymbol{x}, t]}{\delta f_0}\tilde{f} + \tilde{C}, \tag{3a}$$

where the collisional change rate C is also separated into the mean and fluctuation part $C = \bar{C} + \tilde{C}$. The first term in RHS of Eq. (3a) indicates the driving by the spatial inhomogeneity of mean parameters, and the second term shows a new mechanism [9]. The second term in the RHS represents the *change rate* of distribution function by heating process, and it directly affects the fluctuations without the change in f_0. This term jumps at the on/off of heating process, so that the effect of on/off of heating can immediately influence the fluctuations, before the slower change of the mean f_0 takes place. Note that the second term in the RHS of Eq. (3a) is a symbolic representation. As was discussed in ref. [10], the functional derivative of S with respect to the distribution is an operator, not a scalar coefficient. It is noted that the second term in the RHS is the lowest order term, which can change at the onset of heating without delay. In other terms in the RHS of Eq. (3a), the coefficients to the fluctuating fields cannot change without variation of mean plasma parameters, i.e., the immediate effect of heating at the onset is not included explicitly.

The equation for mean distribution,

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla + \frac{e_s}{m_s}(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})\cdot\nabla_v\right)f_0 = -\frac{e_s}{m_s}\left\langle\tilde{\boldsymbol{E}}\cdot\nabla_v\tilde{f}\right\rangle + S + C, \tag{3b}$$

includes the influence of turbulent transport and the source (the first and second terms in the RHS, respectively). The separation of Eq. (3b) from Eq. (3a) is based on the symmetry consideration: In this article, quantities in Eq. (3b) are treated constant on a magnetic surface, while those in Eq. (3a) are varying on a magnetic surface. Some aspects of cross-scale nonlinear interactions [2, 3] are noted at the end of this article.

In the following, impacts on the turbulence and on mean distribution in f_0 are explained from the view point that the turbulence can be influenced directly by the onset of heating. This point is the main difference of the consideration here from conventional arguments, where either the heating changes the mean distribution function via Eq. (3b) (which then modifies turbulence) or the change of turbulence and transport results from the change of mean plasma parameters. This article illustrates the non-equilibrium parameter that denotes the distance associated with timescale mixing, which is induced by the new coupling term in Eq. (3a).

3. Mixing of Timescales and Non-Equiribrium Distance

In the conventional arguments, the second term in the RHS of Eq. (3a) is neglected, and the time scale separation is applied to the system of Eq. (3). The characteristic time scales for changes for the global parameters (temperature, etc.), mean distribution function and fluctuations, τ_{global}, τ_{f0}, τ_{cor}, respectively, are separated, $\tau_{\mathrm{global}} \gg \tau_{\mathrm{f0}} \gg \tau_{\mathrm{cor}}$. With this assumption, by neglecting the first term in the RHS of Eq. (3a), the evolution of symmetric part of the mean distribution function (in the time scale of τ_{f0}) is described by

$$\frac{\partial}{\partial t}f_0 = \frac{1}{v^2}\frac{\partial}{\partial v}\left[-\alpha v^2 f_0 + \frac{1}{2}\frac{\partial}{\partial v}(\beta v^2 f_0) + \frac{\langle P\rangle}{4mn}v^2\frac{\partial}{\partial v}f_0\right], \tag{4a}$$

where α and β are the drag and diffusion in the velocity space owing to the collisional process, respectively, $\langle P\rangle$ indicates the mean absorption power by rf heating, and n and m are number density and particle mass, respectively [13]. (The suffix to specify the particle species is dropped.) The Stix's ξ-parameter was introduced as

$$\xi = \frac{\langle P\rangle}{3nT}\tau_s, \tag{4b}$$

where τ_s is the slowing-down time. This parameter is a measure to specify the deformation of the mean distribution function from the Maxwell distribution. In this sense, the ξ-parameter is one measure of non-equilibrium state. Then the transport equation of global parameters is constructed, in which the turbulent transport coefficient is introduced based on the closure model.

In the new approach, in which the second term in the RHS of Eq. (3a) is kept, the new control parameter emerges. This second term is symbolically denoted as

$$\frac{\delta S[f_0; \boldsymbol{v}, \boldsymbol{x}, t]}{\delta f_0}\tilde{f} = \gamma_{\mathrm{heat}}(P)\tilde{f}. \tag{5}$$

This crude estimate of the impact of operator by a scalar coefficient is introduced for an analytic insight of the problem. An analysis was developed by taking an example of trapped particle mode [9]. Combining Eqs.(3a) and (5), the perturbed electron density was calculated as

$$\frac{\tilde{n}_e}{n} = \left(1 - \sqrt{\frac{2r}{R}}\right)\frac{e\tilde{\phi}}{T_e} + \sqrt{\frac{2r}{R}}\int dv^3\frac{\left(\omega - \omega_e^T\right)f_0}{\omega - \omega_D + i\nu_{\mathrm{eff}} - i\gamma_{\mathrm{heat}}}\frac{e\tilde{\phi}}{T_e}, \tag{6}$$

where the first term in the RHS is the contribution of the transit particles and the second term is that of trapped particles, with

$$\omega_{*\mathrm{e}}^{T}(E) = \omega_{*} + \omega_{*}\eta_{\mathrm{e}}\left(v^{2}/2v_{\mathrm{th}}^{2} - 3/2\right),$$
$$\eta_{\mathrm{e}} = (\mathrm{d}\ln T_{\mathrm{e}}/\mathrm{d}x)\,(\mathrm{d}\ln n_{0}/\mathrm{d}x)^{-1}\ .$$

As a result that the direct effect of heating appears in the denominator in Eq. (6), the growth rate of trapped particle mode is modified as [10]

$$\gamma_{\mathrm{TPM}} = \frac{r}{4R}\frac{\omega_{*}^{2}}{\nu_{\mathrm{eff,e}} - \gamma_{\mathrm{heat,e}}} - \nu_{\mathrm{eff,i}} + \gamma_{\mathrm{heat,i}}\,. \tag{7}$$

The particle flux by the fluctuating velocity $\tilde{V}_{\mathrm{r}}$, $\langle \tilde{n}_{\mathrm{e}}\tilde{V}_{\mathrm{r}}\rangle$, is calculated as

$$\langle \tilde{n}_{\mathrm{e}}\tilde{V}_{\mathrm{r}}\rangle = -\frac{nT_{\mathrm{e}}k_{\theta}}{eB}\left|\frac{e\tilde{\phi}}{T_{\mathrm{e}}}\right|^{2}\sqrt{\frac{2r}{R}}\mathrm{Im} \times \int \mathrm{d}v^{3}\frac{\left(\omega - \omega_{*\mathrm{e}}^{T}\right)f_{0}}{\omega - \omega_{\mathrm{D}} + i\nu_{\mathrm{eff}} - i\gamma_{\mathrm{heat}}}\,. \tag{8}$$

This result shows that the influence of the heating directly appears in the denominator of response function, so that the transport flux changes immediately when the heating power changes in time, without waiting the elapse time of the transport. The magnitude of the direct impact of the heating on the transport process is measured by the ratio between γ_{heat} and the decorrelation rate of the fluctuation, $\gamma_{\mathrm{heat}}\tau_{\mathrm{cor}}$, or

$$\Gamma_{\mathrm{heat}} = \frac{\gamma_{\mathrm{heat}}}{\chi_{\mathrm{N}}k^{2}}, \tag{9}$$

where χ_{N} is the turbulent transport coefficient. This is because the higher order nonlinear effects determine the width of the resonance of the propagator in Eq. (6) or (8), if renormalized. The control parameter Γ_{heat} in Eq. (9) is compared to the ξ-parameter in Eq. (4b). The new control parameter in Eq. (9) is explicitly rewritten as

$$\Gamma_{\mathrm{heat}} = \frac{\delta S\left[f_{0};\boldsymbol{v},\boldsymbol{x},t\right]}{\delta f_{0}}\frac{1}{\chi_{\mathrm{N}}k_{\perp}^{2}}. \tag{10}$$

The coefficient $\delta S/\delta f$ has the similar parameter dependence as $\langle P\rangle/nT$ in Eq. (4b). While such a similarity between ξ and Γ_{heat} exists, the main difference is that the parameter Γ_{heat} denotes the direct impact of heating (sources) on turbulence, so that the turbulent transport can vary immediately after the source changes in time.

The term (9) indicates that the direct influence of heating on turbulence is more effective for perturbations with longer correlation time. Based on this consideration, the response of long-range fluctuations (which are linearly stable and driven by nonlinear processes) was studied in [10]. By use of the fluid model and introducing the response coefficient γ_{h} by the relation,

$$\gamma_{\mathrm{h}}\tilde{p} \equiv \frac{\delta P}{\delta p}\tilde{p}\,, \tag{11}$$

where P and p are the heating power density and pressure, respectively, one obtaines the direct influence of the plasma heating on the nonlinearly-excited long-range fluctuations as [10]

$$I = \frac{1}{1 - \Gamma_{\mathrm{h}}}I_{0}\,, \tag{12}$$

where I is the normalized density of fluctuation energy of interest, and the control parameter

$$\Gamma_{\mathrm{h}} = \frac{\gamma_{\mathrm{h}}}{\chi_{\mathrm{N}}k_{\perp}^{2}} = \frac{\delta P}{\delta p}\frac{1}{\chi_{\mathrm{N}}k_{\perp}^{2}}\,, \tag{13}$$

is the counter part of the parameter (9) in the fluid modelling, and I_{0} is the mean intensity in the absence of the heating effect. Note that the normalizing time $(\chi_{\mathrm{N}}k_{\perp}^{2})^{-1}$ depends on the correlation length of the fluctuation of the interest. This dependence causes the additional timescale mixing through cross-scale nonlinear interactions. The control parameter and Γ_{heat} is proportional to the heating power (if other parameters are common). It is shown that, before the changes of pressure and its gradient, the turbulent intensity increases after the onset of heating if near $\gamma_{\mathrm{h}} > 0$. The similarity and difference between Γ_{h} and ξ is analogous to the case of Γ_{heat}. Thus, the parameter Eq. (13), together with (10), plays a role of measure that specifies the distance from thermodynamic equilibrium.

The relation (12) shows that the impact on fluctuation intensity becomes stronger as the heating power increases. The relation (12) was obtained in the limit of small Γ_{h}. The enhancement of fluctuation is shown to be prominent if $\gamma_{\mathrm{h}} \sim \chi_{\mathrm{N}}k_{\perp}^{2}$. Experimental observation has also shown that the increment of fluctuation intensity and jump in the hysteresis increase more rapidly than the increment of heating power [4]. Equation (12) is in qualitative agreement with experimental observation. However, Eq. (12) shows a singularity at $\gamma_{\mathrm{h}} \sim \chi_{\mathrm{N}}k_{\perp}^{2}$, although the singularity does not appear in experimental observations.

This singularity is resolved by considering the nonlinear damping of the excited mode. Following the Kadomtsev's argument in [14], the evolution of the fluctuation intensity follows the equation

$$\frac{\partial}{\partial t}I = -\left(\gamma_{\mathrm{damp}} - \gamma_{\mathrm{h}}\right)I - \omega_{2}I^{2} + \varepsilon\,, \tag{14}$$

where $\gamma_{\mathrm{damp}} = \chi_{\mathrm{N}}k_{\perp}^{2}$ is the damping rate of the fluctuation (in the absence of heating effect), the term $\omega_{2}I^{2}$ denotes the damping rate by self-nonlinear effect, and ε is the spontaneous excitation as was deduced in [10]. The mean energy density and the spontaneous emission term is related as $\varepsilon = \gamma_{\mathrm{damp}}I_{0}$, which gives the stationary solution Eq. (12) in the limit of small fluctuation amplitude. Equation (14) gives the stationary solution

$$I = \frac{\Gamma_{\mathrm{h}} - 1 + \sqrt{(\Gamma_{\mathrm{h}} - 1)^{2} + 4I_{0}\omega_{2}\chi_{\mathrm{N}}^{-1}k_{\perp}^{-2}}}{2\omega_{2}\chi_{\mathrm{N}}^{-1}k_{\perp}^{-2}}\,. \tag{15}$$

In the limit of small Γ_h, Eq. (12) is recovered. In the limit of stronger heating, $\Gamma_h \gg 1$, one has

$$I \sim \gamma_h/\omega_2 . \quad (16)$$

The result of Eq. (15) resolves the singularity in Eq. (12). The transition from Eq. (12) to Eq. (16) takes place near $\Gamma_h \sim 1$.

4. Summary

We here study the direct influence of heating on the turbulence and turbulent transport, by paying the attention to the coupling term between source and fluctuations in the phase space dynamics. The mixing of the timescale happens. The conventional ordering of time scale separation, $\tau_{global} \gg \tau_{f0} \gg \tau_{cor}$, is violated, where τ_{global}, τ_{f0}, τ_{cor}, are the characteristic time scales for the changes of global parameters, mean distribution function and fluctuations, respectively. This timescale mixing is one of the characteristic features of the far-nonequilibrium plasmas. The measure of the 'distance' is introduced as Eq. (10) or Eq. (13), which is compared to the Stix's ξ parameter that characterizes the deviation from Maxwellian distribution.

This article discusses the coupling term between source and fluctuations in the phase space dynamics. This coupling seems to be important in understanding experimental observations [4]. The long-range nonlinear coupling between microscopic fluctuations at two radial locations, the spatial distance between which are much longer than the auto-correlation length of microfluctuations, has been identified experimentally [15]. However, the immediate influence of the heating power on the turbulent transport (as illustrated by the hysteresis) in [4] seems to be unexplainable in the framework of the real-space coupling. A global turbulence simulation has been performed: Non-diffusive responses were simulated, but the observed hysteresis was not reproduced in the simulation [16]. More emphasis on study of the phase space dynamics is necessary for the understanding of turbulent plasmas.

In search of understanding of observations [4–8] along the line of thought in this article, extensions of the model are necessary. The first issue is the importance of the cross-scale nonlinear interactions in turbulence. As is shown by Eq. (9), the direct influence of heating (that is modelled in this article) can appear more easily for meso-scale or long-range fluctuations, which are known to modify microscopic turbulence via cross-scale nonlinear interactions [2, 3]. Thus, the extension of the present consideration into the framework of multiple-scale turbulence is necessary. The second issue is the evaluation of the parameter γ_{heat}. In principle, the parameter γ_{heat}, which denotes the direct influence of heating on turbulent transport, can be negative. In such a case, the heating reduces the turbulence transport. One possibility for the case of negative γ_{heat} is the Ohmic heating. This is possible to understand, because the increase of plasma temperature reduces the Ohmic heating. The other possibility is that the operator $\delta S/\delta f$ has a negative eigenvalue, although a concrete example has not yet been demonstrated. The evaluation of γ_{heat} is a future important task to examine the relevance of the model.

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