Nonlinear Stability of Externally Induced Magnetic Islands in Multi-Helicity Helical Systems

Seiya NISHIMURA

Kobe City College of Technology, Kobe 651-2194, Japan

(Received 4 January 2015 / Accepted 9 February 2015)

Self-healing (spontaneous shrinkage) of externally induced magnetic islands is a critical issue in helical systems, where helical ripple-induced neoclassical viscous torques play essential roles. In this study, effective helical ripple rates of magnetic fields in multi-helicity helical systems are revisited. In a typical parameter regime of the Large Helical Device, effective helical ripple rates are sensitive to magnetic axis positions. An extended theory of the self-healing taking into account effective helical ripple rates is firstly developed. It is newly found that self-healing thresholds considerably depend on magnetic axis positions, which is due to neoclassical viscous torques depending on effective helical ripple rates.

Keywords: magnetohydrodynamics, neoclassical transport, resonant magnetic perturbation, magnetic island, helical system

DOI: 10.1585/pfr.10.1203014

In magnetic confinement fusion with toroidal devices, magnetic islands are produced by externally induced resonant magnetic perturbations (RMPs) through forced magnetic reconnection. In the Large Helical Device (LHD), spontaneous shrinkage of RMP-induced magnetic islands has been observed, known as self-healing [1, 2], which is a critical issue for divertor configurations with magnetic islands. Historically, magnetic reconnection in helical systems has been investigated in the context of the curvature-driven tearing mode [3–5]. It has been pointed out that resonant Pfirsch-Schlüter current, perturbed bootstrap current, and polarization current are hopeless to explain the self-healing of large magnetic islands with bifurcation characteristics in low-β regimes [1, 2], where β is a ratio of total plasma pressure to magnetic pressure. For these reasons, a new mechanism of the self-healing was required. By analogy with tokamaks [6], it has been attempted to explain the self-healing by shielding effects of helical ripple-induced neoclassical flows [7–9]. In helical systems, a magnetic axis position is one of key parameters to control neoclassical transport, where magnetic axis shifts are associated with multi-helicity effects of background magnetic fields. It is known that optimal magnetic axis shifts drastically improve neoclassical transport [10–13], which is caused by changes of effective helical ripple rates [11, 13]. Because a self-healing threshold is determined by a balance between island-induced electromagnetic torques and helical ripple-induced neoclassical viscous torques [7–9], changes of effective helical ripple rates might directly affect the self-healing threshold. In the present study, we check this hypothesis, which has not been addressed in previous works.

author's e-mail: n-seiya@kobe-kosen.ac.jp

Model equations used in this study are originated from those in our previous work [7]. We consider a helical plasma with an averaged minor radius a and an averaged major radius R_0, which is mainly produced by helically winding coils with a pole number l and a pitch number N. In the following, (r, θ, z) indicate cylindrical coordinates, where r is the averaged minor radial position, θ is the poloidal angle, and z is the distance in the toroidal direction. We consider a resonant mode at the rational surface $r = r_s$. A single-helicity RMP with poloidal and toroidal mode numbers (m, n) is considered, which produces a vacuum island with a radial width w_v and phase angles $(\theta, z) = (\theta_0, z_0)$. The model is composed of an island width evolution equation, an island phase evolution equation, and a poloidal flow evolution equation. The poloidal flow is due to an $E \times B$ drift. In writing the model equations, we consider a so-called Rutherford regime, no curvature limit, and the toroidal currentless limit. Considering a stationary state of evolution equations, which corresponds to a penetration state of RMP, we obtain a balance relation of electromagnetic torques and viscous torques as [7]

$$\frac{k_{\theta s} v_{As}^2}{4L_s^2} \left(\frac{w_v}{4}\right)^4 \left(-\Delta_0'\right) \sin(2\Delta\Theta) = \frac{\mu}{\lambda} V_s^{neo}, \tag{1}$$

where we omitted electron diamagnetic drift, because electron pressure is flatten inside large magnetic islands, $\Delta\Theta = \Theta - \Theta_0$ is the magnetic island phase angle measured from those of vacuum islands, i.e., $\Theta_0 = m\theta_0 - nz_0/R_0$, Δ_0' (< 0) is the tearing mode stability parameter, $k_{\theta s} = m/r_s$, v_{As} is the Alfvén velocity, L_s is the magnetic shear length, μ is the poloidal momentum diffusivity, V_s^{neo} is the ion neoclassical flow velocity, $\lambda = (\mu/\nu_s^{neo})^{1/2}$ is the typical scale length of neoclassical flows, and ν_s^{neo} is the ion neoclassical damping rate. The ion neoclassical damping rate is defined by $\nu_s^{neo} = (f\epsilon_t^2\epsilon_h^{3/2}V_{\perp i}^2/\rho_i^2\nu_i)|_{r_s}$, where f is the numerical factor of order unity, $\epsilon_t = r/R_0$, ϵ_h is the helical ripple rate (relative magnitude of helical ripples of magnetic fields), $V_{\perp i}$ is the ion toroidal drift velocity, ρ_i is the ion Larmor radius, and ν_i is the ion collision frequency. In writing Eq. (1), we consider the following assumptions: influence of magnetic islands on the neoclassical viscosity is negligible since magnitude of helical ripples due to magnetic islands is much smaller than that of background magnetic fields; ion neoclassical viscosity is dominant; the ion neoclassical damping rate shows so-called $1/\nu$ dependence; viscous torques are evaluated in an anomalous viscosity dominant regime [7], where $\lambda \gg w/2$. The RHS of Eq. (1) has a fixed value for a given equilibrium, while, the LHS of Eq. (1) has the maximum absolute value when $\Delta\Theta = \pm\pi/4$ [8]. Thus, Eq. (1) with $|\sin(2\Delta\Theta)| = 1$ gives the threshold of existence of locked, large magnetic islands as

$$\left(\frac{w_v}{4}\right)^4 = \left(\frac{\mu\nu_s^{neo}}{v_{As}^2}\right)^{1/2}\left|\frac{4L_s^2V_s^{neo}}{\Delta_0' k_{\theta s}v_{As}}\right| \propto \epsilon_h^{3/4}. \quad (2)$$

In other words, the self-healing occurs when w_v becomes smaller than that given by Eq. (2). Thus, the self-healing threshold of RMP amplitude is a monotonic increasing function of ϵ_h.

Next, multi-helicity effects of background magnetic fields on the torque balance are discussed. We assume that magnitude of background magnetic fields in helical systems is approximated by $B/B_0 = 1 - \epsilon_t\cos\theta + \sum_{j=0,\pm1}\epsilon_h^{(j)}\cos[(j+l)\theta - N(z/R_0)]$. In neoclassical transport, trapped particles enhance radial particle and heat fluxes. According to a theory in Ref. [10], the radial particle flux in the $1/\nu$ regime is proportional to a factor S. In our case, S is given by

$$\frac{S}{S_0} = \int_0^{2\pi}\frac{d\theta}{\pi}h^{3/2}\left[\sin^2\theta - \frac{2G_2}{G_1}\frac{\epsilon_h^{(0)}}{\epsilon_t}\sin\theta\frac{\partial h}{\partial\theta} + \frac{G_3}{G_1}\left(\frac{\epsilon_h^{(0)}}{\epsilon_t}\right)^2\left(\frac{\partial h}{\partial\theta}\right)^2\right], \quad (3)$$

where $h = (C^2 + D^2)^{1/2}/\epsilon_h^{(0)}$, $C = \epsilon_h^{(0)} + (\epsilon_h^{(-1)} + \epsilon_h^{(+1)})\cos\theta$, $D = (\epsilon_h^{(-1)} - \epsilon_h^{(+1)})\sin\theta$, $G_1 = 16/9$, $G_2 = 16/15$, and $G_3 = 0.684$. Note that the sign of ϵ_t is different from that in Ref. [10]. In the absence of the sideband Fourier components of background magnetic fields, i.e., $\epsilon_h^{(\pm1)} = 0$, S is reduced to $S_0 = G_1\pi\epsilon_t^2\left(\epsilon_h^{(0)}\right)^{3/2}$. In general cases, we express S as $S = G_1\pi\epsilon_t^2\epsilon_{eff}^{3/2}$, where ϵ_{eff} is an effective helical ripple rate which involves multi-helicity effects [11, 13]. In other words, the effective helical ripple rate is evaluated as

$$\frac{\epsilon_{eff}}{\epsilon_h^{(0)}} = \left(\frac{S}{S_0}\right)^{3/2}. \quad (4)$$

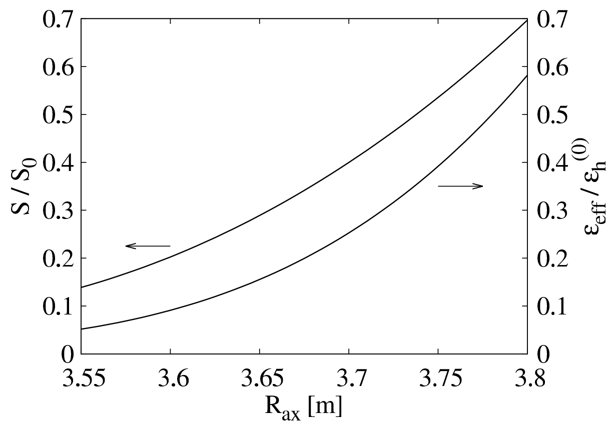

Fig. 1 Magnetic axis position dependence of S/S_0 and $\epsilon_{eff}/\epsilon_h^{(0)}$.

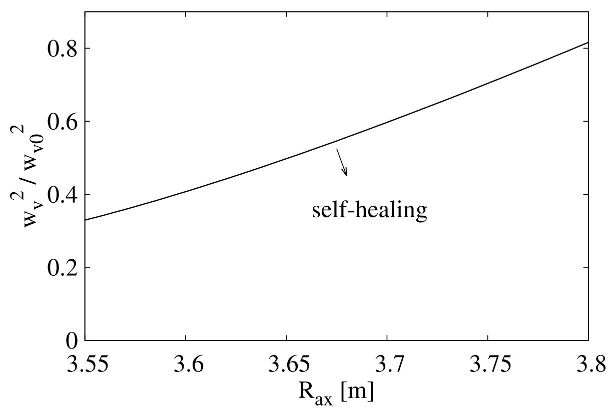

Fig. 2 Self-healing threshold of w_v^2/w_{v0}^2 (proportional to RMP amplitude) with different values of R_{ax}.

Considering Eqs. (2)–(4), the self-healing threshold of w_v^2 (proportional to RMP amplitude) is written as

$$\frac{w_v^2}{w_{v0}^2} = \left(\frac{\epsilon_{eff}}{\epsilon_h^{(0)}}\right)^{3/8} = \left(\frac{S}{S_0}\right)^{9/16}, \quad (5)$$

where w_{v0} is defined by the self-healing threshold in the absence of the sideband Fourier components as $w_{v0} = w_v|_{S=S_0}$. In order to calculate Eq. (3), values of $\epsilon_h^{(\pm1)}/\epsilon_h^{(0)}$ are sampled from numerical calculations of the magnetic fields in Ref. [12], which correspond to LHD experiments with different values of magnetic axis positions R_{ax}. Note that a relation $|R_{ax} - R_0|/R_0 < a/R_0 \ll 1$ is typically satisfied, and changes of equilibrium quantities are negligible in comparison with that of ϵ_{eff}. Using data of $\epsilon_h^{(\pm1)}/\epsilon_h^{(0)}$ at $r/a = 0.84$ in cases of $R_{ax} = 3.6$ [m] and $R_{ax} = 3.75$ [m] [12], fitted curves for R_{ax} dependence of $\epsilon_h^{(\pm1)}/\epsilon_h^{(0)}$ are obtained based on the first-order Lagrange interpolating polynomials, where $r/a = 0.84$ is regarded as the rational surface. In addition, the value of $\epsilon_t/\epsilon_h^{(0)}$ is not sensitive to R_{ax} [12], therefore, $\epsilon_t/\epsilon_h^{(0)} = 0.6$ is used. Figure 1 shows the R_{ax} dependence of S/S_0 and $\epsilon_{eff}/\epsilon_h^{(0)}$, where the fitted curves of $\epsilon_h^{(\pm1)}/\epsilon_h^{(0)}$ are used. It is found that S/S_0 and $\epsilon_{eff}/\epsilon_h^{(0)}$ strongly depend on R_{ax}. Figure 2 shows the R_{ax} dependence of the self-healing threshold of w_v^2/w_{v0}^2. It is found that strong dependence of ϵ_{eff} on R_{ax} results in the over two times difference of the threshold value of the RMP amplitude.

In summary, it is newly found that the self-healing threshold of externally induced magnetic islands substantially depends on magnetic axis positions. This work explores a methodology to control the self-healing by effective helical ripples of background magnetic fields. Extension of the theory to more general cases and detailed comparison with experimental observations are left as future works.

I would like to thank for useful discussions with Y. Narushima, K. Ichiguchi, S. Toda, M. Yokoyama, and S. Satake. This work is partially supported by Grant-in-Aid for Research Activity Start-up (No. 26889064). I also acknowledge a collaboration program of National Institute for Fusion Science.

[1] N. Ohyabu *et al.*, Plasma Phys. Control. Fusion **47**, 1431 (2005).
[2] Y. Narushima *et al.*, Nucl. Fusion **48**, 075010 (2008).
[3] J.R. Cary and M. Kotschenreuther, Phys. Fluids **28**, 1392 (1985).
[4] T. Hayashi, T. Sato and A. Takei, Phys. Fluids B **2**, 329 (1990).
[5] A. Bhattacharjee, T. Hayashi, C.C. Hegna, N. Nakajima and T. Sato, Phys. Plasmas **2**, 883 (1995).
[6] R. Fitzpatrick, Nucl. Fusion **33**, 1049 (1993).
[7] S. Nishimura, S. Toda, M. Yagi and Y. Narushima, Phys. Plasmas **19**, 122510 (2012).
[8] C.C. Hegna, Nucl. Fusion **51**, 113017 (2011).
[9] R. Fitzpatrick and F.L. Waelbroeck, Phys. Plasmas **19**, 112501 (2012).
[10] K.C. Shaing and S.A. Hokin, Phys. Fluids **26**, 2136 (1983).
[11] V.V. Nemov, S.V. Kasilov, W. Kernbichler and M.F. Heyn, Phys. Plasmas **6**, 4622 (1999).
[12] S. Murakami *et al.*, Nucl. Fusion **42**, L19 (2002).
[13] M. Yokoyama and K.Y. Watanabe, Nucl. Fusion **45**, 1600 (2005).