On Kinetic Resistive Wall Mode Theory with Sheared Rotation*)

Junya SHIRAISHI, Naoaki MIYATO[1)] and Go MATSUNAGA

Japan Atomic Energy Agency, Naka 311-0193, Japan

[1)]*Japan Atomic Energy Agency, Rokkasho 039-3212, Japan*

(Received 21 November 2013 / Accepted 18 February 2014)

To study toroidal rotation shear effect on Resistive Wall Mode (RWM) stability, kinetic RWM formulation is extended to include general equilibrium rotation. By starting from the guiding-center Lagrangian with sheared rotation, an energy functional of kinetic resonance is generalized. Based on the generalized energy functional, a new dispersion relation is derived in the large aspect ratio limit. Numerical analysis of the new dispersion relation indicates that the rotation shear can reduce the growth rates of the RWMs.

Keywords: resistive wall mode, kinetic theory, rotation, rotation shear, tokamak

DOI: 10.1585/pfr.9.3403027

1. Introduction

Advanced tokamaks, which aim to confine high-β (above no-wall limit) plasmas steadily, need to stabilize Resistive Wall Modes (RWMs) that limit the achievable β value. The RWMs originate from unstable external kink modes, whose growth rates are significantly reduced by the effect of eddy currents flowing in a resistive wall located close to a plasma surface. One candidate of the RWM stabilization method is plasma rotation that brings about mode-particle resonance, especially resonance between the RWMs and particles' drift motion. Kinetic RWM theory [1] including the mode-particle resonance shows that slow plasma rotation (comparable to particles' drift frequencies) can stabilize the RWMs.

In the kinetic RWM theory, the stability is determined by the dispersion relation [2],

$$-i\omega\tau_w^* = -\frac{\delta W_f^\infty + \delta W_k}{\delta W_f^b + \delta W_k}, \tag{1}$$

where ω is the eigenvalue, τ_w^* is the modified wall diffusion time, δW_f^∞ is a fluid energy functional (source or sink) without a wall, δW_f^b is a fluid energy functional with ideal wall location $r = b$ (r is the radial coordinate, and b is the wall minor radius normalized by the plasma minor radius a). The energy functional of kinetic resonance is denoted by δW_k. Without kinetic effects, equation (1) reduces to $-i\omega\tau_w^* = -\delta W_f^\infty/\delta W_f^b$. Since δW_f^∞ and δW_f^b are real, the "fluid" RWM is unstable for $\delta W_f^\infty < 0$ and $\delta W_f^b > 0$. As indicated in (1), the imaginary part of δW_k works as a damping effect for the RWMs. Hence, self-consistent computation of δW_k is one of the most important ingredients in the kinetic RWM theory.

This study is motivated by experimental investigation of the RWM dynamics in JT-60U, which has revealed that rotation shear also plays an important role for RWM stabilization [3]. Standard formulation introduces the equilibrium rotation by using a Doppler-shifted eigenvalue [2], hence the rotation shear effect does not appear explicitly. It should be emphasized that the guiding-center Lagrangian underlies the formulation of δW_k as shown in Ref. [4]. The previous formulation employs the Lagrangian without rotation. In this study, we reformulate δW_k by starting from the guiding-center Lagrangian with sheared equilibrium rotation [5] to investigate how the formulation is affected by including the rotational modification to the Lagrangian.

The remainder of this paper is organized as follows. In Sec. 2, we derive the perturbed distribution function of the drift-kinetic equation by starting from the guiding-center Lagrangian with sheared rotation. In Sec. 3, we derive the new δW_k, which has additional energy functionals compared with the previous theory. Section 4 shows the numerical analysis of the dispersion relation based on new δW_k, which shows that the rotation shear stabilizes the RWM. This paper is summarized in Sec. 5.

2. A Solution to the Perturbed Drift Kinetic Equation with Sheared Rotation

As noted in the previous section, the guiding-center Lagrangian plays a central role in the kinetic RWM theory. We start from the guiding-center Lagrangian of a particle with charge Q and mass M including the sheared equilibrium rotation $\boldsymbol{D}(\boldsymbol{R})$ ($\boldsymbol{R}$ is the guiding-center position) [5],

$$\begin{aligned} L &= L(\boldsymbol{R}, v_\parallel, y, \alpha, \dot{\boldsymbol{R}}, \dot{v}_\parallel, \dot{y}, \dot{\alpha}, t) \\ &= \left[Q\boldsymbol{A}(\boldsymbol{R}, t) + Mv_\parallel \hat{b}(\boldsymbol{R}, t) + M\boldsymbol{D}\right] \cdot \dot{\boldsymbol{R}} \\ &\quad + \frac{y}{\Omega_c(\boldsymbol{R}, t)}\dot{\alpha} - E_t, \end{aligned} \tag{2}$$

where $v_\parallel$ is the parallel component of particle velocity in

author's e-mail: shiraishi.junya@jaea.go.jp

*) This article is based on the presentation at the 23rd International Toki Conference (ITC23).

the moving frame with $\boldsymbol{D}$, $y = \mu B$ is the perpendicular energy in the moving frame [$\mu = Mv_\perp^2/(2B)$ is the magnetic moment with the perpendicular component of particle velocity in the moving frame $v_\perp$ and the magnetic field amplitude B], α is the gyro-angle, the dot indicates the total derivative by time t, $\boldsymbol{A}$ is the vector potential that gives the magnetic field as $\boldsymbol{B} = \nabla \times \boldsymbol{A}$, $\hat{b} = \boldsymbol{B}/B$ is the unit normal vector parallel to the magnetic field, $\Omega_c = QB/M$ is the cyclotron frequency, and $E_t = M/2|v_\parallel \hat{b} + \boldsymbol{D}|^2 + y + Q\Phi(\boldsymbol{R}, t)$ is the total energy with the scalar potential Φ that gives the electric field as $\boldsymbol{E} = -\partial_t \boldsymbol{A} - \nabla\Phi$.

Variations of (2) by independent variables yield the Euler-Lagrange equations as the parallel motion $U := v_\parallel + D_\parallel = \hat{b}\cdot\dot{\boldsymbol{R}}$ where $D_\parallel = \boldsymbol{D}\cdot\hat{b}$ is the parallel component of the equilibrium rotation, the gyro-motion $\dot{\alpha} = \Omega_c$, conservation of the magnetic moment $\dot{\mu} = 0$, and the equation of guiding-center motion

$$\dot{\boldsymbol{R}} = U\hat{b} + \frac{\hat{b}}{M\Omega_c} \times \{-Q\boldsymbol{E} + \mu\nabla B + M\left[(\boldsymbol{C}\cdot\nabla)\boldsymbol{C} + \partial_t \boldsymbol{C}\right]\}, \tag{3}$$

where $\boldsymbol{C} = v_\parallel \hat{b} + \boldsymbol{D} = U\hat{b} + \boldsymbol{D}_\perp$ is the generalized velocity vector with the perpendicular component of the equilibrium rotation $\boldsymbol{D}_\perp = \boldsymbol{D} - D_\parallel \hat{b}$. We have assumed the perpendicular component of the equilibrium rotation can be approximated by $E\times B$ rotation as $\boldsymbol{D}_\perp \sim \boldsymbol{V}_E = \boldsymbol{E}\times\hat{b}/B$.

To investigate the perturbation due to the kinetic response, we employ the collision-less drift-kinetic equation,

$$\partial_t f + \dot{\boldsymbol{R}}\cdot\nabla f + \dot{v}_\parallel \partial_{v_\parallel} f + \dot{y}\partial_y f = 0, \tag{4}$$

where $f = f(\boldsymbol{R}, v_\parallel, y, t)$ is the distribution function in the lowest order of ϵ where ϵ is the ratio of gyro-radius and scale length. By linearizing (4) and integrating by time, we obtain the formal solution to the linearized drift-kinetic equation as

$$f^{(1)} = P_\phi^{(1)}\partial_{P_\phi} F + Q\Phi^{(1)}\partial_{E_t} F - \frac{\mu B^{(1)}}{B}\partial_\mu F + h^{(1)}, \tag{5}$$

where the superscript (1) indicates the perturbed part, $P_\phi = \partial_{\dot{\phi}} L$ is the canonical angular momentum with the toroidal angle ϕ, and $F = F(P_\phi, E_t, \mu)$ is the equilibrium distribution function. The non-adiabatic part $h^{(1)}$ satisfies $\mathrm{d}h^{(1)}/\mathrm{d}t = \partial_{E_t} F \partial_t L^{(1)} - \partial_{P_\phi} F \partial_\phi L^{(1)}$.

The linearized guiding-center Lagrangian is calculated as

$$L^{(1)} = Q\boldsymbol{A}^{(1)}\cdot\dot{\boldsymbol{R}} - \mu B^{(1)} - Q\Phi^{(1)}. \tag{6}$$

We relate the linearized electromagnetic fields in (6) to the ideal magnetohydrodynamic (MHD) perturbation. Even with the equilibrium rotation, the perturbation of the magnetic field is written as $\boldsymbol{B}^{(1)} = \nabla\times(\boldsymbol{\xi}_\perp \times \boldsymbol{B})$. Then the perturbation of the vector potential reads $\boldsymbol{A}^{(1)} = \boldsymbol{\xi}_\perp \times \boldsymbol{B}$ with an appropriate gauge condition. Linearization of the ideal Ohm's law leads to $\Phi^{(1)} = -\boldsymbol{\xi}\cdot\nabla\Phi$. Using the above equations, equation (6) can be written as

$$L^{(1)} = -M\boldsymbol{\xi}_\perp\cdot\left[(\boldsymbol{C}\cdot\nabla)\boldsymbol{C}\right] + \mu B\left(\nabla\cdot\boldsymbol{\xi}_\perp + \boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp\right), \tag{7}$$

where $\boldsymbol{\kappa} = (\hat{b}\cdot\nabla)\hat{b}$ is the magnetic curvature. Setting $\boldsymbol{D} = 0$ (i.e, $\boldsymbol{C} = v_\parallel \hat{b}$) and $\boldsymbol{V}_E = O(\epsilon)$ in (3) and (7) recovers the standard results [4].

We introduce the coordinate system r-θ-ϕ, where θ is the well-defined poloidal angle. The guiding-center motion reads $\dot{r} = \dot{\boldsymbol{R}}\cdot\nabla r$, $\dot{\theta} = \dot{\boldsymbol{R}}\cdot\nabla\theta$, and $\dot{\phi} = \dot{\boldsymbol{R}}\cdot\nabla\phi$. Bounce time is defined as $\tau_b = \oint \mathrm{d}\tau = \oint \dot{r}^{-1}\mathrm{d}r = \oint \dot{\theta}^{-1}\mathrm{d}\theta$, where $\oint$ refers to the integral along the closed orbit. Bounce-averaging is defined as $\langle X\rangle = \tau_b^{-1}\oint X(\tau)\mathrm{d}\tau$. The perturbed guiding-center Lagrangian can be treated as a function of t as

$$L^{(1)}(t) = \hat{L}^{(1)}(r(t), \theta(t))\mathrm{e}^{-i\omega t}\mathrm{e}^{-in\phi(t)}, \tag{8}$$

where n is the toroidal mode number. Using (8), a standard integration technique along the unperturbed orbit leads to

$$h^{(1)} = \sum_l (\omega - n\omega^*)\left(\partial_{E_t} F\right) Y_l \times \frac{\mathrm{e}^{-i[\omega + n\omega_d + (l+\tilde{\alpha}nq)\omega_b]t}}{\omega + n\omega_d + (l+\tilde{\alpha}nq)\omega_b}, \tag{9}$$

where l is the poloidal harmonic number of bounce motion, $\omega^* = \partial_{E_t} F/\partial_{P_\phi} F$, $Y_l = \langle \tilde{L}^{(1)}\mathrm{e}^{i(l+\tilde{\alpha}nq)\omega_b t}\rangle$ is the bounce-averaged bounce harmonics of the oscillating part of the perturbed Lagrangian $\tilde{L}^{(1)} = L^{(1)}\mathrm{e}^{i(\omega+\omega_d)t}$ where $\omega_d = \langle \dot{\boldsymbol{R}}\cdot\nabla(\phi - q\theta)\rangle$ is the drift frequency and q is the safety factor, $\tilde{\alpha} = 0$ (1) for trapped (passing) particles, and $\omega_b = 2\pi/\tau_b$ is the bounce frequency.

3. Reformulating the Energy Functional of Kinetic Resonance

To compute the energy functional of kinetic resonance, we need to calculate the perturbed stress tensor. Since $v_\parallel$ and $v_\perp$ are defined in the frame moving with $\boldsymbol{D}$, the stress tensor in the rest frame reads

$$\mathsf{P} = \int \mathrm{d}\boldsymbol{v} f\left[M\boldsymbol{C}\boldsymbol{C} + \frac{Mv_\perp^2}{2}\left(\mathsf{I} - \hat{b}\hat{b}\right)\right], \tag{10}$$

where I is the identity tensor. Linearizing (10), we get

$$\mathsf{P}^{(1)} = \mathsf{P}_\parallel^{(1)} + p_\perp^{(1)}\left(\mathsf{I} - \hat{b}\hat{b}\right), \tag{11}$$

where $\mathsf{P}_\parallel^{(1)} = M\int \mathrm{d}\boldsymbol{v} f^{(1)}\boldsymbol{C}\boldsymbol{C}$ is the perturbed parallel pressure tensor, and $p_\perp^{(1)} = \int \mathrm{d}\boldsymbol{v}(Mv_\perp^2/2)f^{(1)}$ is the perturbed perpendicular pressure. In derivation of (11), we have assumed $(p_\parallel - p_\perp)/B^2 = 0$ where $p_\parallel = \int \mathrm{d}\boldsymbol{v} M(v_\parallel + D_\parallel)^2 F$ is the equilibrium parallel pressure and $p_\perp = \int \mathrm{d}\boldsymbol{v}(Mv_\perp^2/2)F$ is the equilibrium perpendicular pressure. This condition can be satisfied for isotropic thermal particles.

Computing $(1/2)\int \mathrm{d}\boldsymbol{x}\boldsymbol{\xi}^*\cdot(\nabla\cdot\mathsf{P}^{(1)})$ with (5), (9), and (11) (the asterisk indicates the complex conjugate), and neglecting $\xi_\parallel$ and $O(\epsilon)$ terms, we obtain

$$\frac{1}{2}\int \mathrm{d}\boldsymbol{x}\boldsymbol{\xi}^*\cdot(\nabla\cdot\mathsf{P}^{(1)}) = \delta W_{fp} + \delta W_k, \tag{12}$$

where $\delta W_{fp} = -(1/2)\int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{v} L^{(1)*}[f^{(1)} - h^{(1)}]$ is the adiabatic response, while

$$\delta W_k = \sum_l \frac{1}{2}\int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{v}\,(\omega - n\omega^*)\,(-\partial_{E_t}F) \times \frac{|Y_l|^2}{\omega + n\omega_d + (l+\tilde{\alpha}nq)\omega_b}, \tag{13}$$

is the energy functional of kinetic resonance. We note that the adiabatic response δW_{fp} in (12) reduces to the standard fluid pressure response in the zero-banana-width limit [4]. In what follows, we assume the zero-banana-width limit, which enable us to substitute the fluid energy functionals for the adiabatic response.

When the Maxwellian equilibrium distribution function is employed, i.e., $F = N(r)[M/(2\pi T(r))]^{3/2}\exp(-E_t/T(r))$ where $N(r)$ is the number density and $T(r)$ is the temperature, equation (13) reduces to

$$\delta W_k = \sum_l \frac{1}{2}\int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{v}\left(-\frac{\partial F}{\partial E_t}\right)Q_l \times \left|\left\langle \mathrm{e}^{i(l+\tilde{\alpha}nq)\omega_b t}L^{(1)}\right\rangle\right|^2. \tag{14}$$

The mode-particle resonance factor Q_l reads

$$Q_l = \frac{\omega + n\omega_{*N} + (E_t/T - 3/2)\,n\omega_{*T}}{\omega + (l+\tilde{\alpha}nq)\,\omega_b + n\omega_d}, \tag{15}$$

where ω_{*N} and ω_{*T} are the diamagnetic drift frequencies due to the density and temperature gradients. If we add the Doppler-shift effect by putting $\omega \to \omega + n\omega_E$ where ω_E is the $E\times B$ frequency, equations (14) and (15) are formally identical to the previous ones (e.g., [6]). However, our expression has some difference. Firstly, the perturbed guiding-center Lagrangian in (14) is different from the standard one [see (7)]. Secondly, the resonant operator contains the total energy (not kinetic energy), and we have ω_E only in the denominator through ω_d. Especially, as shown in the next section, the generalization of $L^{(1)}$ gives the additional kinetic energy functionals.

4. Numerical Analysis of the Dispersion Relation

A new aspect of our theory is contained in the "generalized" curvature,

$$(\boldsymbol{C}\cdot\nabla\boldsymbol{C}) = U^2\boldsymbol{\kappa} + U\boldsymbol{f}_1 + \boldsymbol{f}_0, \tag{16}$$

where $\boldsymbol{f}_1 = (\hat{b}\cdot\nabla)\boldsymbol{D}_\perp + (\boldsymbol{D}_\perp\cdot\nabla)\hat{b}$ is the sum of the Coriolis acceleration and the parallel acceleration, and $\boldsymbol{f}_0 = (\boldsymbol{D}_\perp\cdot\nabla)\boldsymbol{D}_\perp$ indicates the centrifugal acceleration. In (16), we have omitted the terms parallel to $\hat{b}$ since they vanish in (7). Considering the large aspect ratio plasmas, we obtain $\boldsymbol{\xi}_\perp\cdot\boldsymbol{f}_1 = -2\xi_r \mathrm{d}\Phi/\mathrm{d}r/(qR_0B_0)$ and $\boldsymbol{\xi}_\perp\cdot\boldsymbol{f}_0 = -\xi_r(\mathrm{d}\Phi/\mathrm{d}r)^2/(rB_0^2)$ where R_0 is the major radius and B_0 is the magnetic field amplitude at the magnetic axis. Also we obtain the analytic formulas for bounce and drift frequencies as $\omega_b = (v_{th}/R_0)(\sigma\sigma_1/q)\hat{E}_k W$ and $\omega_d = \omega_E + (\rho_L/r)(v_{th}/R_0)q\hat{E}_k D$ where v_{th} is the thermal speed, σ is the sign of U, $\sigma_1 = 1$ $(\sigma/2)$ for passing (trapped) particles, $\hat{E}_k = E_k/T$ is the normalized kinetic energy with $E_k = E_t - MD_\perp^2/2 - Q\Phi$, and $\rho_L = v_{th}/\Omega_c$ is the Larmor radius. The non-dimensional frequencies W and D can be found in Ref. [6], which depends on $\varepsilon_r = r/R_0$ and the pitch angle variable $\lambda = \mu B_0/E_k$. The diamagnetic frequencies due to the density and temperature gradients are $\omega_{*N} = (2rB_0\mathrm{d}r/\mathrm{d}\psi)(\rho_L/r)(v_{th}/R_0)(1/4\varepsilon_r)[-(r/N)\mathrm{d}N/\mathrm{d}r]$ and $\omega_{*T} = (2rB_0\mathrm{d}r/\mathrm{d}\psi)(\rho_L/r)(v_{th}/R_0)(1/4\varepsilon_r)[-(r/T)\mathrm{d}T/\mathrm{d}r]$.

We focus on the trapped ions with a dominant bounce harmonic $l = 0$, i.e., $l + \tilde{\alpha}nq = 0$. Also we assume the eigenfunction satisfies the condition $\nabla\cdot\boldsymbol{\xi}_\perp = -2\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp$. Then (14) and (15) yield three energy functionals, $\delta W_k = \delta W_{k1} + \delta W_{k2} + \delta W_{k3}$. Here

$$\delta W_{k1} = 2\pi^{3/2}\int_0^a \mathrm{d}r X(r)\varepsilon_r P\,|\langle\xi_R\rangle|^2 \times \int_{1/(1+\varepsilon_r)}^{1/(1-\varepsilon_r)} \mathrm{d}\lambda \frac{(2-\lambda)^2}{W} I_{5/2}(r,\lambda), \tag{17}$$

is the energy functional similar to the previous δW_k where $P = NT$ is the pressure, $X(r) = \exp(-(MV_E^2/2 + Q\Phi)/T)$ and

$$I_\alpha(r,\lambda) = \int_0^\infty \mathrm{d}\hat{E}_k \hat{E}_k^\alpha \mathrm{e}^{-\hat{E}_k} Q_0. \tag{18}$$

We remark that there are two additional energy functionals due to the generalized Lagrangian (7) with (16) as

$$\begin{aligned}\delta W_{k2} &= 2\pi^{3/2}\int_0^a \mathrm{d}r X(r)\varepsilon_r P\frac{MR_0}{T} \\ &\times \left(\langle\xi_R\rangle^* \langle\boldsymbol{\xi}_\perp\cdot\boldsymbol{f}_0\rangle + \langle\boldsymbol{\xi}_\perp\cdot\boldsymbol{f}_0\rangle^*\langle\xi_R\rangle\right) \\ &\times \int_{1/(1+\varepsilon_r)}^{1/(1-\varepsilon_r)} \mathrm{d}\lambda\frac{(2-\lambda)}{W} I_{3/2}(r,\lambda), \\ \delta W_{k3} &= 2\pi^{3/2}\int_0^a \mathrm{d}r X(r)\varepsilon_r P\frac{M^2R_0^2}{T^2}\left|\langle\boldsymbol{\xi}_\perp\cdot\boldsymbol{f}_0\rangle\right|^2 \\ &\times \int_{1/(1+\varepsilon_r)}^{1/(1-\varepsilon_r)} \mathrm{d}\lambda\frac{1}{W} I_{1/2}(r,\lambda).\end{aligned} \tag{19}$$

The energy integration (18) can yield the imaginary part through the pole of Q_0. We emphasize that the standard drift-kinetic RWM theory overlooked these two additional energy functionals since the guiding-center Lagrangian is described without rotation.

In what follows, we assume that the rotation is slow but finite with the ordering $|\omega_d| \ll |\omega + n\omega_E|$, which is valid for thermal particles. In this frequency region, the Landau resonance damping, which is significant for very low rotation in the range $|\omega + n\omega_E| \ll |\omega_d|$, is not essential [6]. In this case, from (17) and (19) we obtain

$$\begin{aligned}\delta W_{k1} &= 2\pi^{3/2}\int_0^a \mathrm{d}r X(r)\varepsilon_r P\,|\langle\xi_R\rangle|^2\, G_{5/2}(r), \\ \delta W_{k2} &= 2\pi^{3/2}\int_0^a \mathrm{d}r X(r)\varepsilon_r P\frac{MR_0}{T} \\ &\times \left(\langle\xi_R\rangle^*\langle\boldsymbol{\xi}_\perp\cdot\boldsymbol{f}_0\rangle + \langle\boldsymbol{\xi}_\perp\cdot\boldsymbol{f}_0\rangle^*\langle\xi_R\rangle\right) G_{3/2}(r), \\ \delta W_{k3} &= 2\pi^{3/2}\int_0^a \mathrm{d}r X(r)\varepsilon_r P\frac{M^2R_0^2}{T^2}\times\left|\langle\boldsymbol{\xi}_\perp\cdot\boldsymbol{f}_0\rangle\right|^2 G_{1/2}(r),\end{aligned} \tag{20}$$

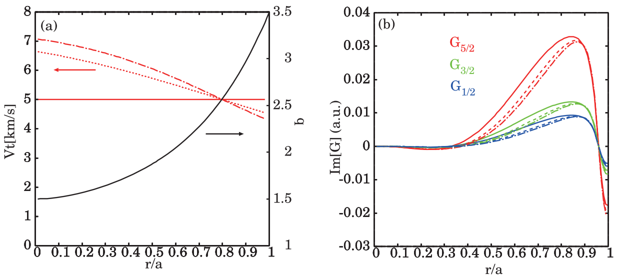

Fig. 1 (a) A fixed safety factor profile (black) and various rotation profiles (red). (b) Radial profiles of G factors for various rotation profiles.

where $G_\alpha(r) = \int d\lambda(2-\lambda)^{\alpha-1/2} I_\alpha/W$. Since G_α has the analytical expression with the aid of the Gamma function, we can easily compute the imaginary part of the G factor with the typical fluid RWM growth rate $\omega = i\gamma_f$. Figure 1 (a) shows a fixed safety factor profile and various rotation profiles, and Fig. 1 (b) indicates the radial profiles of imaginary parts of G factors for various rotation profiles. The solid, dotted, and broken lines in Fig. 1 (b) correspond to the rotation profiles in Fig. 1 (a). From Fig. 1 (b), the imaginary parts of the new energy functionals cannot be neglected compared with the previous one. Hence the new energy functionals are expected to work as a damping effect for RWMs.

For full computation of δW_k, the eigenfunction of external kink modes is required. We employ the cylindrical model with flat safety factor and density profiles and a parabolic pressure profile. In this case, we can use the analytic expression of the kink eigenfunction as $\boldsymbol{\xi}_\perp = am(r/a)^{m-1}(\hat{r} + i\hat{\theta})e^{im\theta}/F_0$ where m is the poloidal mode number and $F_0 = (m - nq_0)a/(R_0q_0)$. Then $\langle\xi_R\rangle$ and $\langle\xi_r\rangle$ in (20) can be analytically calculated.

We investigate the dispersion relation (1). In cylindrical theory, we obtain $\tau_w^* = \tau_w[1-(b/a)^{-2m}]/m$ where $\tau_w = \mu_0 bd/\eta$ is the wall diffusion time (μ_0 is the vacuum permeability, d is the wall width, and η is the wall volume resistivity). Also, the fluid energy functionals δW_f have analytic formulas [7]. As shown in Fig. 1 (a), to study the rotation shear effect on RWM stability, we have varied the rotation profiles at $r/a = 0.8$ where the G factor and the eigenfunction become large. The parameters are $q_0 = 2.2$, $m/n = 3/1$, and $a/R_0 = 1/3$. Figure 2 shows the RWM growth rates normalized by $1/\tau_w$ as functions of wall location for various δW_k. The green lines indicate $\delta W_k = \delta W_{k1}$ corresponding to the previous formulation, the red lines denote $\delta W_k = \delta W_{k1} + \delta W_{k2} + \delta W_{k3}$ as the new self-consistent formulation, and the blue lines show the fluid RWM with $\delta W_k = 0$. The solid, dotted, and broken lines correspond to the rotation profiles in Fig. 1 (a). As clearly shown in Fig. 2, in the new formulation, the RWM growth rates are significantly decreased by increasing the rotation shear, while the results by the previous theory is not so sensitive to the rotation shear.

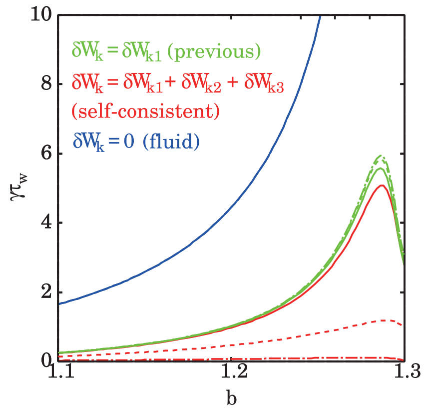

Fig. 2 RWM growth rates normalized by the inverse of the wall diffusion time as functions of wall location.

5. Summary

In summary, we have revisited the formulation of the energy functional of kinetic resonance δW_k, which plays an important role in the RWM dispersion relation. Previous formulations use the guiding-center Lagrangian without rotation. By starting from the guiding-center Lagrangian with sheared rotation, we have shown that the energy functional of kinetic resonance has two additional functionals originating from the coupling of magnetic curvature and the centrifugal acceleration. We have shown that the new δW_k has a significant imaginary part, which is related to the kinetic damping of RWMs. Also, by using cylindrical theory, we have numerically shown that the new formulation indicates the tendency that the rotation shear stabilizes the RWM, which is qualitatively consistent with experiment results. As a final remark, we note that for quantitative analysis by the present theory, we need to implement the present formulation in the computation in tokamak geometry, such as MINERVA/RWMaC code [8]. The extension to the tokamak geometry will be reported elsewhere.

The authors are grateful to Dr. Y. Kamada, Dr. T. Ozeki, Dr. S. Ide, and Dr. M. Yagi for their support. This work was supported by KAKENHI 24760708.

[1] B. Hu and R. Betti, Phys. Rev. Lett. **93**, 105002 (2004).
[2] G.Z. Hao, A.K. Wang, Y.Q. Liu and X.M. Qiu, Phys. Rev. Lett. **107**, 015001 (2011).
[3] G. Matsunaga, Y. Sakamoto, N. Aiba, K. Shinohara, M. Takechi *et al.*, Fusion Energy 2008 (Proc. 22nd IAEA Fusion Energy Conf., Geneva (IAEA, Vienna)) IAEA-CN-165/EX/5-2(2008).
[4] F. Porcelli, R. Stankiewicz and W. Kerner, Phys. Plasmas **1**, 470 (1994).
[5] A.J. Brizard, Phys. Plasmas **2**, 459 (1995).
[6] Y. Liu, M.S. Chu, C.G. Gimblett and R.J. Hastie, Phys. Plasmas **15**, 092505 (2008).
[7] J.A. Wesson, Nucl. Fusion **18**, 87 (1978).
[8] J. Shiraishi, N. Aiba, N. Miyato and M. Yagi, Fusion Energy 2012 (Proc. 24th IAEA Fusion Energy Conf. Geneva (IAEA, Vienna)) IAEA-CN-197/TH/P3-19.