Comprehensive Analysis on the Role of Helical Movement of the Magnetic Axis in Stellarators*)

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The physical confinement properties of stellarators are analyzed for the neo-classical transport and the magnetic well. Three typical concepts of stellarators (LHD, Wendelstein 7-X and TJ-II) are compared in terms of the helical movement of cross sections of the last closed magnetic surface. It is shown that this geometric element strongly determines the confinement properties of both non-planar axis and planar axis stellarators.

Keywords: stellarator, magnetic configuration, helical axis, non-planar axis, Fourier modes, boundary shape, magnetic axis

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1. Introduction

The shapes of magnetic surfaces of stellarators have a wide variety of configurations. It is true not only for the theoretical configuration design works but also for the existing experimental devices in the world. One of the primary reasons comes from the three-dimensional structure of the stellarator configurations in contrast to the tokamak world with a much simpler axi-symmetric shape. However the more essential reason is that the stellarator community has not made sufficient efforts of comprehensive and comparative studies for the advantages and disadvantages of many stellarator configurations. We have not obtained the scientific bases for discussing a unique solution of the stellarator device for the future reactor.

One of classification of stellarator configurations is to divide them into two groups of the planar-axis structure and the non-planar-axis structure. The shape of magnetic axis for the former group is close to a simple circle and the shape for the latter one is three-dimensional shape (sometimes it is said that the axis shape has the helical excursion). It is interesting that two largest stellarators in the world, LHD and Wendelstein 7-X, represent these two groups, respectively. However in our study of analyzing LHD magnetic field configuration [1], we found that a helical movement of the magnetic axis of LHD device plays a big role in determining the different confinement characteristics of the LHD configurations with magnetic axis shift [2].

For discussing the difference between many stellarator configurations or the characteristics of the target configuration, the Fourier modes of boundary shape of the configuration are very useful information. It is because of the mathematical fact that a complete three-dimensional equilibrium is obtained based on the boundary information with two scalar variables of the pressure and the current (or the rotational transform) as functions of minor radius. In this paper, we examine the physical roles of all Fourier modes of boundary shape taking examples of LHD configurations in the experiments and select a small number of important Fourier modes that are necessary to determine the physical characteristics of the configurations. In the process of this study, we found that the role of the Fourier mode giving the helical movement of the boundary, which makes the helical movement of the magnetic axis, is very large compared to other modes. It was also found that this fact is common for all other stellarators. In this paper, we compare the confinement properties of stellarators in the world in terms of this helical movement term of the Fourier modes. The equilibria discussed in the paper are vacuum configurations.

2. Expression of Boundary Shape

In the magnetic confinement fusion research, commonly used expression of boundary shape is the one that is used in the VMEC code [3]. R and Z coordinates of each point on the boundary of a torus are defined as functions of two angle variables θ and ϕ, which are expressed by the Fourier series of cosine and sine functions as follows.

$$R(\theta, \phi) = \sum r(m, n) \cdot \cos(m\theta - n\phi),$$
$$Z(\theta, \phi) = \sum z(m, n) \cdot sin(m\theta - n\phi).$$

We started the comprehensive analysis of stellarator geometry by analyzing three LHD configurations, which have been used in the experiments as typical operational modes of different confinement properties (neo-classical transport and the MHD stability). They are called configurations with R_{ax} = 3.6 m, 3.75 m and 3.9 m, where R_{ax} is the major radius of magnetic axis position. Because

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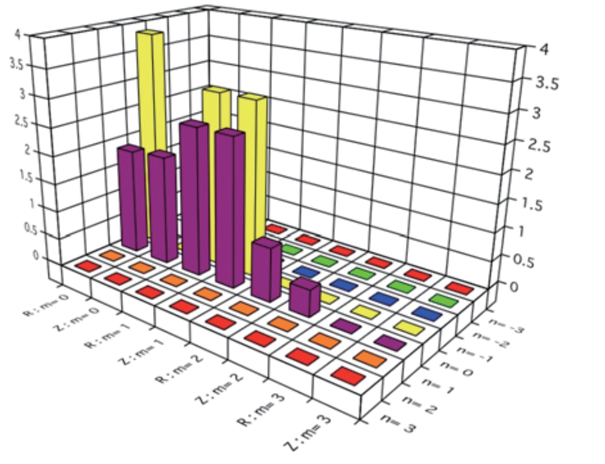

Fig. 1 Distribution of reduced number of Fourier modes of LHD R_{ax} = 3.6 m configuration. The $\log_{10}$ values of absolute amplitudes are plotted.

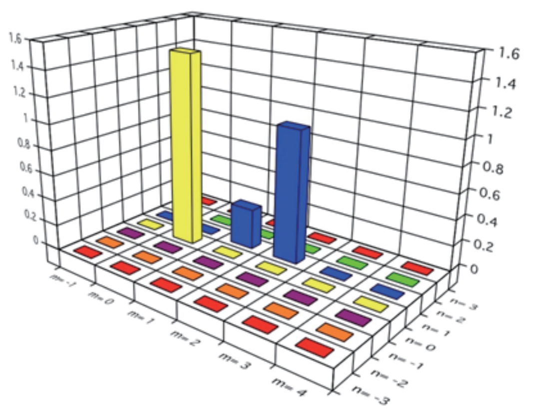

Fig. 2 Distribution of Fourier modes of LHD R_{ax} = 3.6 m configuration in complex number formula.

the VMEC code is very fast in calculating equilibria, we usually take as many Fourier modes as more than 100 to describe accurately the boundary shape of plasmas in the experiments. However we followed opposite way in this paper to reduce the number of Fourier modes for the purpose of finding the essential Fourier modes to determine the confinement properties. We found that the 9 modes are sufficient to configure these configurations [2]. Figure 1 shows a distribution of amplitudes of reduced number of Fourier modes of R_{ax} = 3.6 m configuration. Since the variation of amplitude is large, amplitudes are plotted in the logarithmic scale and abstract values are used eliminating sign of values. Amplitudes of columns are in a relative scale.

As is shown in Fig. 1, the roles of Fourier coefficients for R and Z are physically almost equivalent. Then we take another way of expression of boundary shape using complex numbers as below [4], which gives much simpler plot of the Fourier mode distribution shown in Fig. 2.

$$R(\theta, \phi)+iZ(\theta, \phi) = \exp(i\theta) \sum \Delta_{mn} \exp(-im\theta+in\phi).$$

In this Fourier mode expression, all Fourier elements are normalized to the minor radius that is shown as a tall yellow column in the figure(Δ_{00}) and much simpler relations between the Fourier elements and the geometry of a torus are given. A taller blue column (Δ_{21}) expresses a rotating elliptical cross section, which is the basic helical structure. A shorter blue column (Δ_{11}) shows the helical movement of plasma cross section that makes also the helical movement of the magnetic axis, which is the subject of this paper. This Δ_{11} is the key element to give differences of confinement properties for three LHD configurations.

3. Role of Helical Movement of Magnetic Axis in LHD Configurations

In order to understand the role of the helical movement of magnetic axis in LHD, we vary the amplitude of this Fourier element in the calculations of equilibria for wider range than the variation taken in the experiment. Figure 3 shows the dependence of the effective helical ripple at two minor radii of 1/3 and 2/3 as functions of Δ_{11} Fourier element. The effective helical ripple is a measure of the neo-classical transport that expresses the dependence of the transport coefficient on the magnetic configuration [5].

Three arrow positions indicate different neo-classical transport values of LHD configurations. It is shown that a positive value of Δ_{11} gives the improved neo-classical transport. For finite beta equilibria, since the Shafranov shift of the magnetic axis increases the effective ripple in the range of $\Delta_{11} < 0.1$, the curves in Fig. 3 is shifted to the right.

Δ_{11} changes also the magnetic well, which is another important confinement property of LHD configuration. Figure 4 shows the dependence of magnetic well depth on Δ_{11} as well as the (normalized) specific volume ($\mathrm{d}V/\mathrm{d}Psi$) at the plasma edge, which is a measure of the MHD stability (higher value indicate the poor stability).

The normalized specific volume and the magnetic well depth are defined in the formula.

Specific volume: $SV(r) \equiv \mathrm{d}V(r)/\mathrm{d}\Psi(r)$
  V: volume of flux tube
  Ψ: magnetic flux in the tube
Normalized specific volume at edge:
  $\mathrm{d}V/\mathrm{d}Psi = SV(r = a)/SV(r = 0)$
Magnetic well depth:
  $\mathrm{Well} = -\,[SV(r = r_{\min}) - SV(r = 0)]/SV(r = 0)$
  $r_{\min}$: minor radius for minimum value of $SV(r)$.

The geometric meaning of a positive value of Δ_{11} is that the major radius of the vertically elongated cross section of the magnetic surface is larger than that of horizontally elongated cross section. It is general characteristics of LHD type stellarators with a conventional plasma shape that the magnetic well is created for the negative Δ_{11} and the neo-classical transport is improved for the positive Δ_{11}.

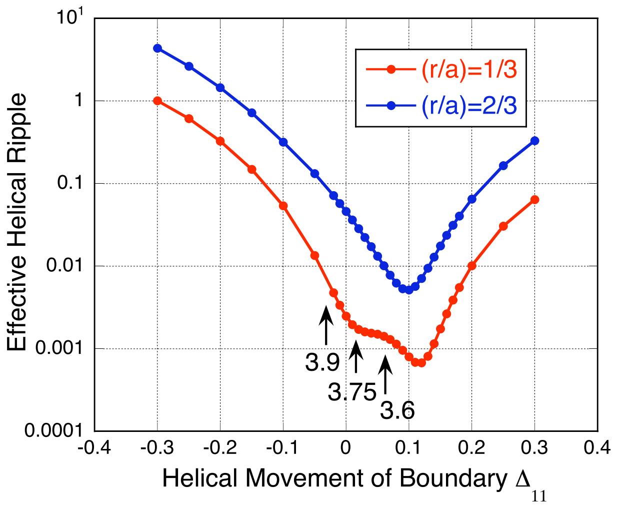

Fig. 3 Dependence of effective helical ripple on helical movement of the plasma boundary. Arrow positions correspond to three typical LHD configurations.

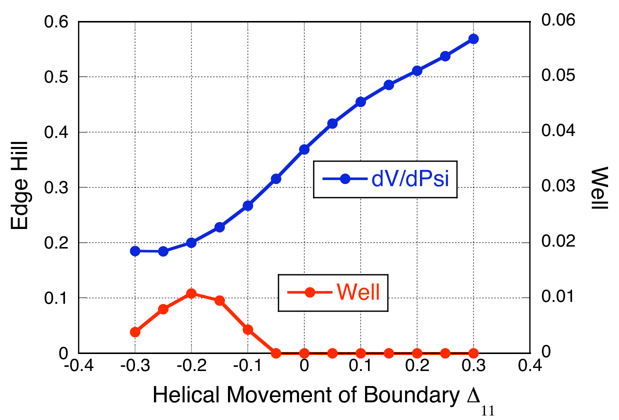

Fig. 4 Dependence of dV/dPsi at plasma edge and magnetic well depth on Δ_{11}.

However there are optimum levels of the helical movement of the boundary for both properties and a too much movement does not help the confinement.

For finite beta equilibria, the Shafranov shift always helps to create the magnetic well. The upper limit of Δ_{11} to give the magnetic well (-0.05 in Fig. 4) is increased.

4. Role of Helical Movement of Magnetic Axis in Other Stellarators

We apply these analytical processes on different type of stellarators. The distribution of Fourier modes of the boundary shape of Wendelstein 7-X is shown in Fig. 5 with the same definition as Fig. 2.

Compared to the LHD case, many additional elements are included. Among them, important elements that give special geometric features of Wendelstein 7-X are Δ_{-1-1} (taller violet column) for the crescent (bean) shape and Δ_{32} (green column) for the triangular shape. It has a larger helical movement of boundary Δ_{11} than LHD.

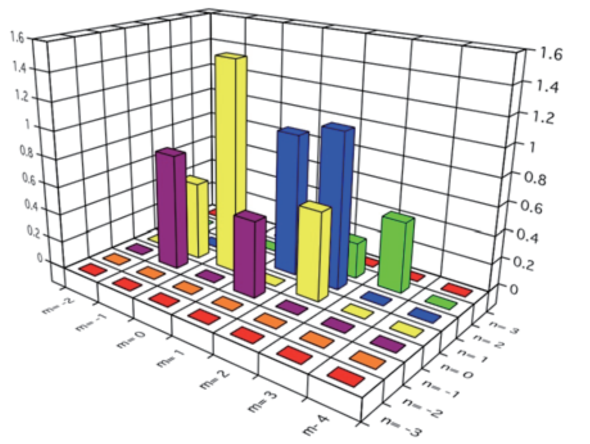

Fig. 5 Distribution of Fourier modes of Wendelstein 7-X stellarator.

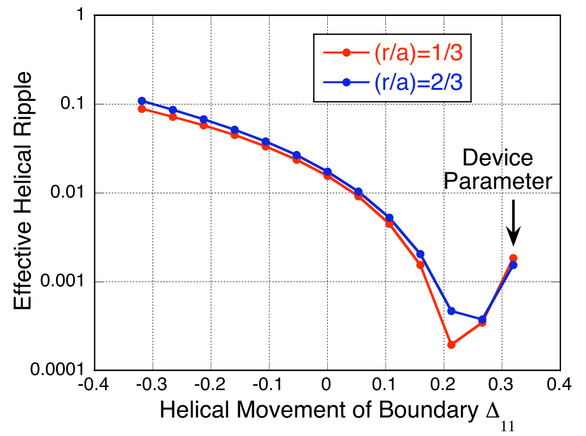

Fig. 6 Dependence of effective helical ripple on helical movement of the plasma boundary for W 7-X.

Figure 6 shows the dependence of the effective helical ripple on Δ_{11}. Since the profile of the effective helical ripple is flat for W 7-X, values for two minor radii are very close. The actual value of Δ_{11} for the vacuum configuration in device design is shown by arrow. Because of the Shafranov shift in the high beta equilibrium of W 7-X, the minimum level of the neo-classical transport would be obtained in the experiments.

The magnetic well depth and the specific volume at the boundary are shown in Fig. 7 as functions of Δ_{11}. Contrary to the LHD case, the magnetic well is produced in the positive range of Δ_{11}. It owes to the contributions of additional Fourier modes especially Δ_{-1-1} and Δ_{32}. Therefore W 7-X device has the good neo-classical transport and the MHD stability at the same time.

Finally we analyze the magnetic configuration of TJ-II stellarator [6], which is based on another stellarator concept of 'Heliac'. As is shown in Fig. 8, the distribution of the Fourier modes of boundary shape is very unique. It has a rotating crescent shape cross section and because the shape does not change for all toroidal position, all Fourier

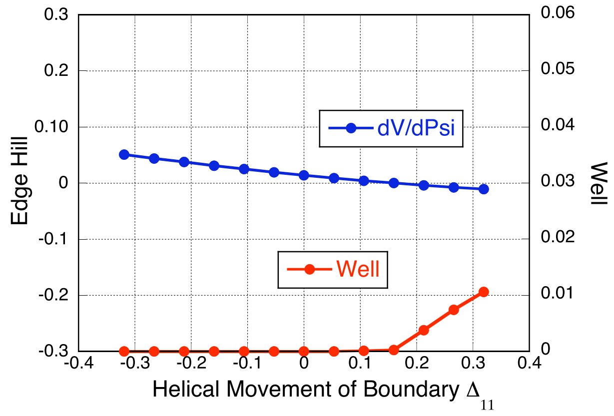

Fig. 7 Dependence of dV/dPsi at plasma edge and magnetic well depth on Δ_{11} for W 7-X.

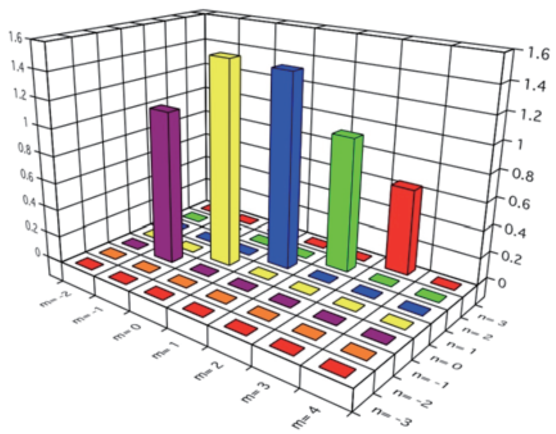

Fig. 8 Distribution of Fourier modes of TJ-II stellarator.

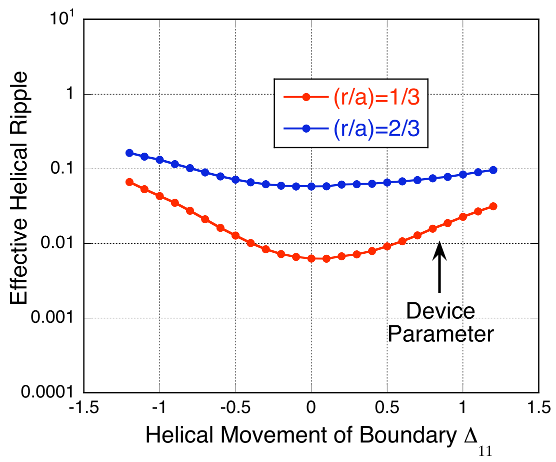

Fig. 9 Dependence of effective helical ripple on helical movement of the plasma boundary for TJ-II.

elements has the same m and n mode numbers.

The dependence of the effective helical ripple on Δ_{11} is shown in Fig. 9. Contrary to LHD and W 7-X cases, the variation of the neo-classical transport is small and the Δ_{11}

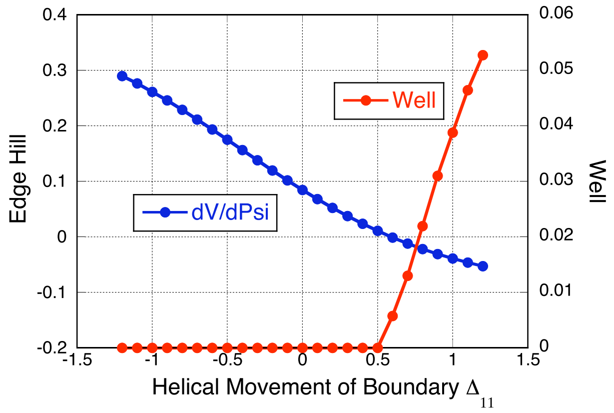

Fig. 10 Dependence of dV/dPsi at plasma edge and magnetic well depth on Δ_{11} for TJ-II.

value chosen as a device design parameter does not fit to the optimum point for the transport.

Figure 10 shows the dependence of the magnetic well depth and the specific volume at the plasma edge for TJ-II. It is shown that, in the TJ-II device design, the creation of the strong magnetic well is one of the most important physics targets.

5. Conclusion

By using a complex formula of Fourier mode expressions for the plasma boundary shape, the relation between the Fourier elements and the geometric features become clearer. Using such Fourier mode expressions, the dependence of the effective helical ripple and the magnetic well on the helical movement of the plasma boundary cross section was investigated for three different concepts of stellarators. It is commonly observed that a positive value of Fourier element of the helical movement is preferable for the improved neo-classical transport. However, the creation of the magnetic well depends on the cross section shapes. In finite beta equilibria, general dependences of the helical effective ripple and the magnetic well do not change but the curves in the figures are shifted. In conclusion, the helical movement of the magnetic axis is the most important key element for understanding the confinement characteristics of all stellarators of both planar and non-planar-axis.

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