Statistics of Clusters of Ionized Regions in the Simulation of Discharge using a Percolation Model*)

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We have developed a simulation code based on the percolation model to understand the stochastic behavior of discharge, which is observed in natural phenomena such as lightning as well as experimental discharges used for light sources and environmental technologies. We have developed a model of ionization and attachment to determine the distribution of ionized region in the gas, and carried out calculation of spatial and temporal evolution of discharge in SF_6. In addition, we discuss computational algorithms used to analyze the structure of the ionized region and its statistics in the plasma. The work presented in this article will also be useful for analyzing properties of the plasmas with spatially non-uniform structure.

Keywords: gas discharge, streamer, percolation, simulation, cluster

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1. Introduction

Discharge is a plasma phenomenon, which has been of interest not only as a subject of basic research, but also as a basis for a variety of applications. However, the sudden formation of a complex path, which is mainly observed in high-pressure discharges, has not yet been fully explained [1, 2]. For instance, discharge does not simply propagate along the electric field line, instead producing paths with branching and detouring. Recently, in a study of lightning discharge, ionization in air by cosmic rays has been shown to have a significant effect in initiating the discharge [3]. At certain conditions, electrons produced by cosmic rays can escape, traveling large distances to produce random ionized regions in the medium. We use a percolation model [4], assuming that the discharge occurs when connection is established between random ionized regions in the medium. The model reproduces such stochastic behaviors of discharges [5, 6]. In this paper, we discuss a method of modeling discharge as well as an application to the discharge in SF_6 for the calculation of initial partial discharge to the growth of a stepped leader [7]. We chose SF_6 because it is widely used in high voltage technologies and also because the discharge in SF_6 exhibits stochastic behaviors of breakdown voltage and turn-on time, as well as in its spatial and temporal structure [1]. Furthermore, we present an algorithm for analyzing the structure of the cluster of ionized region for the investigation of spatial and temporal properties of the discharge.

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2. The Model

2.1 Percolation model

The percolation model is a cell simulation model. The discharge medium is divided into cells. Two states - insulator and conductor ones - are considered for each cell, which correspond to neutral gas and plasma. This is accomplished by simplifying the atomic and molecular processes in the discharge medium.

Cells form a network of the electric circuit as shown in Fig. 1 (a). By solving the circuit equation, we can find the discharge current. We consider each cell as a circuit element, consisting of a serially connected plasma resistor and a spatial capacitor. A switch is connected in parallel to

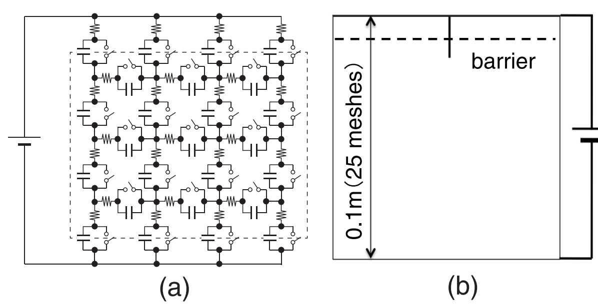

Fig. 1 (a) An equivalent circuit of the percolation model for the calculation of discharge current. (b) Geometry of the calculation using two-dimensional, 25 × 25 cells. DC voltage is applied between the upper and lower electrodes. A 3 unit long tip is placed at the center of the upper electrode.

the capacitor, which is turned on when the corresponding region of the medium is ionized and turns into the plasma. We ignore spatial inductance because we mainly consider laboratory-scale discharges.

We calculate the temporal evolution of discharge current in the medium. Figure 1 (b) shows the geometry of the calculation using 25 × 25 cells in the two-dimensional geometry. The voltage is applied between the upper and lower electrodes, and the upper electrode has a small protrusion. Each cell is assumed to ionize randomly; the cell to be ionized is chosen with some probability, and the distribution of ionized regions is updated. Subsequently, the discharge current is obtained by calculating the circuit equation.

2.2 Calculation of cluster statistics

In the percolation model, the ionized region formed by one or more ionized cell is called a cluster. In the investigation of the discharge, the position, size and shape of the clusters are of the interest. Therefore, we need an appropriate data format for representing the structure of clusters and the methods for manipulating the data.

2.2.1 Definition of a cluster

The clusters can be represented by a list of cells, however, using this simple representation, we must investigate the arrangement of the ionized areas over the entire calculation region at each step, when an ionized cell is produced or removed. In the present calculation, a cluster is determined by the collection of cells which are connected below the head cell, as shown in Fig. 2 (a). Each cell has zero or one parent cell and zero or more child cells. If the cell has no parent cell, it is defined as the head cell, which represents the cluster. This representation applies to any arrangement of the ionization region; usually more than one representation stands for an arrangement. The cluster, to which each cell belongs, can be found by finding the parent cell until the head cell is found. The number of clusters in the system can be found by counting the head cells. The size of the cluster can be calculated recursively by counting the number of children cells from the head cell.

2.2.2 Adding a cell

In the simulation of the percolation model, ionized cells may be added or removed at each time step, thus the statistics of the cluster need to be updated. When an ionized cell is added, we investigate the state of adjacent cells. If there are no ionized cells among them, the new cell is determined to be a new head cell, representing a new cluster. If a connection to an ionized cell, that is a cluster, is found, the new cell becomes a child of the adjacent ionized cell (Fig. 2 (b)).

In the case of more than one connection being found, if two adjacent cells are a part of the same cluster as shown in Fig. 2 (c) no more connection changes are needed, because each cell must have only one parent cell. However, connections to two clusters are found as shown in Fig. 2 (d), the new cell becomes the part of the either one of the two clusters, which refers to parent cluster, and then the other cluster is connected to the parent cluster as in Fig. 2 (e), by reversing the direction of the connection, i.e. switching the parent and child, recursively until its head.

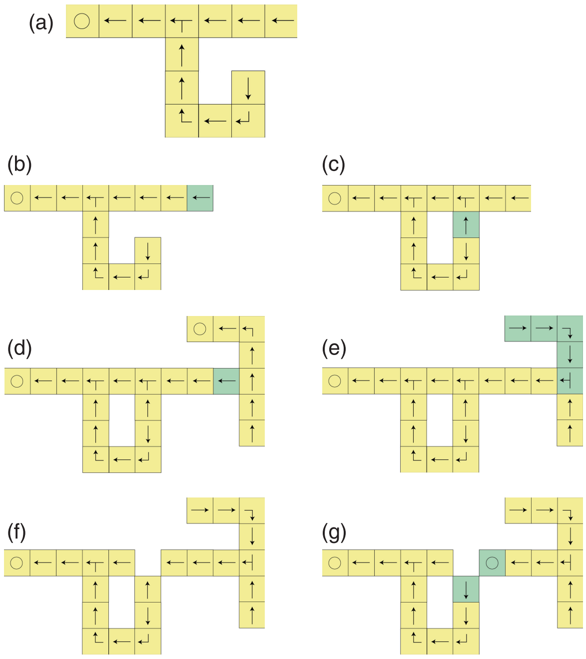

Fig. 2 The structure of the cluster of the ionized region, which is used in the present percolation model. (a) A cluster is represented by its head cell, and each cell has either zero or one parent cell and zero or more child cells. The operations needed for the maintenance of a cluster when adding a cell (b)-(e) and removing a cell (f,g) are shown.

2.2.3 Removing a cell

If an ionized cell is removed, the connection of the adjacent cells, which are the children cells of the removed cell, need to be investigated. If another connection to the existing cluster is found, the cell becomes the part of that cluster (Fig. 2 (f)). If no connections to the existing clusters are found, the cell becomes the head, representing a new cluster. This corresponds to a situation in which a cluster is separated into two clusters (Fig. 2 (g)).

3. Result

Figures 3 and 4 show the calculated current for the simple percolation, where a constant ionization rate for each neutral gas cell is applied. This calculation shows a known result, which is given by the theory of percolation, namely that breakdown occurs when the population of the ionized region exceeds the theoretical limit. Note that we consider the spatial capacity here, so that displacement current appears after each ionization event for each cell. As

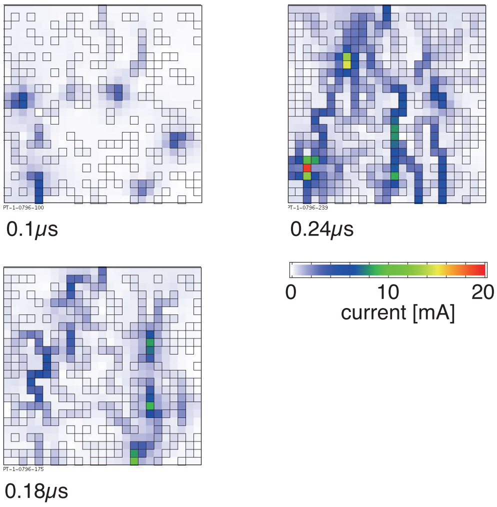

Fig. 3 The temporal evolution of the discharge path in simple percolation.

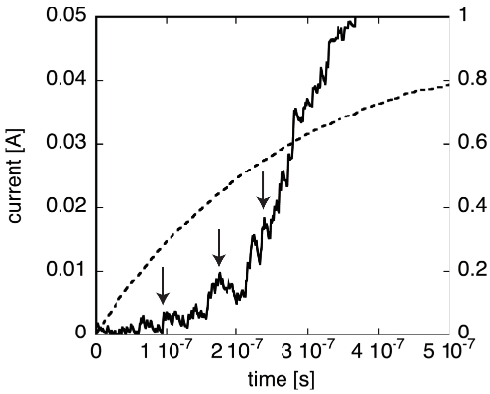

Fig. 4 The temporal evolution of the discharge current (solid line) and population of the ionized region in the discharge medium (dashed lines). Arrows indicate the time (0.1, 0.18, and 0.24 μs) for which the discharge path is shown in Fig. 3.

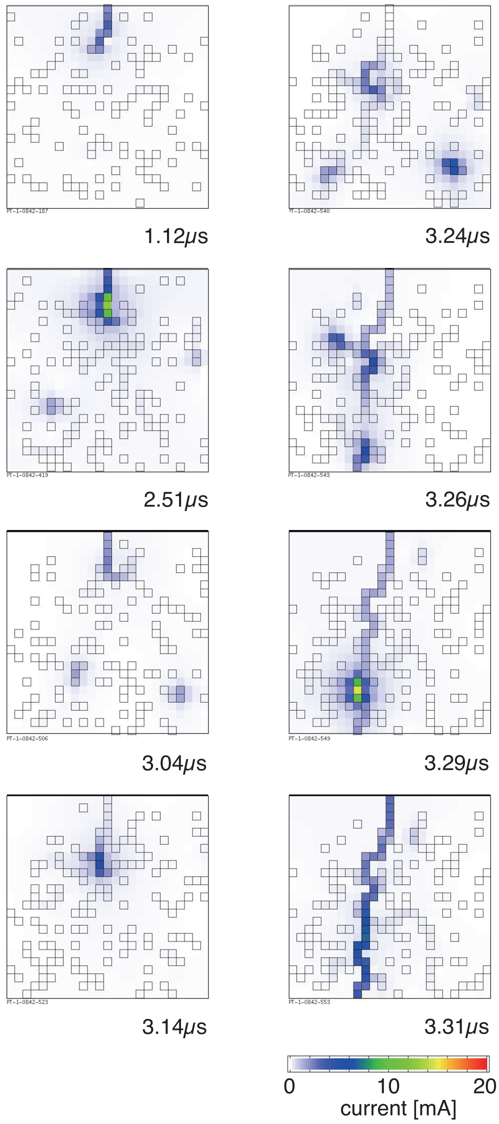

Fig. 5 Propagation of the streamer calculated taking the dependence of probability of ionization and attachment to the electric field into account.

shown in Fig. 3, the discharge path appears abruptly, and growth of the streamer is not observed.

Subsequently, we have carried out calculations for discharge in SF_6 gas, by taking the coefficient of ionization and attachment [8], the latter being the main process of electron loss in SF_6. The ionization and attachment coefficient become equal at the critical electric field of 360 Td. The ionization coefficient α per molecule, in 10^{-18} cm^2, is proportional to, and the attachment coefficient η is inversely proportional to the applied electric field E as,

$$\alpha = 35\frac{E}{360Td}, \tag{1}$$

$$\eta_0 = \frac{35}{E/360Td}. \tag{2}$$

Based on these ionization and attachment coefficients, we calculate the probability of production and elimination of the ionized region as a function of the local electric field. According to eqs. 1 and 2, the electron density in the medium and thus the probability of ionization increases as the square of the applied electric field, resulting in the growth of the streamer at its tip, where the strength of the electric field is enhanced. However, this also leads to the discharge path disappearing rapidly after its creation, because the electric field inside the channel is reduced, which suppresses ionization and enhances the attachment. Thus, we suppress the probability of attachment inside the channel.

$$\eta = \frac{\eta_0}{1 + j/j_0}. \tag{3}$$

As the discharge current exists, plasma temperature increases resulting in the reduction of the rate of attachment, which will contribute to sustain the channel.

Figure 5 shows the calculated temporal evolution of the discharge path. In this calculation, the electric field

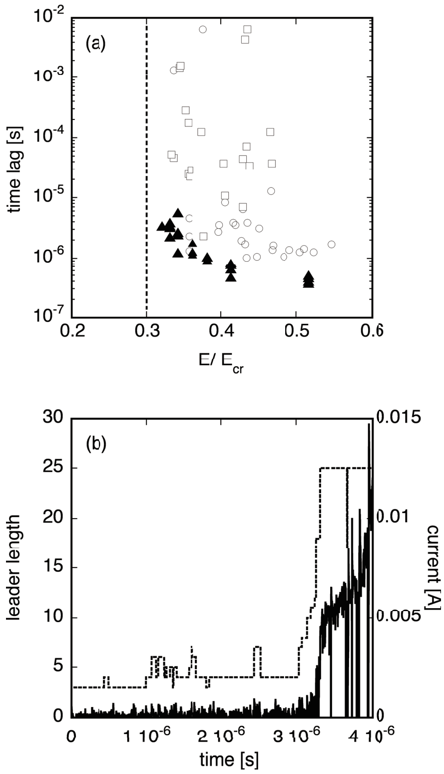

Fig. 6 The time lag of the discharge as a function of applied electric field normalized by the critical field (a). Open squares and circles correspond to experimental values in ref. [7]. Triangles are from present calculations. (b) The discharge current (solid line) and length of the stepped leader (dashed line) for the same calculation as displayed in Fig. 5.

dependence of ionization and attachment probabilities as well as the suppression of attachment are taken into account. Until $t = 3.0\,\mu s$, short streamers appear repeatedly near the tip of the electrode, which then starts to grow and breakdown occurs immediately. Any ionization or attachment event causes a current pulse, due to charging or discharging the spatial capacitors. The calculation reproduces initial partial discharge to the growth of the stepped leader, which is observed in experiments [7]. It has been shown experimentally that, as in this calculation, the probability of ionization increases as the electric field, and the ionization is more likely to occur at the tip of the electrode. When the streamer starts to grow, an enhancement of the electric field occurs at its tip, and the growth of the streamer is accelerated. Since the calculation is probabilistic, the results differ from one calculation to the next, however, repeated calculations show the trend of the breakdown. Figure 6 (a) shows the time for breakdown increasing as the applied voltage is decreased, and no breakdown occurs below the threshold voltage, which reproduces the experimental trend.

Figure 6 (b) shows the temporal evolution of the discharge current and the leader length, with steplike behavior. This arises from the fact that the cluster of ionized region grows not only from the tip of the electrode but inside the medium, along the electric field line and connecting with the region starting from the electrode.

4. Summary

We have presented a simulation of discharge based on the percolation model. We model the ionization and attachment of electrons in SF_6 gas, which is useful for reproducing the experimental properties of the discharge. To provide a technology useful for large-scale simulations, we also discuss the representation of the cluster of ionized region in the simulation and the associated computational methods, which are used in the simulation to investigate the temporal and spatial development of the discharge. The computer simulation program has been written in Java, based on object oriented programming using simple recursive algorithms, which will be described in detail elsewhere. The present method will also be useful for calculating the statistics of clusters from experimentally observed images.

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