Viscosity Effects on Explosive Growth Dynamics of the Nonlinear Resistive Tearing Mode*)

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A collapse of the X-point occurs above a critical island width, $\Delta' w_c$, in the resistive tearing mode for large instability parameter, Δ', leading to current sheet formation [N.F. Loureiro *et al.* Phys. Rev. Lett. **95**, 235003 (2005)]. In this study, we analyze this problem by including viscosity effects on the onset of the X-point collapse and the explosive nonlinear growth dynamics of the reconnected flux. While explosive growth seems to be independent of viscosity in the magnetic Prandtl number regime $Pr < 1$, a transition behavior is revealed at $Pr \approx 1$ for the viscosity dependence of $\Delta' w_c$, for the X-point collapse as well as the linear tearing instability. A secondary instability analysis, which included quasi-linear modifications of the equilibrium current profile due to the zonal current, shows that current peaking is plausibly responsible for the onset of the X-point collapse and the explosive growth of reconnected flux, which leads to the current sheet formation.

Keywords: resistive tearing mode, viscosity, X-point collapse, critical island width, secondary instability analysis

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1. Introduction

Magnetic reconnection [1, 2] is a ubiquitous plasma process, which is probably responsible for the main mechanism behind many astrophysical phenomena, such as solar flares, and sawtooth crashes in magnetic fusion plasmas. Early tearing mode theory, such as the Sweet–Parker model based on the single fluid resistive magnetohydrodynamics (MHD), gives low reconnection rates that are an order of magnitude slower than observations. To explain the fast reconnection process, resistive tearing mode theories, other than the Hall MHD or anomalous resistivity theories, have been extended to the regime with a large instability parameter Δ' and low resistivity η. During the nonlinear evolution of a tearing mode, an explosive growth of reconnected flux has been observed after the X-point collapse [3]. However, the effects of viscosity were not considered in that study.

Viscosity is a dissipative mechanism that is as important as resistivity in many applications. Furthermore, the viscosity is not always weaker than resistivity in laboratory and astrophysical plasmas because micro-scale turbulence may cause an anomalous viscosity [4–6]. The anomalous viscosity is usually larger than the collisional viscosity and strongly depends on the plasma temperature. Consequently, the magnetic Prandtl number $Pr = \mu/\eta$, (where η represents the viscosity) sharply increases with plasma temperature and can be estimated to be $Pr \propto T^3$. For example, for a typical fusion plasma of $T \sim 1\,\mathrm{keV}$, Pr can easily be of the order of 100.

Porcelli [7] highlighted the role of viscosity by carrying out a comprehensive linear analysis of the tearing mode. It has been shown that in the limit of low Δ', the usual tearing scaling of the growth rate $\gamma \sim \eta^{3/5}$, is modified in the presence of a finite viscosity and scales as $\gamma \sim \eta^{2/3} Pr^{-1/6}$, which is called the visco-tearing mode. In the limit $\Delta' \to \infty$, the growth rate scales as $\gamma \sim \eta^{1/3} Pr^{-1/3}$, which is called the visco-resistive kink mode. These scalings have been numerically confirmed in different regimes [8]. Stability analyses of the visco-resistive tearing mode showed a small threshold of Δ' at moderate values of η and $Pr \sim O(1)$ [9]. Moreover, viscous effects are important in tearing modes with shear flows [10, 11].

In this study, we simulate the nonlinear evolution of the resistive tearing mode including viscosity effects. The focus is on the Pr dependence of the critical island width $\Delta' w_c$ for the X-point collapse and on the explosive growth of reconnected flux, which show a transition at $Pr \approx 1$. The growth rate of the reconnected flux in the speed-up stage remains unaffected until $Pr > 1$, after which it decreases with viscosity, as predicted by Park *et al.* [12]. A secondary instability analysis is proposed to understand the explosive growth of reconnected flux, which corresponds to the nonlinear current sheet formation.

2. Model and Simulation

The nonlinear tearing mode can be simulated using reduced MHD (RMHD) equations in slab geometry as follows

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$$\partial\psi/\partial t = -[\phi, \psi] + \eta\nabla^2\psi, \quad (1)$$

$$\partial(\nabla^2\phi)/\partial t = -[\phi, \nabla^2\phi] + [\psi, \nabla^2\psi] + \mu\nabla^2(\nabla^2\phi), \quad (2)$$

which describe the evolution of the magnetic flux ψ and stream function ϕ. The total magnetic field is $B = B_z e_z + e_\times \nabla\psi$, where e_z is a unit vector along the guiding field. The equilibrium configuration is given by $\psi_0(x) = 1/\cosh^2(x)$ [3]. A finite difference method is employed in the x-direction with box size $[-5, 5]$ and mesh number 2048. Fourier transformation is applied in the y-direction with total poloidal mode numbers in the range $m = 30$ - 90 and box size $[0, 2\pi L_Y]$. The key parameters in the simulations are η, μ, and Δ', which is closely related to the wave number $k_Y = m/L_Y$.

Nonlinear simulations are carried out for different Δ'. When Δ' is low, kinetic and magnetic energies show linear growth followed by the Rutherford regime. The magnetic island is saturated at a small width. However, for large enough Δ', the evolution of both magnetic and kinetic energies shows abrupt growth after the Rutherford regime instead of the saturation. A typical example is shown in Fig. 1 (a) for $Pr = 1$. The abrupt growth process is more clearly evidenced by the nonlinear growth rates of both the kinetic and magnetic energies, as shown in Fig. 1 (b). The instantaneous nonlinear growth rates pass through a minimum and grow again due to the X-point collapse, instead of saturating. Once the island width exceeds a certain critical value, w_c, the X-point configuration collapses, leading to the formation of a current sheet. The critical island width is important in understanding the explosive growth behavior of the tearing mode since it may act as a precursor to the speed-up stage.

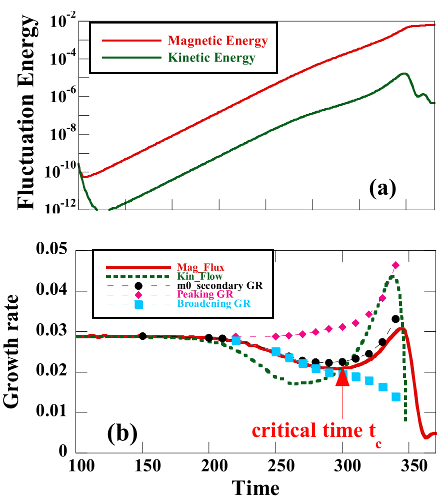

Fig. 1 (a) Evolution of kinetic and magnetic energies for a typical tearing mode. (b) Evolution of nonlinear growth rates (solid and dashed) and secondary growth rates (circle, square, and diamond). $\mu = \eta = 2.8 \times 10^{-4}$, $\Delta' = 17.3$.

3. Viscosity Dependence of Tearing Mode Dynamics

3.1 Viscosity dependence of linear growth rates

Firstly we investigate the linear growth rate scaling with viscosity, by performing linear simulations for given values of Δ' and η over a broad range of viscosities. Figure 2 shows the linear growth rate dependence on viscosity in terms of Pr, for four fixed values of η. Increasing the viscosity generally reduces linear growth rates due to the dissipation effect, which opposes the resistive destabilization [7]. However, the linear growth rate scaling shows a slight transition at $Pr = 1$. The linear growth rates for $Pr > 1$ decrease with the viscosity faster than those in the region $Pr < 1$; similar to observations in the Harris current sheet configuration [8]. Such behavior of the linear tearing mode may affect nonlinear evolution of the magnetic island, which is to be discussed below. The scaling of the linear growth rate versus Pr is measured to be $\gamma_{\text{lin}} \sim Pr^{-1/5}$, which differs somewhat from the theoretical prediction of $\gamma_{\text{lin}} \sim Pr^{-1/6}$, that applies for very small Δ' and large Pr [9].

3.2 Viscosity dependence of critical island width

The X-point collapse signifies the onset of the explosive growth of nonlinear resistive tearing mode at a critical island width, $\Delta' w_c$, which corresponds to the minimum instantaneous growth rate of the magnetic flux in Fig. 1 (b) ($t \sim 300$). To study the viscosity effect on the nonlinear evolution of the magnetic island, we perform nonlinear simulations in which the viscosity is varied in a way similar to the parametric scan done for Fig. 2. In Fig. 3, the critical island width $\Delta' w_c$ is plotted against the Pr for different η with a given Δ'; the figure shows a linear dependence of $\Delta' w_c$ on Pr (or viscosity). However, $\Delta' w_c$ is inversely proportional to Pr (or viscosity) for $Pr < 1$, while it is proportional to Pr (or viscosity) for $Pr > 1$. An evident

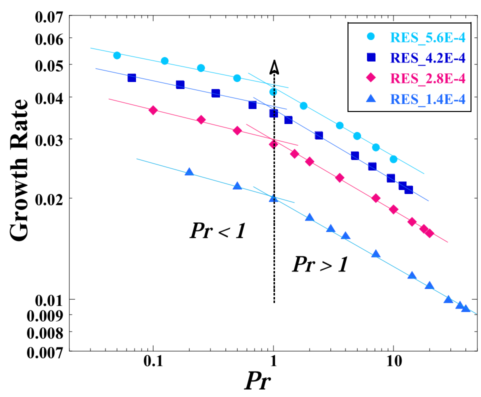

Fig. 2 Linear growth rates versus Pr for four different resistivities at $\Delta' = 17.3$. The reference dashed line with arrow labels the scaling transition at $Pr = 1$.

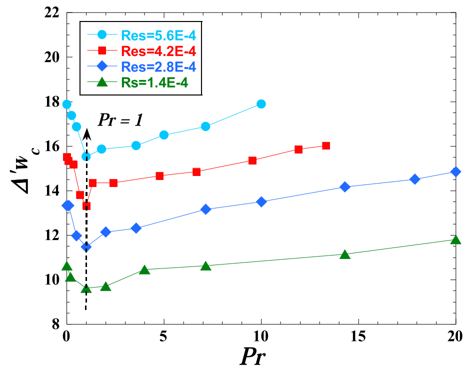

Fig. 3 Critical island width $\Delta' w_c$ versus Pr for four different resistivities at $\Delta' = 17.3$.

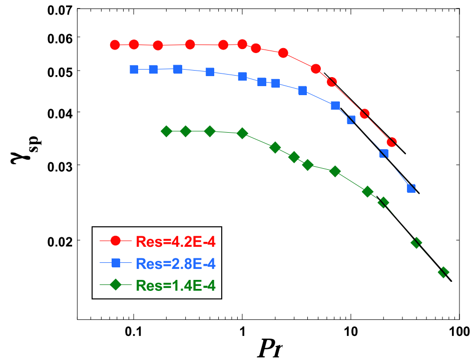

Fig. 4 Growth rates of reconnected flux in the explosive growth phase versus Pr for three different resistivities. The reference solid lines mark the power scaling for $Pr \gg 1$.

scaling transition occurs at $Pr = 1$. For a given viscosity, the critical width $\Delta' w_c$ increases as the resistivity increases, similar to the behavior of the resistive tearing mode [3].

The underlying mechanism of the transition behavior $Pr = 1$ at may be qualitatively understood as different responses of the magnetic island evolution to viscosity. For $Pr < 1$, the viscosity primarily increases the width of the resistive layer so the X-point collapse occurs easily, probably leading to a small $\Delta' w_c$ for current sheet formation. However, a large viscosity mainly slows inflows and outflows to dampen the tearing mode. The dissipation mechanism reduces the reconnected flux, leading to a wider island threshold necessary for the X-point collapse. This tendency is consistent with the scaling laws of the linear growth rates in Fig. 2, which show a weak stabilization for $Pr < 1$ and strong dissipation for $Pr > 1$.

3.3 Viscosity effects on magnetic reconnection

The X-point collapse starts after the island width exceeds a critical value, leading to abrupt growth of both kinetic and magnetic energies, as shown in Fig. 1. It is interesting to investigate viscosity effects on the magnetic reconnection in this explosive growth phase. Park *et al.* [12] predicted that the viscosity could modify the Sweet–Parker type reconnection, leading to a scaling of the reconnection rate as $\dot{\psi}_s \sim \eta^{1/2} Pr^{-1/4}$ for $Pr \gg 1$. Here $\dot{\psi}$ is the change rate of the magnetic flux at the X-point.

To more clearly reveal the viscosity effects on the magnetic reconnection, we evaluate the scaling of the growth rate of reconnected flux, γ_{SP} [3], which is defined by $\psi_{rec} - \psi_{coll} = \exp(\gamma_{SP}(t - t_c))$. Here, ψ_{rec} is the reconnected flux, which is measured as the difference between the maximal and minimal fluxes through the X-point along the current sheet. ψ_{coll} is the reconnected flux at time t_c, which corresponds to the X-point collapse. For the case without viscosity, we confirm the Sweet–Parker scaling, $\gamma_{SP} \sim \eta^{1/2}$. However, the viscosity modifies this scaling by changing the exponent 1/2. Most importantly, γ_{SP} is independent of Pr (or viscosity) for $Pr < 1$ over a wide range of resistivities, as shown in Fig. 4. Values of γ_{SP} moderately decrease when $Pr \approx 1$. However, the viscosity effect is prominent for $Pr \gg 1$, leading to the scaling law $\gamma_{SP} \sim Pr^{-1/4}$ over a wide resistivity range. Hence, similar to the linear growth rate and critical island width, γ_{SP} in the explosive growth phase also exhibits a transition behavior at $Pr \approx 1$.

4. Secondary Instability Analysis for Explosive Growth Dynamics

Although we have discussed the viscosity dependence of the reconnection rate in the abrupt nonlinear growth phase, the physical mechanism leading to explosive growth is still an open issue. Here, we conduct a secondary instability analysis to explore the origin of this problem. It is proposed that, after the Rutherford regime, the equilibrium current is modified by the zonal current [13, 14], and this may cause a secondary instability. Such instability may possibly trigger the X-point collapse for the current sheet formation. The nonlinear positive feedback of current sheet formation leads to explosive growth of the reconnected flux. To testify this idea, a stability analysis is carried out in a quasi-linear equilibrium involving the zonal current, which is taken directly from the nonlinear simulations [15, 16]. This means that the new equilibrium, as shown in Fig. 5 (a), is composed of the initial current (or flux) at time $t = 0$ and the zonal current (flux) at time $t = t_0$ during the explosive growth phase, i.e.,

$$\psi_E(x, y, t) \sim \psi_0(x, t = 0) + \psi_{m=0}(x, t = t_0), \tag{3}$$

$$J_E(x, y : t) \sim J_0(x, t = 0) + J_{m=0}(x, t = t_0). \tag{4}$$

We perform quasi-linear simulations using Eq. (1) and (2). We found that the secondary growth rate due to the zonal current modification is comparable with the instantaneous nonlinear growth rate of magnetic flux in the explosive growth phase, as shown in Fig. 1 (b). This suggests that the secondary instability may plausibly be responsible for the trigger of the X-point collapse and the explosive growth of the nonlinear tearing mode, leading to current

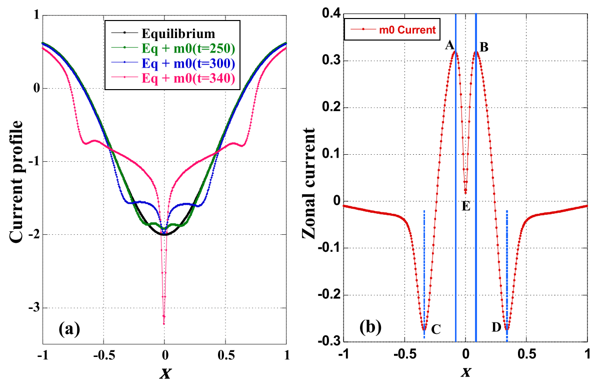

Fig. 5 (a): Current profiles modified by the zonal current at different times. (b) Structure of zonal current at time $t = 300$, corresponding to the nonlinear simulation in Fig. 1. The reference solid lines locate the local peaking region. Both reference dashed and solid lines (left or right) label the broadening regions.

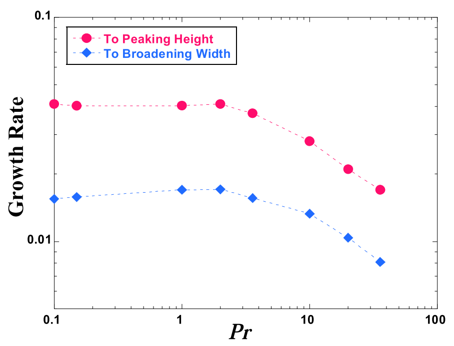

Fig. 6 Growth rates of the peaking height δ_h and broadening width δ_w versus Pr during the explosive growth phase. $\varDelta' = 17.3$ and $\eta = 2.8 \times 10^{-4}$.

sheet formation.

To clarify why the quasi-linear current modification could provide a mechanism to drive the explosive growth, we examine the roles of local and global structures of the zonal current. In Fig. 5 (a), the current profile tends to peak near the resonant surface and to be broad far from the resistive layer, as the magnetic grows explosively. We separately inspected the effects of current peaking and broadening on the secondary instability. The two parts are identified in Fig. 5 (b), by the two reference solid lines, which correspond to the locations with zero radial gradients in the zonal current. The peaking and broadening effects are represented by the peaking height δ_h (the difference in the zonal current amplitude between points A and E), and the broadening width δ_w (the projection in the x-direction between points A and C or points B and D). Nonlinear simulations show that the instantaneous δ_h and δ_w in the explosive growth phase exponentially grow (The figure is not presented here). The growth rates plotted in Fig. 6 decrease with increasing Pr for $Pr > 1$ but are almost independent of Pr (or viscosity) for $Pr < 1$. By separately including the

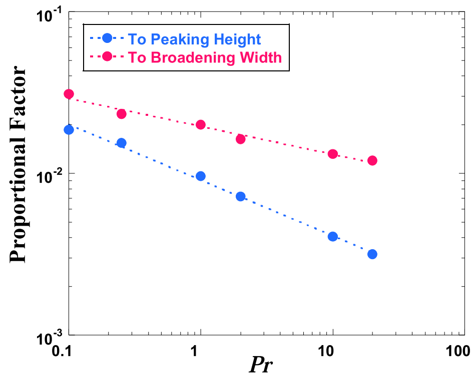

Fig. 7 Proportional factor of the secondary growth rates to peaking height or broadening width for cases with only peaking or broadening effect. $\varDelta = 17.3$ and $\eta = 2.8 \times 10^{-4}$.

peaking and broadening parts in the secondary instability analyses, calculations show that local current peaking remarkably destabilizes the secondary tearing mode, while global broadening plays a strong stabilizing role, as illustrated in Fig. 1 (b). Growth rates of the secondary instability are proportional (or inversely proportional) to δ_h (or δ_w) for a given viscosity. The combination of these two processes may lead to an explosive growth (i.e., exponential of exponential growth) of fluctuations. The proportional factor for the secondary growth rate to δ_h (or δ_w) decreases with increasing Pr, as shown in Fig. 7 for $\varDelta' = 17, 3$ and $\eta = 2.8 \times 10^{-4}$, and shows no remarkable transition behavior with changes in Prandtl number. This may partially correspond to the scaling of linear growth rates in Fig. 2, which depends on Pr but with a slight transition for $Pr < 1$. Comparisons with the nonlinear growth rate of the instantaneous δ_h and δ_w, as shown in Fig. 6, may imply that the explosive growth of the resistive tearing mode is purely nonlinear process in nature, although the quasi-linear secondary instability may provide a plausible trigger mechanism. Further nonlinear analysis is in progress.

5. Conclusions

We performed a detailed investigation of viscosity effects on the linear and nonlinear evolution of resistive tearing modes over a wide range of parameters. We found that both the linear growth rate and critical island width for the X-point collapse exhibit a transition behavior at $Pr \approx 1$. In the explosive growth phase of magnetic reconnection, viscous effects are significant only for large Prandtl numbers $Pr > 1$; this validates the results of Park *et al.* [12]. The secondary instability analysis with quasi-linear current profile modifications due to the zonal current suggest that current peaking around the rational surface may be a plausible cause for the trigger of the X-point collapse and the explosive growth of the reconnected flux, which leads to the nonlinear current sheet formation.

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