Identification of Quasi-Periodic Nonlinear Waveforms in Turbulent Plasmas

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(Received 18 November 2013 / Accepted 24 January 2014)

A new method is presented for identifying waveforms of fluctuations in turbulent plasmas. The method is based on heartbeat analysis in which the convolution of a waveform is obtained by employing the phase tracking method. Phase tracking is performed by correlating raw time-series data with a template waveform; the template is evaluated through iteration procedure. The method is applied to fluctuations in a PANTA plasma, and the nonlinear waveform and its distribution of periods are obtained.

Keywords: fluctuation, phase tracking, plasma turbulence, conditional averaging

DOI: 10.1585/pfr.9.1201016

The identification of nonlinear waveforms is an essential task in the study of turbulent plasmas. The waveform, which includes information on higher harmonics (i.e., amplitude as well as phase), is the fundamental information that reflects nonlinearity in plasmas. A standard experimental approach is the convolution (i.e., conditional averaging) of time-series data. This approach has been applied to turbulent plasmas in the LMD-U device, and this has led to identification of large-amplitude solitary wave structures [1]. However, simple convolution within a prescribed period is not always sufficient because the period of oscillations can change over time. Intermittent variations in fluctuation periods, which often occur in turbulent plasmas, require a new method for identifying nonlinear waveforms.

In this article, a new method is presented for identifying the waveforms of perturbations in turbulent plasmas. Convolution of a waveform is performed by employing the phase tracking method, which is based on heartbeat analysis [2]. The method is applied to fluctuations in PANTA plasmas [3], and the nonlinear waveform and its distribution of periods are obtained.

At this point, we explain the new method. Figure 1 (a) illustrates a time series of observed fluctuations for a floating potential, which was observed in PANTA plasmas [1]. In this case, the perturbation is quasi-periodic, as can be seen in the power spectrum in Fig. 1 (b). In this dataset, the central frequency occurs at 1.1 kHz, but the half width is large. Owing to this large half width, which denotes temporal variations in the phase of the observed waveform, the standard convolution does not suffice.

Herein, we propose to perform phase tracking by analyzing the correlation of the time-series data with a template; the template is constructed by iteration. In this case, the template converges after only a few iterations.

We form an initial template, $f_{j=0}(t')$ $(-T/2 < t' < T/2)$, by choosing a cosine function having the fundamental period of the raw fluctuation signal; here, T is the period of the fundamental frequency. The final template is constructed by iteration; j is the number of iterations. Figure 1 (c) shows an example of the template at the second iteration ($f_2(t')$). The correlation of the raw signal $F(t)$ with the j-th template $f_j(t')$ is calculated by

$$C_j(t) = \int_{-T/2}^{T/2} (F(t-t') - \bar{F}(t)) \cdot (f_j(t') - \bar{f}_j)\mathrm{d}t' / \sigma_F(t)\sigma_f, \quad (1)$$

where

$$\bar{F}(t) = \int_{-T/2}^{T/2} F(t-t')\mathrm{d}t', \quad \bar{f}_j = \int_{-T/2}^{T/2} f_j(t')\mathrm{d}t',$$

$$\sigma_F^2 = \int_{-T/2}^{T/2} \left(F(t-t') - \bar{F}(t)\right)^2 \mathrm{d}t',$$

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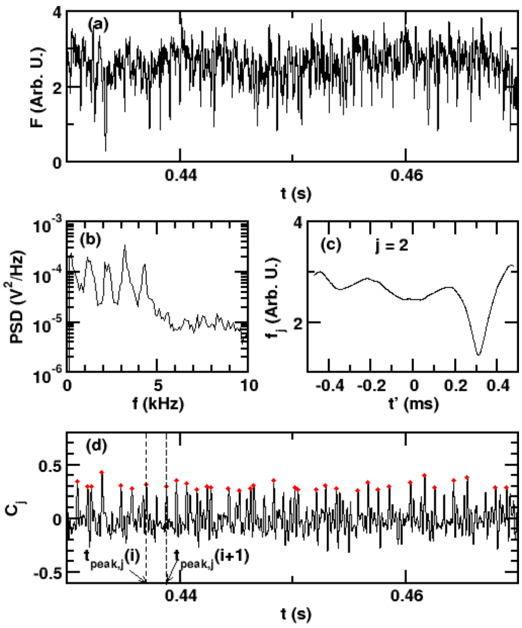

Fig. 1 Intermediate results during an analysis. (a) Reference signal, (b) its power spectrum density (PSD), (c) nonlinear waveform (template) at the second iteration, and (d) cross-correlation function between $F(t)$ and $f_j(t')$. Red circles in (d) denote detected peaks.

$$\sigma_f^2 = \int_{-T/2}^{T/2} \left(f_j(t') - \bar{f}_j\right)^2 \mathrm{d}t'.$$

We introduce a threshold for "peaks" in $C_j(t)$ and identify peaks in $C_j(t)$, as shown in Fig. 1 (d). The times of these peaks are used to track the phase. The i-th peak time for the j-th iteration, $t_{\mathrm{peak},j}(i)$, is used for conditional averaging:

$$f_{j+1}(t') = \sum_i F(t - t_{\mathrm{peak},j}(i))/n,$$
$$(-T/2 \le t' = t - t_{\mathrm{peak},j}(i) \le T/2), \qquad (2)$$

where n is the number of peak times. This provides the waveform, which is projected from the raw data with the help of the template. The iteration of the template is performed until f_{j+1} converges.

Figure 2 (a) shows the waveforms (templates) obtained after the first, second, third, and tenth iterations. Convergence is fast. The number of peaks in the correlation $C_j(t)$ (within a time window of 200 ms) for these iterations are shown in Fig. 2 (b). In Fig. 2 (a), the correlation for $j = 0$ (C_0) is similar to a cosine function; thus, it significantly differs from that for $j > 0$. The number of peaks in C_j converges for iterations $j \ge 2$. This method also provides the temporal evolution (change) in the phases of nonlinear waves. The peak in the time series of $C_j(t)$ is used to identify the phase of nonlinear waves in turbulent fluctuations. Figure 3 shows the analysis of the period of oscillations. The instantaneous period is obtained by measuring the time difference between two consecutive peaks in $C_j(t)$. The distribution of instantaneous periods in Fig. 3 shows a dominant peak at the fundamental frequency. However, the peak at twice the fundamental period is also pronounced. Thus, this analysis provides a method for finding period doubling, which might occur in an intermittent manner.

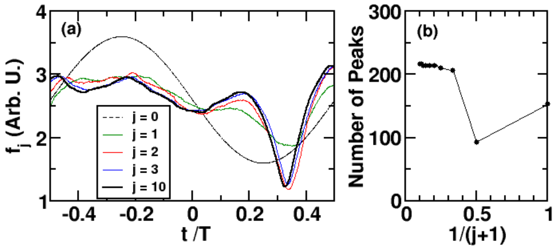

Fig. 2 Convergence of the template. (a) Templates at four different iterations and (b) the number of peaks in C_j as a function of iterations.

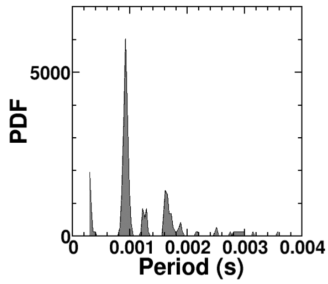

Fig. 3 Normalized probability density function (PDF) of period.

In this article, a new method for identifying nonlinear waveforms of fluctuations in turbulent plasmas is explained. The convolution of a waveform is performed by employing the phase tracking method. An initial guess for the nonlinear waveform (template) is made and then refined on subsequent iterations; convergence is found to occur quickly. The method was applied to fluctuations observed in the PANTA plasma, and the nonlinear waveform and the distribution of periods were evaluated. This method will be useful, e.g., for studies on turbulent transport [4] and period doubling. Results from such applications will be reported in future publications.

This work is partly supported by the Grant-in-Aid for Scientific Research of JSPF, Japan (21224014, 23244113, and 23360414) and the collaboration programs of the RIAM of Kyushu University and Asada Science Foundation.

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