Stability of Double Tearing Mode in the Presence of Shear Flows*)

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The stability of two eigen states of double tearing mode (DTM) with symmetric or antisymmetric islands in the presence of shear flows is numerically simulated based on a reduced MHD model in slab geometry. For given antisymmetric flow profile, a degenerated state is observed at a critical flow amplitude v_c. Below v_c, the shear flow stabilizes the DTM with antisymmetric islands and destabilizes the other one through distorting the magnetic flux mainly governed by the global effect of flow profile. Above v_c, the degenerated state bifurcates into two eigen states with the same growth rate but opposed propagating direction. These two eigen modes show single tearing mode structure due to one of two islands is prevented by the Alfvén resonance (AR). However, the AR can destabilize the DTMs through enhancing the inflow to the X-point of the remaining island, then competing with the stabilization of local flow shear, leading to distinctive features of DTM eigen states.

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1. Introduction

Recently, the evolution of the double tearing mode (DTM), which is a very violent current driven resistive magnetohydrodynamics (MHD) instability, has received much attention since it may be responsible for the fast reconnection in the astrophysical plasmas and the major disruption in magnetic fusion plasmas. Theoretical investigations show that the DTM consists of two different island states with antisymmetric or symmetric islands located on each current sheet, corresponding to eigen states with even or odd parity of magnetic perturbations [1], which are referred to as even or odd DTM. Generally, the even DTM is more unstable than the odd one which is usually ignored in previous studies. The explosive growth phase during the nonlinear evolution of the DTM, leading to the deterioration of global equilibrium and confinement, has been explained as a new type of secondary instability [2–4]. On the other hand, the off-axis sawtooth crashes caused by the nonlinear DTM reconnection have been observed in tokamak experiments [5]. In order to suppress such events, sheared plasma flows/rotations have been considered to be one of plausible candidates. It has been observed that the shear flow between two current sheets can decouple the two islands, then suppress the island growth effectively [6]. More recently, a new mechanism, namely, Alfvén resonance (AR), has been proposed to understand the stabilization effect of the shear flow on the DTM in cylinder geometry [7]. Thus, understanding the role of the shear flow in suppressing the DTM instability is of significant importance. Particularly, for the linear DTM in the presence of antisymmetric poloidal shear flow, Voslion et al. have simulated the evolution of the DTM as an initial-value problem [8]. It has been observed that weak shear flow under a critical value v_c stabilizes the DTM. In this paper, we focus on the fluctuation characteristics of both even and odd DTM eigen states with different external shear flow, as well as the role of local flow shear and AR in the stabilization/destabilization process.

2. Physical Model and Analysis Method

The evolution of the DTM in the presence of shear flows can be numerically simulated based on a reduced resistive MHD (RMHD) model in slab geometry [9]. The linearized RMHD equations,

$$\begin{aligned}\partial_t \nabla_\perp^2 \tilde{\phi} = -v_{eq}\partial_y \nabla_\perp^2 \tilde{\phi} + \partial_y \tilde{\phi} \partial_x^2 v_{eq} \\ + B_{eq}\partial_y \nabla_\perp^2 \tilde{\psi} - \partial_y \tilde{\psi} \partial_x^2 B_{eq},\end{aligned} \tag{1}$$

$$\partial_t \tilde{\psi} = -v_{eq}\partial_y \tilde{\psi} + \partial_y \tilde{\phi} B_{eq} + \eta \nabla_\perp^2 \tilde{\psi}, \tag{2}$$

govern the linear behavior of the magnetic flux, ψ, and electric potential, ϕ. They are subject to the magnetic field and flow through $\boldsymbol{B} = B_0\hat{z} + \hat{z}\times\nabla\psi$ and $\boldsymbol{v} = \hat{z}\times\nabla\phi$, where B_0 is a strong constant guiding magnetic field and $\hat{z}$ is the unit vector in z direction. In these equations, the coordinates x and y, magnetic field $\boldsymbol{B}$, velocity $\boldsymbol{v}$, and time t are normalized by a, B_0, v_A and τ_A, where a is the scale length of equilibrium magnetic field, $v_A = B_0/\sqrt{4\pi\rho_0}$ is the Alfvén velocity and $\tau_A = a/v_A$ is the Alfvén time. The resistivity η is normalized as $\eta/v_A a$. For the equilibrium magnetic

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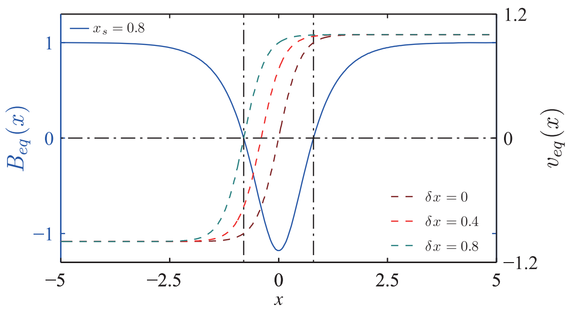

Fig. 1 Equilibrium magnetic field $B_{\mathrm{eq}}(x)$ with $x_{\mathrm{s}} = 0.8$ (solid curves) and shear flow profiles $v_{\mathrm{eq}}(x) = v_0 \tanh(\kappa(x + \delta x))$ with $\delta x = \{0,\ 0.4,\ 0.8\}$ (dashed curves).

field, a double Harris current sheet configuration [10],

$$B_{\mathrm{eq}} = 1 - (1 + b_{\mathrm{c}})\mathrm{sech}(\zeta x), \tag{3}$$

is employed as shown in Fig. 1, where b_{c} is chosen to keep the local magnetic shear $\hat{s} = B'_{\mathrm{eq}}(x_{\mathrm{s}}) = \pi/2$, ζ is taken to satisfy $\mathrm{sech}(\zeta x_{\mathrm{s}}) = 1/(1 + b_{\mathrm{c}})$. Here two current sheets, flowing oppositely along the z direction, exist at $\pm x_{\mathrm{s}}$. The shear flow, $v_{\mathrm{eq}}(x)$, is expressed as

$$v_{\mathrm{eq}} = v_0 \tanh[\kappa(x + \delta x)], \tag{4}$$

also shown in Fig. 1. Here v_0 is the amplitude of the shear flow, κ determines the flow shear and δx is used to modify the local flow shear on $x = -x_{\mathrm{s}}$, left current sheet. In the following, $x_{\mathrm{s}} = 0.8$, $\eta = 5 \times 10^{-4}$, $\kappa = 2$ and $\delta x = [0,\ 0.8]$ are set. In this work, the RMHD Eqs. (1) and (2) are solved by employing the eigenvalue analysis using an eigenvalue solver. Note that usually the DTM with the lowest mode number, $m = 1$, is most unstable, which is also responsible to the magnetic island formation. Hence, the $m = 1$ mode is mainly discussed in this work.

3. Results of Eigenvalue Analysis

An eigenvalue solver is developed to solve the linearized RMHD Eqs. (1) and (2). Figure 2 plots the linear growth rates and real frequencies of two branches of the DTM as a function of the flow amplitude v_0 with $\delta x = 0$ as a typical case. Note that in this case a degenerated state of two DTMs is observed at a critical flow amplitude v_{c}, below and above which the behavior of the eigen modes is totally different. To elucidate the influence of the shear flow, in Fig. 3 we pictorialize the magnetic islands structures of the two DTM branches for cases with the flow amplitude below and above v_{c}, corresponding to the points A-D marked in Fig. 2. Below the critical value v_{c}, the even and odd DTM structures can still be traced. We label the two branches to the even and odd DTMs in terms of the structure in the case without external flow. In this region, the even DTM is more unstable than the odd one [1], but they all keep static with zero real frequency as shown in the inset of Fig. 2 (b). Interestingly, the odd DTM is destabilized by the shear flow. The stabilization or destabilization

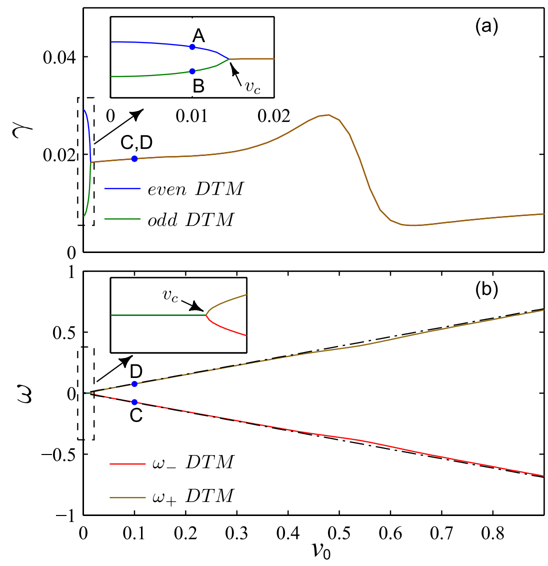

Fig. 2 Linear growth rates (a) and real frequencies (b) of the even and odd DTMs as a function of the flow amplitude v_0 for $\delta x = 0$. The dot-dashed lines are reference lines in terms of the Doppler shift frequency $\omega = kv(\pm x_{\mathrm{s}})$.

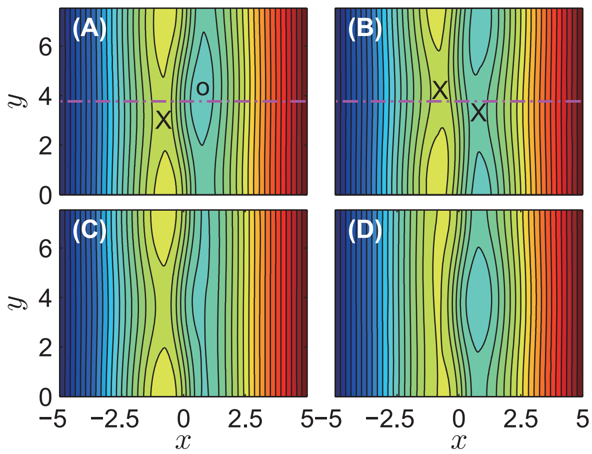

Fig. 3 Contour plots of the island structures corresponding to the cases A∼D marked in Fig. 2. The marks X and O represent the X-point and O-point of the island.

of the even or odd DTM by weak antisymmetric shear flow results from the distortion of the magnetic islands, which is represented by the relative shift of two islands along the direction of the flow as shown in Fig. 3 (cases A and B with $v_0 = 0.01$). And the island deformation between two current sheets due to the relative shift can also be observed.

As the flow increases further, above v_{c}, the growth rates of two eigen modes become the same. And the real frequencies have the same value but opposed sign, which are consistent with the Doppler shift frequency in the current sheets, $\omega = kv(\pm x_{\mathrm{s}})$, where $v(\pm x_{\mathrm{s}})$ are the flow amplitude in the current sheets, as shown in Fig. 2 (b) by the dot-dashed lines. We can not trace the even and odd DTMs anymore due to the existence of the degenerated state at

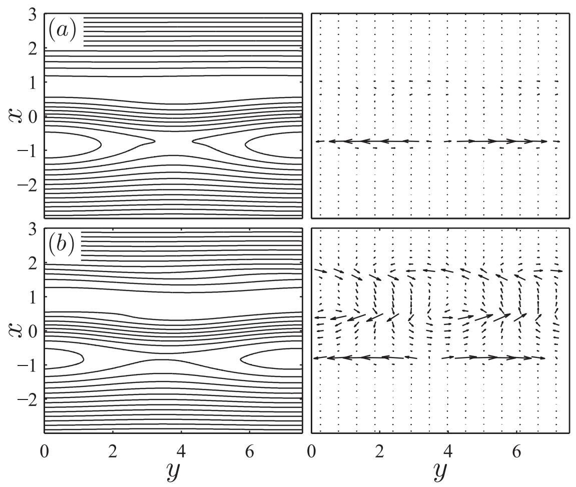

Fig. 4 The island structures (left panel) and corresponding plasma flow patterns (right panel) for different flow amplitude (a) $v_0 = 0.1$, (b) $v_0 = 0.4$.

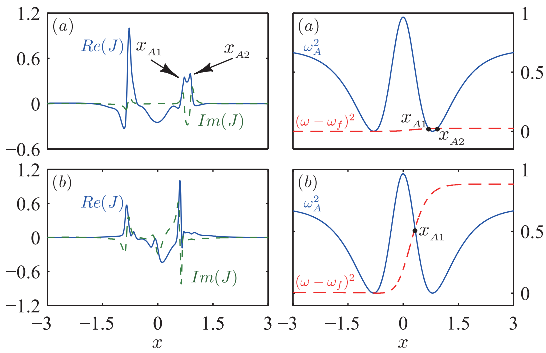

Fig. 5 Radial structures of the perturbed current of the DTM ($J = \nabla_\perp^2 \psi$) with negative frequency (left panel) and the corresponding Alfvén resonance condition (right panel) (a) $v_0 = 0.1$, (b) $v_0 = 0.6$.

$v = v_c$. Thus, in this region, the two branches of DTM are referred to as the positive and negative frequency DTMs according to the eigenvalue of each branch. Interestingly, eigenvalue analysis shows that the DTMs are characterized by a destabilized parameter window, $0.3 < v_0 < 0.55$, then are sharply stabilized. This is identified to result from an so-called AR occurring at one side of the DTM. As the DTM islands start to propagate when the flow amplitude above v_c, each eigen mode may suffer from AR around one current sheet through the relation $(\omega - \omega_f)^2 - \omega_A^2 = 0$. Here $\omega_f = kv_{eq}$ and $\omega_A = kv_A$ are the Doppler shift frequency and Alfvén frequency, respectively. In the sense, the DTM is mainly characterized by a single tearing mode structure (STM-type DTM) since one island is prevented by the AR, as shown in Figs. 3 (C) and (D). Although the AR can suppress the local island to stabilize the DTMs, the appearance of two resonance layers at $x = x_{A1}$ and $x = x_{A2}$, as shown in Fig. 5 (a), produces a new fluctuation structure like a global mode, which may destabilize the DTMs.

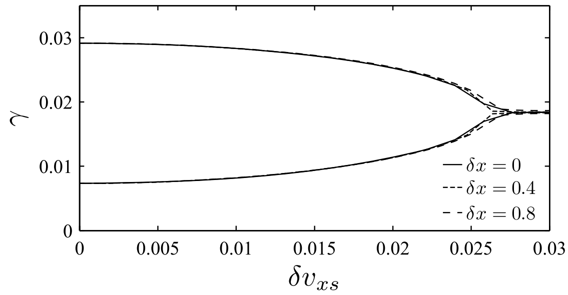

Fig. 6 Growth rates of the DTMs as a function of δv_{xs} for $\delta x = \{0,\ 0.4,\ 0.8\}$ with weak flow.

Most importantly, the plasma flow is amplified around the AR layers as shown in Fig. 4, which may provide an additional inflow to the X-point of the remaining island of the STM-type DTM. The inside AR layer supplies the additional flow to the X-point and the outside one gives sustainment effect for the inside AR layer. This process may be similar to the so-called forced reconnection in thecollisionless tearing mode theory [11], in which the additional flow can enhance the magnetic reconnection process at the X-point. As increasing the shear flow, the inside AR layer shifts inwards approaching the remaining island, supplying stronger enhancement. And the outside AR layer moves outwards until it disappears, as shown in Fig. 5 (b). In this case, the plasma flow loops lose the sustainment from the outside. As a result, the additional inflow to the X-point of the remaining island dramatically decreases, corresponding to a sharp reduction of the growth rate of the DTMs, as shown in Fig. 2.

So far, we have discussed the antisymmetric external flow in respect to the surface $x = 0$, which means almost no flow shear near the current sheets and the characteristics of the DTM affected by the local flow shear can not be specifically revealed. However, the flow shear is one of the most important effects of the shear flow. To examine the role of the local flow shear on the DTM stability, we shift the flow shear layer to approach one current sheet through adjusting the parameter δx in Eq. (4). Bordered on $v_0 = v_c$, we analyze the influence of the flow shear separately.

Firstly, weak flow with $v_0 < v_c$ is considered. Generally speaking, the shear flow can evidently deform the magnetic island structure of the tearing mode through local flow shear near the current sheet [12], then stabilize it. However, it is noticed here that for the DTMs, the global effect of the shear flow seems to be more relevant to the stabilization or destabilization of the DTMs. Such global effect is represented by the difference of the shear flow amplitude between two current sheets, $\delta v_{xs} = v(x_s) - v(-x_s)$, which determines the relative shift between two islands. For the DTMs, the relative shift between two islands makes the inflow to the X-point on one current sheet weakened for the even DTM and enhanced for the odd one, leading to the stabilization or destabilization. Such observations

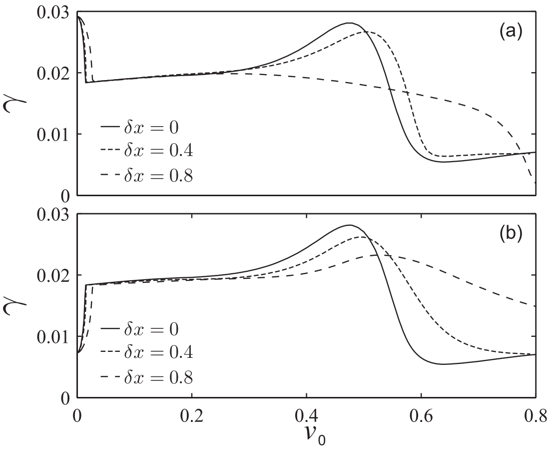

Fig. 7 Growth rates of the DTMs as a function of flow amplitude v_0 for $\delta x = \{0,\ 0.4,\ 0.8\}$.

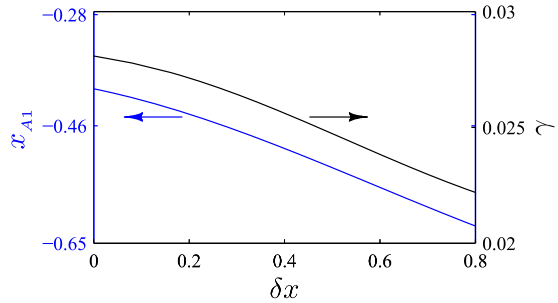

Fig. 8 Comparison between the growth rates of the odd DTM branch in Fig. 7 (b) and the position of the inside Alfvén resonance layer as function of δx for $v_0 = 0.47$.

may show that the island distortion of the DTMs due to the relative shift between two islands is much more effective than that due to the local flow shear around the current sheet [13]. To further examine this notion, Fig. 6 shows the growth rates of the even and odd DTMs as a function of δv_{xs} for three cases $\delta x = \{0,\ 0.4,\ 0.8\}$ in the region of $v_0 < v_c$. The observation with almost the same growth rates shows that the local flow shear around the current sheet, $x = -x_s$, does not significantly contribute to the stabilizing or destabilizing mechanism for weak flow.

However, for strong flow, the DTMs are mainly characterized by STM-type island structure due to one of two islands is prevented by the AR. The local flow shear near the current sheet of the remaining island can distort the magnetic island directly to stabilize it, competing with the destabilizing effect of the AR at opposite side. Figure 7 plots the growth rates and real frequencies of two DTM branches as a function of the flow amplitude v_0 for three cases $\delta x = \{0,\ 0.4,\ 0.8\}$. Obviously different from the case $\delta x = 0$, the DTMs in cases $\delta x = \{0.4,\ 0.8\}$ show different characteristics, i.e. no degenerated state so that we can still trace the even and odd DTMs. Meanwhile, they all propagate with a Dopper shift frequency. Let's discuss two DTM branches one by one.

The branch corresponding to the even DTM in Fig. 7 (a) shows a strong stabilizing tendency as the local flow shear increases in the current sheet of the remaining island ($x = -x_s$), e.g. $\delta x = 0.8$. In this case, the flow shear layer is located on the same side of the remaining island, i.e. opposed side of the AR layers, so that the local flow shear can directly distort the island, leading to a strong stabilization. On the other hand, the branch corresponding to the odd DTM in Fig. 7 (b) shows a destabilizing trend quite similar to the case with $\delta x = 0$. For this branch, the flow shear layer is located on the opposite side of remaining island ($x = x_s$), i.e. the same side of AR layers. As a result, the local flow shear become difficult to influence the island. Hence, this STM-type DTM branch is mainly destabilized due to the plasma flow enhanced by the ARs. And the slight stabilization between three cases observed in Fig. 7 (b) may be due to the integrated effect of all stabilizing and destabilizing mechanisms. As referred before, the inside AR current sheet could enhance the inflow to the X-point of the remaining island on opposite side, thereby destabilizing the STM-type DTM. So the distance between the inside AR layer and the remaining island may quantify this destabilizing mechanisms. To examine this notion, Fig. 8 gives comparison between the growth rates of the branch corresponding to the odd DTM with the remanding island is located at $x = x_s = 0.8$, in Fig. 7 (b) and the position of the corresponding inside AR current sheet as a function $\delta x = [0\ 0.8]$ for $v_0 = 0.47$. The similar tendency may evidence the importance of the inside AR layer, especially its position. In addition, it is noted that from Figs. 7 (a) and (b) the branch corresponding to even DTM is stabilized and the one corresponding to odd DTM is destabilized to become most unstable mode by such kind of asymmetric shear flow. This may imply an importance of the odd DTM eigen state in the nonlinear dynamics of the DTM fluctuation, which was almost neglected in previous studies due to less unstable characteristics.

4. Summary

In this work, we have systematically revisited the DTM instabilities in the presence of shear flows. Eigenvalue analysis has been carried out based on reduced resistive MHD model in slab geometry. The eigenmode characteristics of the DTMs in the presence of shear flows are specified. For a given weak shear flow below v_c, at which a degenerated DTM eigen state is observed, the even (or odd) DTM is stabilized (or destabilized) by the distortion of magnetic islands mainly due to the relative shift of two islands in the direction of flow around the current sheets. In this case, the local flow shear play a less important role. As the shear flow increases, $v_0 > v_c$, the degenerated DTM eigen state bifurcates into two eigen modes with STM-type island structure. Most importantly, the DTMs can be destabilized in a parameter window. It is identified that the Alfvén resonance can separate one equilibrium current sheet into two resonance current sheets, and the inside one may provide an additional enhanced inflow to the X-point of the remaining island on the opposite side. Meanwhile, the local flow shear plays a remarkable stabilizing role when it can distort the island directly, competing with the destabilizing mechanism of the Alfvén resonance. In addition, it is testified that the odd DTM can become the most unstable one when the stabilization of flow shear become dominant, which may imply an importance of the odd DTM eigen state in the nonlinear dynamics of DTM fluctuations.

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