Simulation Study of ECCD by GNET with Momentum Conserving Collisional Operator*)

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The conservation of the momentum during particle collisions is important for studying the electron cyclotron current drive (ECCD). Two momentum conserving collision models are considered applying an iterative method and implemented to GNET code, in which the drift kinetic equation for energetic electrons are solved in 5-D phase space. The simulation results show a good conservation of the momentum and the calculated ECCD current is larger than the non-conserving one.

Keywords: ECH, ECCD, Heliotron J, Fisch-Boozer effect, Ohkawa effect

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1. Introduction

Electron Cyclotron Current Drive (ECCD) is a reliable methods to drive a plasma current by injecting the electron cyclotron wave. The electron cyclotron wave is in the GHz frequency range and the wave absorption position can be controlled locally by changing the magnetic field strength. Consequently, ECCD can control current profile locally and has been applied for toroidal devices to keep the current profile, to stabilize MHD instabilities and to cancel the bootstrap current in helical systems.

In order to study the physics of ECCDs, we have simulated the current drive by Electron Cyclotron Heating (ECH) on the Heliotron J device using the GNET code [1]. GNET solves the drift kinetic equation for energetic particles in 5D phase space. It has been developed for transport study of high energy electrons in helical systems and has been applied to ECCD analysis in helical systems.

In a previous study [2], we simulated the current drive in the Heliotron J and found qualitative agreement between numerical simulation and experimental measurement. There are two well known ECCD mechanisms: (1) the Fisch–Boozer effect; and (2) the Ohkawa effect [3]. These effects drive the current in opposite directions. The result implies that EC current is driven by the Fisch–Boozer effect and Ohkawa effect.

In the present GNET code, the linear Monte Carlo collision operator is applied. This operator expresses the collisional effect between test particle and background particle only as the pitch angle scattering and energy slow down. This operator does not express the change for the background paricle distribution and ignore the momentum transferred from the test particle to the background. It is pointed out that conserving momentum in electron-electron collisions may have the effect for the current [4], it is necessary to modify the operator to conserve momentum. The Ray tracing code show a large impact of parallel momentum conservation for ECCD simulation [5, 6]. However, in the previous study the finite orbit and radial drift effects are not considered because of a radially local assumption.

In the study presented here, in order to study ECCD quantitatively, we develop collisional operators conserving momentum for GNET. Splitting the collisional operator into two parts, we can consider the test particle's momentum given to the background one. Two momentum conserving collision operator models are considered applying an iterative method and implemented to GNET code. We simulate the ECCD in the Heliotron J plasma with the momentum conserving operators. The results show a good conservation of the momentum and the calculated ECCD current is larger than the non-conserving one.

2. Simulation Model

The GNET code can solve the linearized drift kinetic equation as a (time–dependent) initial value problem based on the Monte Carlo technique in 5D phase space. A technique similar to the adjoint equation for dynamic linearized problems is used and the linearized drift kinetic equation for the deviation from the Maxwellian background, δf, is solved. We can obtain the steady state solution of the distribution function by GNET. In helical system the motion of trapped particles becomes complicated because of the complex 3D magnetic configuration. Therefore, in order to analyze ECH in detail on helical systems, we have to take into account of the radial diffusion of trapped particles. Thus, we must consider the distribution function at least in 5D phase space.

In GNET the gyrophase averaged electron distribution

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function is described as

$$f(\boldsymbol{x}, v_{\|}, v_{\perp}, t) = f_{\max}(r, v^2) + \delta f(\boldsymbol{x}, v_{\|}, v_{\perp}, t), \qquad (1)$$

where $f_{\max}(r, v^2)$ represents a Maxwellian depending on the effective radius r. The linearized drift kinetic equation can be written with the initial condition $\delta f(\boldsymbol{x}, v_{\|}, v_{\perp}, t = 0) = 0$ as

$$\frac{\partial \delta f}{\partial t} + (\boldsymbol{v}_d + \boldsymbol{v}_{\|}) \cdot \frac{\partial \delta f}{\partial \boldsymbol{x}} + \dot{\boldsymbol{v}} \cdot \frac{\partial \delta f}{\partial \boldsymbol{v}} = C^{\mathrm{coll}}(\delta f) + L^{\mathrm{orbit}}(\delta f) + S^{\mathrm{ql}}(f_{\max}), \qquad (2)$$

where $\boldsymbol{v}_{\mathrm{d}}$ is the drift velocity and $\boldsymbol{v}_{\|}(= v_{\|}\hat{\boldsymbol{b}})$ is the parallel velocity. The acceleration term $\dot{\boldsymbol{v}} = \dot{\boldsymbol{v}}_{\|} + \dot{\boldsymbol{v}}_{\perp}$ is given by the conservation of magnetic moment and total energy. C^{coll} and L^{orbit} are the collision operator and the particle loss term respectively. S^{ql} represents the quasi-linear heating term.

We assumed the S^{ql} as

$$S^{\mathrm{ql}} = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left[v_\perp D_{\mathrm{rf}}\left(\frac{v_\perp}{v_{\mathrm{the}}}\right)^2 \times \delta\left(\omega - \frac{2\omega_{\mathrm{ce}}}{\gamma} - k_{\|}v_{\|}\right)\frac{\partial}{\partial v_\perp}f_{\max}\right], \qquad (3)$$

where D_{rf} is the wave diffusion coeficient and $v_{\mathrm{the}} = \sqrt{2T_{\mathrm{e}}/m_{\mathrm{e}}}$ is the electron thermal velocity ω and ω_{ce} are the EC wave frequency and the electron cyclotron frequency, respectively $k_{\|} = \omega n_{\|}/c$ is the prallel component of the wave vector and $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor. The delta function in Eq. (3) is approximated by the Gaussian function which has the broadening factor Δ, and then, Eq. (3) is described as

$$S^{\mathrm{ql}} = -\frac{2D_{\mathrm{rf}}}{v_\perp}\frac{\partial}{\partial v_\perp}\left[\left(\frac{v_\perp}{v_{\mathrm{the}}}\right)^4 \frac{1}{\pi^{1/2}\Delta} \times \exp\left\{-\left(\frac{\gamma(1 - k_{\|}v_{\|}/\omega) - 2\omega_{\mathrm{ce}}/\omega}{\Delta}\right)^2\right\} f_{\max}\right]. \qquad (4)$$

The form of S^{ql} depends of the broading factor.

$C^{\mathrm{coll}}(\delta f)$ represents the effect of collision between test particles and background patricles. In previous study, we approximated the term as linearlized collision operator and the particle collision by the s-species (electron and ions) were given by

$$C^{\mathrm{coll}}_{\mathrm{s}}(\delta f) \sim C(\delta f, f_{\max}) = \frac{1}{v}\frac{\partial}{\partial v}\left[v^2 v_{\mathrm{E}}^2\left(v\delta f + \frac{T_{\mathrm{s}}}{m}\frac{\partial \delta f}{\partial v}\right)\right] + \frac{v_{\mathrm{d}}^{\mathrm{s}}}{2}\frac{\partial}{\partial \lambda}(1 - \lambda^2)\frac{\partial \delta f}{\partial \lambda}, \qquad (5)$$

where $C(\delta f, f_{\max})$ is test particle operator which represents the collisional effect for the test particle, $\lambda = v_{\|}/v$, and v_{E}^2 and $v_{\mathrm{d}}^{\mathrm{s}}$ are the energy transfer rate and the deflection collision frequency by a background of s-species particles, respectively.

In order to conserve the momentum, we assume the particle collision term as

$$C^{\mathrm{coll}}(\delta f) = C(\delta f, f_{\max}) + C(f_{\max}, \delta f), \qquad (6)$$

where $C(f_{\max}, \delta f)$ is the field particle operator which represents the collision effect for the background particles. In this study we consider two models. One is very easy to implement but it does not include the information in the velocity space. We named it as the “simple” model. The other includes all information in the velocity space, but it is now being implemented. We named it as “velocity dependent” model.

In the simple model, we assume a high speed limit and $C(f)$ is expressed as

$$C(f_{\max}, \delta f) = p(\boldsymbol{x}, \boldsymbol{v}) f_{\max}, \qquad (7)$$

where $p(\boldsymbol{x}, \boldsymbol{v})$ is a function of the real space coordinate $\boldsymbol{x}$ and the velocity $\boldsymbol{v}$. $p(\boldsymbol{x}, \boldsymbol{v})$ is determined by calculating the conservation low of the energy and momentum. After some calculations, we obtain

$$p(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{v} \cdot \boldsymbol{p}(\boldsymbol{x}) + \lambda(\boldsymbol{x})\left(\frac{v^2}{v_{\mathrm{the}}^2} - \frac{3}{2}\right), \qquad (8)$$

where

$$\boldsymbol{p}(\boldsymbol{x}) = -\frac{2}{n_0 v_{\mathrm{the}}^2}\int \boldsymbol{v} C(\delta f, f_{\max}) \mathrm{d}\boldsymbol{v}, \qquad (9)$$

$$\lambda(\boldsymbol{x}) = -\frac{2}{3 n_0 v_{\mathrm{the}}^2}\int v^2 C(\delta f, f_{\max}) \mathrm{d}\boldsymbol{v}, \qquad (10)$$

where n_0 means the density of background electrons. Once $p(\boldsymbol{x}, \boldsymbol{v})$ is obtained from Eq. (8), we can calculate $C(f_{\max}, \delta f)$ which compensates the lost momentum and energy from test particle. Then we can consider $C(f_{\max}, \delta f)$ as a new source–sink term.

In the GNET code, if we iteratively calculate until δf converges, we obtain a final profile of $C(f_{\max}, \delta f)$. We label the steady state solution obtained by using S^{ql} as δf_0 and use $C(f_{\max}, \delta f_0)$ which becomes a new source term. The steady state solution of this source term is δf_1. Obtaining δf_1, we can consider the conservation of momentum when we calculate δf_0. However the test particle lost the momentum due to the collision with background plasma in the process of calculating δf_1. Therefore we iteratively calculate δf_n (n is the natural number) as

$$\begin{aligned}
&\frac{\partial \delta f_0}{\partial t} + (\boldsymbol{v}_{\mathrm{d}} + \boldsymbol{v}_{\|}) \cdot \frac{\partial \delta f_0}{\partial \boldsymbol{x}} + \dot{\boldsymbol{v}} \cdot \frac{\partial \delta f_0}{\partial \boldsymbol{v}} - C(\delta f_0, f_{\max}) \\
&\quad = S^{\mathrm{ql}}(f_{\max}) + L^{\mathrm{orbit}}(\delta f), \qquad (11) \\
&\frac{\partial \delta f_1}{\partial t} + (\boldsymbol{v}_{\mathrm{d}} + \boldsymbol{v}_{\|}) \cdot \frac{\partial \delta f_1}{\partial \boldsymbol{x}} + \dot{\boldsymbol{v}} \cdot \frac{\partial \delta f_1}{\partial \boldsymbol{v}} - C(\delta f_1, f_{\max}) \\
&\quad = C(f_{\max}, \delta f_0) + L^{\mathrm{orbit}}(\delta f), \\
&\qquad \vdots,
\end{aligned}$$

until the lost momentum approaches almost to zero. At the same time we evaluate the momentum which the test particle lost and calculate the momentum loss rate from them. We stop the iteration when the momentum loss rate becomes small sufficiently. After the iterative method, we obtain the conserving momentum distribution function by calculating $\sum_i^n \delta f_i$.

3. Simulation Result

In this study we consider the magnetic configuration, heating and plasma parameters as the previous paper [2]. Various magnetic configurations are capable in Heliotron J device by changing the ratio of coil currents. Here we consider bumpiness ϵ_b, which is the parameter characterizing magnetic configurations given by

$$\epsilon_b = B_{04}/B_{00}, \tag{12}$$

where B_{mn} represents the Fourier component of the magnetic field strength in Boozer coordinates, and m and n are the poloidal and toroidal mode numbers, respectively. We assume the configuration $\epsilon_b = 0.01$ at the magnetic flux surface $\rho = 0.67$. The radial heating point is set to $(\rho_0, \phi_0, \theta_0) = (0.1, 45°, 0°)$. We also set the parameters describing the EC resonance condition as follows: EC wave frequency is 70 GHz, $2\omega_{ce}/\omega = 0.98$, $n_\parallel = 0.44$ and $\Delta = 1.0\times10^{-3}$. Figure 1 shows the magnetic field strength along the magnetic axis. Fixing the toroidal angle of heating point as $\phi_0 = 45°$, EC power is deposited at the top of the ripple in this configuration.

We run the GNET iteratively and obtain the steady state solution, δf. Figure 2 (a) shows the firstly obtained distribution function. The distribution becomes asymmetric in $v_\parallel$ at the high energy region. This is because many ECRH accelerated electrons hardly become trapped and the collisional relaxtion of the electron deficit in low energy region is faster than that of the accelerated electrons. As a result, the excess of electrons with positive $v_\parallel$ occured and it is found that the negative toroidal current is driven by the Fisch-Boozer effect. Figure 2 (b) shows the source–sink term to conserve the momentum using the steady state solution δf_0. Then the steady state solution δf_1 is evaluated using the this source–sink term (Fig. 2 (c)) and again the next source–sink term is evaluated (Fig. 2 (d)). In the two source–sink terms we can see the larger distribution in

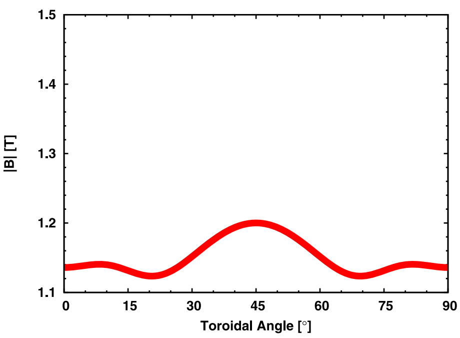

Fig. 1 The magnetic field strength along the magnetic axis.

the positive $v_\parallel$ region and this means the lost momentum have large effect in this region.

Figure 3 shows the momentum loss rate at the each iterative calculation in the simple model. We evaluate the momentum loss at each calculation, and define the momentum loss rate as $p_{loss} = (p_0 - p_n)/(p_0)$, where p_0 is the momentum lost by test particle at first calculation and p_n represents one at the n th iterative caluculation. Figure 3 shows the momentum loss decreases as the iterative calculation advanced and dropped less than 5% of lost momentum than initial simulation. The calculated ECCD current of the simple model is −20.1 kA and the non-conserving one is −18.4 kA. We can see the calculated ECCD current

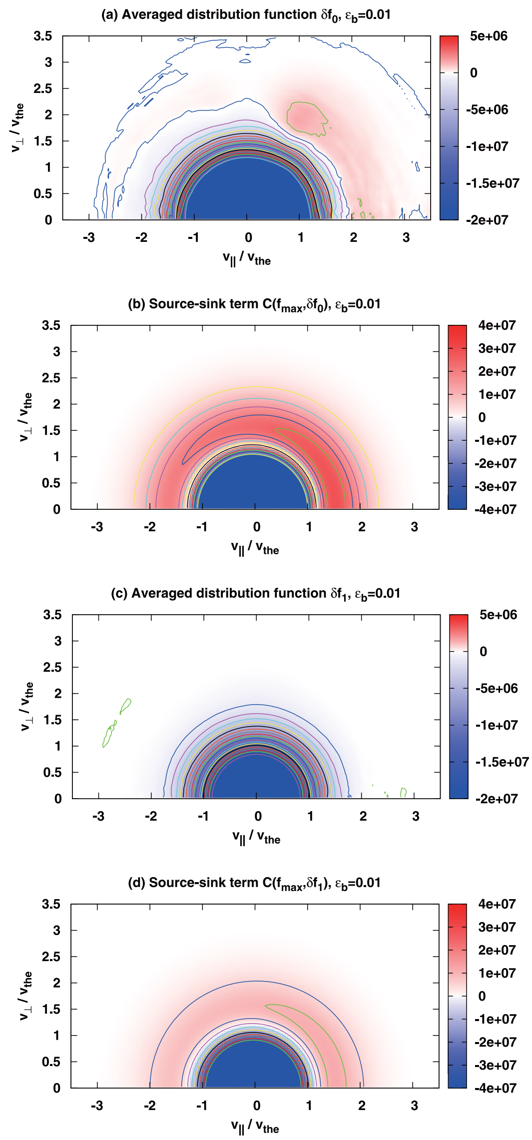

Fig. 2 Flux averaged distribution function of (a) δf_0, (b) source–sink term $C(f_{max}, \delta f_0)$, (c) δf_1 and (d) source–sink term $C(f_{max}, \delta f_1)$. Increase (δf_+) and decrease (δf_-) area from the Maxwellian are colored with red and blue, respectively. These results are calculated in $n_e = 0.5\times10^{19}$ m^{-3}, $T_e = 1.0$ keV, where n_e is electron density and T_e is electron temperature.

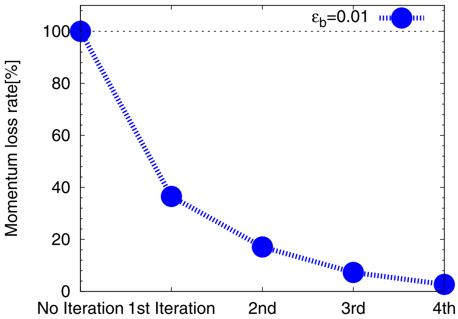

Fig. 3 The momentum loss rate at each iterative calculation.

is larger by 9.2% than that of non-conserving one.

4. Development of Velocity Dependent Model

Though the simple model conserves the momentum, it does not include the exact information in the velocity space. Therefore it is necessary to implement the more exact model. The velocity dependent model is derived from the Fokker–Planck collisional term directly, so it includes exact information more than the simple one.

The field particle operator can be expressed using Legendre polynominals $P_n(\cos\theta)$ as

$$C(f_{\max}, \delta f) = \sum_{n=0}^{\infty} C_n(f_{\max}, \delta f^{(n)}(v)) P_n(\cos\theta), \quad (13)$$

where v is the total velocity of an electron and θ represents the pitch angle. Introducing the Trubnikov-Rosenbluth potential and define $u = \cos\theta$ to simplfy [7, 8], we can describe field particle term $C_n(f_{\max}, \delta f^{(n)}(v))$ as

$$\begin{aligned} C_n(f_{\max}, \delta f^{(n)}(v_e)) &= \Lambda^{e/e} f_{\max} \sum_{i=0}^{\infty} P_i(v_e) \Big[\delta f^{(n)}(v_e) \qquad (14) \\ &+ 2\int_0^{v_e} u^2 \delta f^{(n)}(u) \left\{ \left(n_+ \frac{u^{i+2}}{v_e^{i+1}} - n_- \frac{u^i}{v_e^{i-1}} \right) - \frac{1}{2i+1} \frac{u^i}{v_e^{i+1}} \right\} du \\ &+ 2\int_{v_e}^{\infty} u^2 \delta f^{(n)}(u) \left\{ \left(n_+ \frac{v_e^{i+2}}{u^{i+1}} - n_- \frac{v_e^i}{u^{i-1}} \right) - \frac{1}{2i+1} \frac{v_e^i}{u^{i+1}} \right\} du \Big], \end{aligned}$$

where $n_+ = (i+1)(i+2)/(2i+1)(2i+3)$, $n_- = (i-1)i/(2i-1)(2i+1)$, $v_e = v/v_{\mathrm{the}}$. $\Lambda^{e/e}$ represents the amplitude of field particle term and in this paper it is assumed as $\Lambda_c e^4/m_e^2 \epsilon_0^2$, where Λ_c is coulomb logarithm, e is charge, m_e is mass of an electron and ϵ_0 is permittivity in vacuume. In order to obtain the field particle term $C_n(f_{\max}, \delta f^{(n)}(v))$ which is determined by the obtained pertubed distribution function $\delta f^{(n)}(v)$. We can iteratively calculate $\delta f^{(n)}(v)$ in the same way with the simple model case. After the iterative method, we calculate the complete collision operator according to Eqs. (6) and (13).

Figure 4 shows the first two distribution functions and field particle terms in this procedure. The firstly obtained distribution function generates the first field particle term $C_0(f_{\max}, \delta f^{(0)}(v))$ (Fig. 4 (a)). Then the second distribution

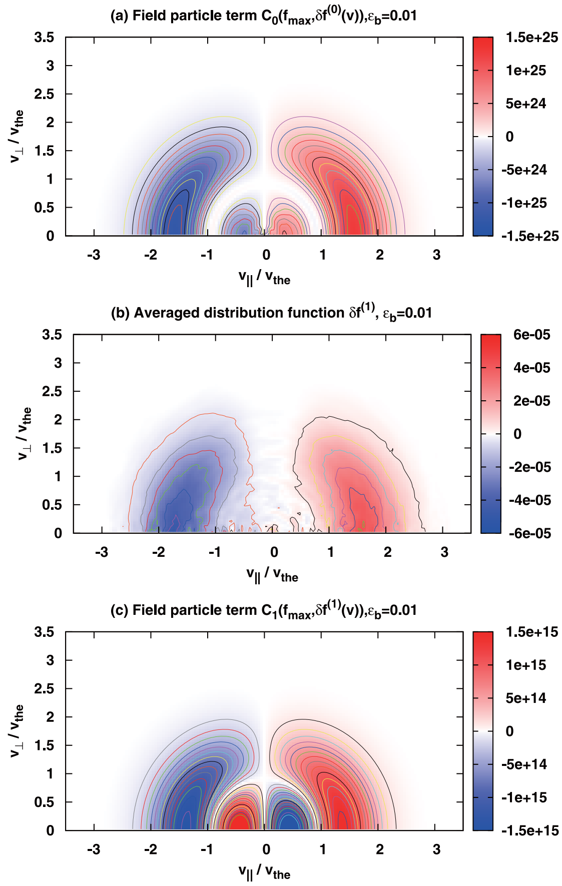

Fig. 4 Flux averaged distribution function of (a) field particle term $C_0(f_{\max}, \delta f^{(0)}(v))$, (b) $\delta f^{(1)}$ and (c) field particle term $C_1(f_{\max}, \delta f^{(1)}(v))$.

function (Fig. 4 (b)) is calculated from the field particle term and the cycle is repeated (Fig. 4 (c)).

The velocity dependent model (Fig. 4) shows the qulitatively similar result in the velocity space. The field particle terms (Figs. 4 (a), (c)), which correspond to the source–sink term in the simple model, show the distribution function will be increased at negative $v_\parallel$ area in high energy region. The steady state solution of field particle term (Fig. 4 (c)) shows the distribution function is increased at positive $v_\parallel$ area, and it means the lost momentum have large effects in this area. The velocity dependent model gives the more exact information in the velocity space, but we have not yet finished implementing this model to GNET.

5. Conclusion

In order to study the ECCD physics on helical plasmas, we have simulated the current drive by ECH in toroidal plasmas using GNET. To evaluate the EC current quantitatively correct, we have improved the collision operator of GNET to conserve the momentum. We have implemented two models; the simple and velocity dependent models. It is easy to implement the simple model and we obtained ECCD current with the momentum conserving. However it does not include the exact information in the velocity space. Therefore we are implementing the velocity dependent model which is expected to include the exact information in the velocity space.

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