Configuration Optimization of a Planar-Axis Stellarator with a Reduced Shafranov Shift[*)]

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Magnetic configurations of LHD experiment are analyzed based on the Fourier modes of the plasma boundary shapes. It was found that a small number of Fourier modes is sufficient to determine the confinement characteristics of different configurations with the magnetic axis shift of LHD. A new configuration is proposed with a different combination of Fourier modes, which has good particle orbits and a favorable property of high beta equilibrium. This solution has a smaller Shafranov shift than the inward-shifted configuration of LHD.

Keywords: stellarator, heliotron, configuration optimization, LHD experiment, Fourier modes, Shafranov shift, boundary shape, planar-axis stellarator

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1. Introduction

In the magnetic confinement fusion research with the helical devices, two major activities are going well and seem to be safe in the research program. LHD experiment is running stably and a new phase of operation with the closed divertor has been started. The machine construction of W7-X is now in the very last phase and the plasma experiment will start within a couple of years. We hope that these two very different paths of stellarator research will be both successful and will be continued to the next step of the experimental test reactor. At the next step, the stellarator research community will be responsible to give a unique design proposal for the stellarator test reactor. Various types of research efforts are necessary to be converted into this important step of making a target design.

The magnetic configurations of LHD have a special characteristic that the control of vertical field, which is called the magnetic axis shift, effectively varies the properties of magnetic configurations. Two distinct properties in such a variation are the magnetic well and the drift orbits of trapped particles. The MHD stability and the high-energy particle confinement strongly depend on these configuration properties respectively. Unfortunately they are not given at the same time in LHD, that is, the magnetic well is created in the configuration with the magnetic axis shifted outward in the torus and the good orbits of trapped particles are obtained for the configuration with the magnetic axis shifted inward. So, an elaborate compromising in selecting the magnetic axis position is necessary for preparing the experimental scenarios and it has been actually successful so far to obtain lots of useful data contributing to the developments of steady state fusion reactors.

Although such a magnetic axis control was successful to obtain a good confinement in the experiments, one of critical issues in the magnetic configuration is a large Shafranov shift for high beta equilibria. While a large Shafranov shift creates the magnetic well, which contributes largely to the MHD stability, the strong deformation of the magnetic surface structure causes the deterioration of the high-energy particle confinement. This problem of LHD configuration is not controlled by the simple vertical field control and a different type of magnetic configuration control is required. In this paper, we describe a new configuration that has a reduced Shafranov shift, which we found in the configuration optimization work considering two conditions of the magnetic well and the trapped particle orbits.

We have been studying the configuration optimization based on the Fourier mode representation of the plasma boundary shape [1]. After analyzing the Fourier modes of three typical LHD configurations with different magnetic axis positions, it was found that a relatively small number of Fourier modes are essential to determine the different confinement characteristics of these configurations. Based on the limited number of Fourier modes, a new space of distributions of Fourier coefficients was explored. As a result of such a study, we found one example of improved configuration with a reduced Shafranov shift. An important feature of this study is that the new confinement characteristics are produced with a small number of Fourier modes, which leaves a room of further steps of optimization in the physical properties and of the flexibility in the magnetic coil design.

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2. Extraction of Essential Fourier Modes

Plasma boundary shapes of the magnetic configurations of LHD experiment are obtained from the free-boundary calculations of the three-dimensional equilibrium code VMEC [2] for the vacuum field. The position of the last closed magnetic surface is determined by the vacuum magnetic field line tracing calculations. By analyzing three sets of two-dimensional Fourier modes for the typical LHD magnetic configurations with different magnetic axis positions (R_{ax} = 3.6, 3.75 and 3.9 m), we found that a small number of modes are responsible to characterize very different confinement properties of these three configurations.

A plasma boundary shape is expressed by the following formula:

$$R(\theta, \phi) = \sum r(m, n) \cdot \cos(m\theta - n\phi),$$
$$Z(\theta, \phi) = \sum z(m, n) \cdot \sin(m\theta - n\phi),$$

θ and ϕ are two (poloidal and toroidal) angle parameters mapped on the boundary surface. R and Z are values of coordinate of each point of the boundary surface of a torus (with angle parameters of θ and ϕ) in a cylindrical coordinate system.

With the vertical magnetic field control for the magnetic axis shift, major radii and minor radii are different for three LHD configurations. We first studied the effects of aspect ratios on the confinement properties by unifying these parameters for three configurations. In terms of Fourier modes, three modes of $r(0, 0)$, $r(1, 0)$ and $z(1, 0)$ are unified. It was confirmed that, with the simple conversion rules that the amplitudes of other Fourier modes are modified in proportion to the change of minor radii, basic parameters and properties of stellarator configurations are conserved [1]. Then we try to extract the essential Fourier modes that give the characteristic confinement properties to three (modified) configurations, which now have the same major radii and the aspect ratios. The results are shown in Figs. 1 (a) and (b) as distributions of the amplitudes of minimum number of Fourier modes for the inward shifted case (R_{ax} = 3.6 m) and outward shifted case (R_{ax} = 3.9 m) of the LHD configurations [3].

The magnetic field configuration configured with 9 components of Fourier modes in Fig. 1 (a) has the same level of good confinement as the inward shifted configuration of LHD and the one with 9 components in Fig. 1 (b) has the same magnetic well structure as the outward shifted one. Figure 2 shows two comparisons of the effective helical ripple profiles between the original LHD configurations and the ones with limited number of Fourier modes shown in Fig. 1. The effective helical ripple is a normalized neoclassical diffusion coefficient [4]. For both inward and outward shifted cases, the profiles calculated for the configurations of 9 components of Fourier modes show the same level of confinement as the original configurations.

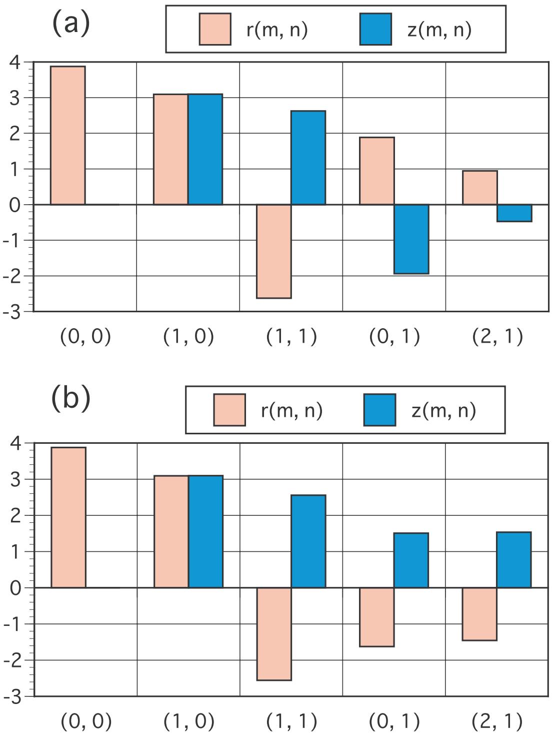

Fig. 1 Amplitudes of Fourier modes of boundary shape for (a) inward shifted configuration and (b) outward shifted configuration. Amplitudes are in logarithmic scales while the polarity reflects the sign of modes.

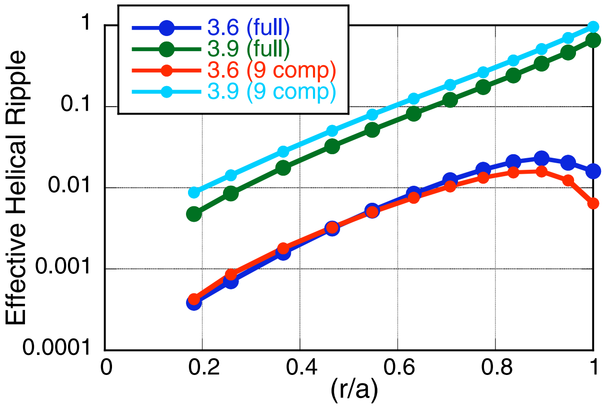

Fig. 2 Effective helical ripple profiles of original LHD configurations and the ones calculated with 9 Fourier components. 3.6 (full) and 3.9 (full) are for the inward and outward shifted LHD configurations respectively. 3.6 (9 comp) and 3.9 (9 comp) are for the configurations of 9 Fourier components.

Because the aspect ratios are unified, Fig. 1 shows clearly the essential difference between the inward and outward shifted configurations of LHD. Polarities are opposite for 4 Fourier components: $r(0, 1)$, $z(0, 1)$, $r(2, 1)$ and $z(2, 1)$ and the amplitudes are different especially for (2, 1) modes (factor of 4).

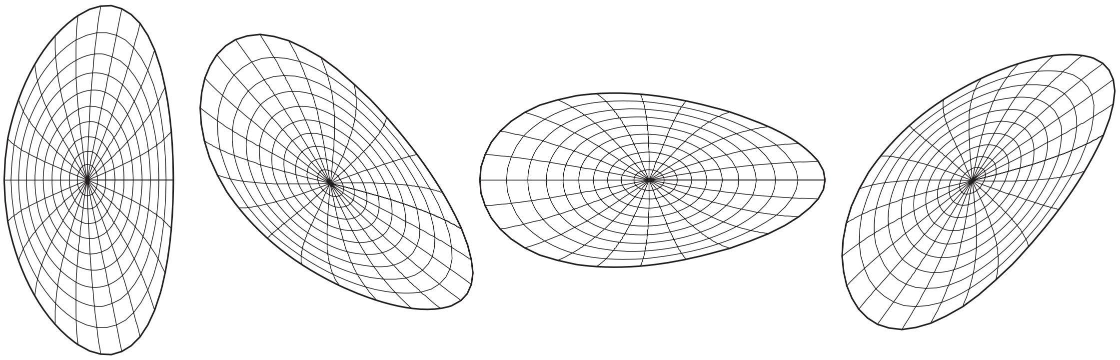

Fig. 3 New configuration with modified distribution of Fourier modes from Fig. 1.

3. A Combination of Inward and Outward Shifted Configurations

For the outward shifted configuration, the effects of (0, 1) mode and (2, 1) mode are investigated on the creation of the magnetic well [5]. The depth of magnetic well is increased with the increase of (0, 1) mode but it saturates and starts to decrease for a very large amplitude of (0, 1) mode. However, the magnetic well depth increases monotonically with the increase of (2, 1) mode and it does not saturate up to the 6 times amplitude of Fig. 1 (b).

In order to find the configuration with simultaneously good confinement and the magnetic well, we investigate a new configuration with the (0, 1) Fourier modes in the polarity of Fig. 1 (a) having the good confinement similar to the inward shifted configuration and the (2, 1) modes in the polarity of Fig. 1 (b) in order to create the magnetic well of outward shifted configuration. Figure 3 shows a configuration with the 1.5 times amplitude of (0, 1) mode of Fig. 1 (a) together with the twice amplitude of (2, 1) mode of Fig. 1 (b). The reason why the amplitudes of these modes are increased is that these modes generally have the competing effects on the confinement and the creation of the magnetic well. Clearest difference of the cross section shape of the new configuration is the triangularity that appears in the first and third cross sections from the left.

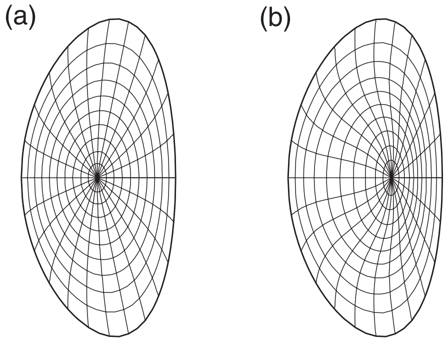

Fig. 4 Magnetic surfaces of (a) vacuum equilibria and (b) 2% averaged beta equilibria of a new configuration.

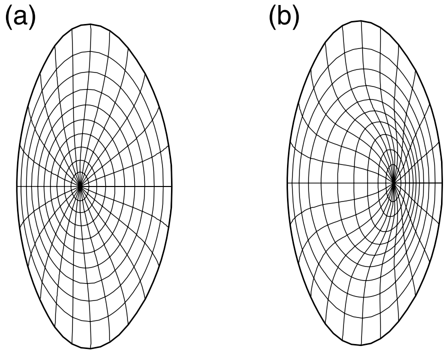

Fig. 5 Magnetic surfaces of (a) vacuum equilibria and (b) 2% averaged beta equilibria of LHD inward shifted configuration.

4. Shafranov Shift of a New Configuration

An effective helical ripple of a new configuration is 4.3×10^{-3} at the 1/3 of minor radius for a zero beta equilibrium, which should be compared to the values of 1.5×10^{-3} and 1.4×10^{-2} for inward and outward shifted configurations, respectively. So it has a confinement between two configurations shown in Fig. 2. It has, however, a good confinement property for the high beta equilibrium. We made fixed-boundary calculations for high beta equilibria with the boundary of the new configuration. Figure 4 shows a comparison between the vacuum and the 2% averaged beta equilibria of the new configuration. For comparison, the equilibria of the LHD inward shifted configuration are also shown in Fig. 5. It is clear that the Shafranov shift is reduced for the new configuration.

The variation of the Shafranov shift with increased beta is shown in Fig. 6. Geometrical position of the magnetic axis in Fig. 5 is calculated by dividing the distance between the left edge and the magnetic axis by the full width of the outer boundary. Figure 6 shows such relative positions of the magnetic axes of the new configuration and the LHD inward shifted configuration as functions of the average beta. For the LHD case, the magnetic axis moves out rapidly for the beta increase from zero to 2% while the axis shift is small for the new configuration.

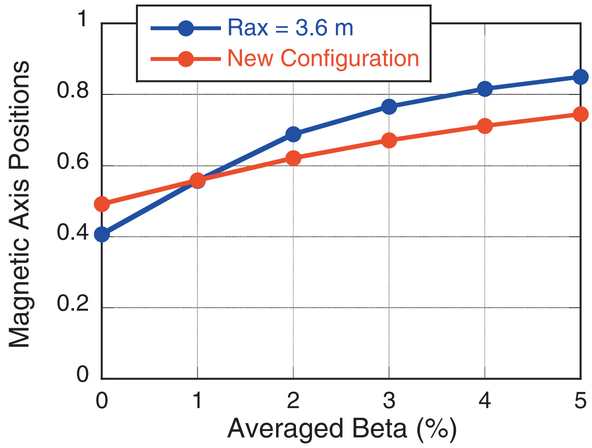

Fig. 6 Relative positions of magnetic axis with increased beta.

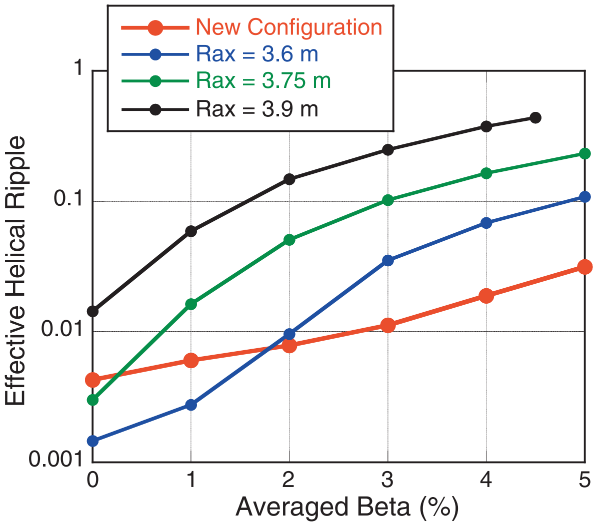

Fig. 7 Variations of effective helical ripple at 1/3 minor radius for new configuration and three LHD configurations as functions of averaged beta.

5. Confinement Properties for High Beta

The effective helical ripple is calculated for the equilibria of average beta values from zero to 5% for the new configuration and the three LHD configurations. It is generally observed for all stellarators that the values of the effective helical ripple increase from the region near the magnetic axis toward the edge region as shown in Fig. 2. From such a profile, values at about 1/3 of minor radius are plotted in Fig. 7 as functions of the averaged beta. Because of the large Shafranov shift of LHD configurations, the effective helical ripple rapidly increases with the beta increase for all three LHD configurations. On the other hand, the increase of the effective helical ripple is slow for the new configuration and for the beta value of 5%, which is a nominal value in the LHD-type reactor design, the effective helical ripple of the new configuration is smaller than any of three LHD configurations.

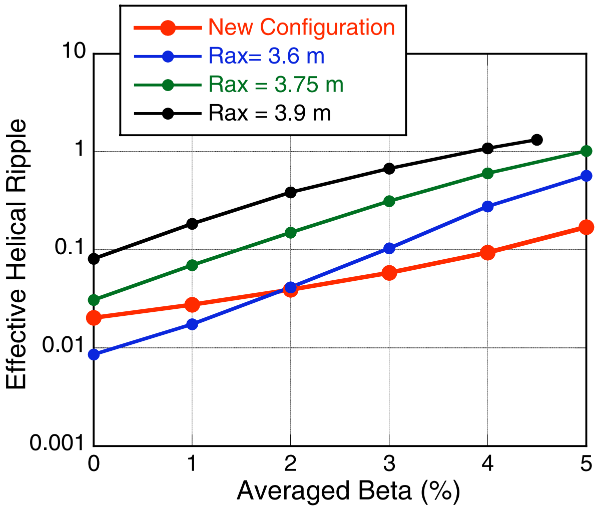

Fig. 8 Variations of effective helical ripple at 2/3 minor radius for new configuration and three LHD configurations as functions of averaged beta.

Figure 8 shows the similar dependence of the effective ripples of four configurations at 2/3 of minor radius as functions of the averaged beta. While the transport at 1/3 of minor radius is relevant to the core plasma confinement, the transport at 2/3 of minor radius is more relevant to the global confinement. It is shown that the calculated global confinement of the new configuration based on the neo-classical transport would be better than LHD configurations for high beta equilibria.

6. Conclusion

On the way of the improvement of the magnetic configuration of LHD, a new configuration is found that has a favorable MHD equilibrium property of the small Shafranov shift. This optimization work is based on the understanding of the relations between differences of the confinement properties of LHD configurations of different magnetic axis positions and the distributions of Fourier modes of the boundary shape. It was found that the essential Fourier modes to give fundamental confinement properties of LHD are (0, 1) modes, which represent the helical axis structure of torus, and (2, 1) modes which give triangle shape (D shape in tokamak). By combining these two modes with polarities of inward and outward shifted configurations of LHD, a new configuration is proposed. This new configuration gives solutions of good confinement for high beta equilibria where the present LHD configurations could not give a good confinement because of the large Shafranov shift.

Since this work is based on the equilibria calculated in the fixed boundary calculation mode of VMEC, the solution for the magnetic coil design is not included. The physics design of magnetic coils is the next step.

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