Higher Harmonics in a Perturbative Transport Experiment

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We have used the convolution method to obtain higher harmonics of the temperature perturbation in a heating modulation experiment on LHD plasma. In comparison with predictions based on the diffusive model, amplitudes of higher harmonics (such as the seventh) decayed much more slowly as they propagated radially. Changes in the time derivative of temperature at the onset and turning-off of heating power remained much sharper after radial propagation than those predicted by the diffusion model.

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1. Introduction

A recent dynamical study of transport properties for magnetically confined plasmas has discovered a hysteresis in the flux-gradient relation [1, 2]. It was found that a simple relation between heat flux across the magnetic surface and the local temperature gradient, such as \( q = -n T \nabla T \), is not as straightforward as the conventional picture assumes. In fact, the heat flux is a multivalued function of the gradient. Hence, the dynamics of temperature perturbations is far from a simple diffusive response with a constant diffusion coefficient. Nonlinear features associated with the hysteresis in the flux-gradient relation should appear in the response of extremely higher harmonics, as has been shown for the propagation of nonlinear waves [3].

In this rapid communication, we report observations of higher harmonics in temperature perturbations induced by modulated electron cyclotron heating (MECH) on the LHD [4]. We found that compared with predictions based on a diffusive response, higher harmonics (such as the seventh) showed much weaker damping in amplitude as they propagate radially. Transient changes in the time derivative at the onset (or turning-off) of heating propagate along a radius with very little smoothing. This work provides a new approach to the study of violations of local closure in transport relations.

2. Experimental Setup

A power modulation experiment was performed on a low-density (central density of \( 1.35 \times 10^{19} \text{ m}^{-3} \)) NBI-heated plasma (balanced injection of 2 MW) that was confined in the LHD (major radius \( R_s = 3.6 \text{ m} \), averaged minor radius \( a = 0.6 \text{ m} \), on-axis magnetic field \( B_{oa} \) of 2.75 T) [5]. The modulation frequency and power of ECH were \( f_{mod} = 25 \text{ Hz} \) and \( P_{mod} = 2 \text{ MW} \), respectively (absorbed in the central core \( \rho \approx 0.2 \), where \( \rho = r/a \) is the normalized minor radius) [2]. The electron temperature was measured at 28 points in the region \( 0.1 \leq \rho \leq 0.9 \) with a time resolution of 4 \( \mu \text{s} \).

3. Analysis of Higher Harmonics

Figure 1 shows the Fourier spectrum of the temperature perturbation: more than ten harmonics are evident in the spectrum. The power spectrum shows that the \( m \)-th harmonic decays as \( p(m f_{mod}) \propto m^\alpha \) where \( \alpha = 3.5 - 4.0 \). By observing the amplitudes of higher harmonics at various radii, the propagation of the temperature perturbation associated with the power modulation can be studied.

If the diffusion equation \( q = -n T \nabla T \) holds, the energy balance for the perturbation takes the form

\[
\frac{d\delta T}{dt} = \chi \nabla^2 \delta T.
\]  

(1)

This expression predicts that the temperature perturbation \( \delta T \cong \exp(-i\omega t + ikx) \) follows the dispersion relation \( k_x = k_r = (\omega/2\chi)^{1/2} \) for \( k = k_r + ik_i \), where \( x \) is the distance from a reference radius. The sign of the wave number is chosen so that the heat wave propagates in the \( x \)-direction. (We do not retain the convective heat flux on the rhs of
Eq. (1) because that term is small in the present experimental condition in which the density is modulated only a little. Even if this term were kept, the conclusion would be unchanged.) In other words, the amplitude of the $m$-th harmonic decays exponentially with the radius,

$$\delta T_m \propto \exp(-k_m x).$$ (2)

For the $m$-th harmonic, $k_m = (m\omega_1/2\chi)^{1/2}$, where $\omega_1$ is the fundamental angular frequency of MECH, $\omega_1 = 2\pi f_{\text{mod}}$. In other words, we have

$$p_m = \delta T_m^2 \propto p_1^z, \quad z = m^{1/2},$$ (3)

i.e., the e-folding length of the $m$-th harmonic is shorter than that of the fundamental mode by a factor of $m^{1/2}$.

Figure 2 shows the radial dependence of the square of the amplitudes of the fundamental, third, and seventh harmonics. From the diffusion model, the predictions for the third and seventh harmonics follow Eq. (3) and are indicated in Fig. 2 by the red and blue lines, respectively. The figure shows that the decay of the amplitudes of higher harmonics is much slower than that predicted by the diffusion model. In contrast, the radial dependence of the higher harmonics is close to that of the fundamental component, as indicated by the thin dotted line in Fig. 2. The slower decay in the amplitudes of higher harmonics indicates that the discontinuous point in the curve $\delta T(t)$ at onset and turning-off of the heating power propagates radially and is smoothed only a little. This temporal response in the temperature perturbation is due to the hysteresis in the flux–gradient relation that was reported in [1, 2].

4. Summary

In comparison with predictions based on the diffusive model, the amplitudes of higher harmonics (such as the seventh) decay more slowly as they propagate radially. This provides a new approach for dynamical studies of transport. Hysteresis in the heat flux can be measured by employing a large number of channels for measuring temperature [2]. In contrast, the radial decay of the amplitude of higher harmonics can be studied using only a few channels. Thus, this analysis may be more conveniently applied to heat modulation studies in many experimental devices.

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