Theoretical Study of Ultra-Relativistic Laser Electron Interaction with Radiation Reaction by Quantum Description*)

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(Received 2 December 2011 / Accepted 23 January 2012)

In the near future, the intensity of the ultra-short pulse laser will reach to 10^{22} W/cm^2. When an electron is irradiated by this laser, the electron's behavior is relativistic with significant bremsstrahlung. This radiation from the electron is regarded as the energy loss of electron. Therefore, the electron's motion changes because of the kinetic energy changing. This radiation effect on the charged particle is the self-interaction, called the "radiation reaction" or the "radiation damping". For this reason, the radiation reaction appears in laser electron interactions with an ultra-short pulse laser whose intensity becomes larger than 10^{22} W/cm^2. In the classical theory, it is described by the Lorentz-Abraham-Dirac (LAD) equation. But, this equation has a mathematical difficulty, which we call the "run-away". Therefore, there are many methods for avoiding this problem. However, Dirac's viewpoint is brilliant, based on the idea of quantum electrodynamics. We propose a new equation of motion in the quantum theory with radiation reaction in this paper.

Keywords: radiation reaction, laser electron interaction, exawatt laser, LAD equation, advanced potential, retarded potential, Dirac equation, QED

DOI: 10.1585/pfr.7.2404010

1. Inroduction

With the rapid progress of the ultra-short pulse laser technologies, the maximum intensities of these lasers have reached the order of 10^{22} W/cm^2 [1, 2]. One laser facility which can achieve such ultra-high intensity is LFEX (Laser for fast ignition experiment) at the Institute of Laser Engineering (ILE), Osaka University [3] and another is the next laser generation project, the Extreme Light Infrastructure (ELI) project [4] in Europe. If an electron is in the strong fields caused by a laser which intensity is larger than 10^{18} W/cm^2, the dynamics of the electron should be described by relativistic equations. The most important phenomenon in this regime is the effect of the ponderomotive force, where an electron is pushed in the propagation direction of the laser. When a charged particle is accelerated, it proceeds in a trajectory accompanied with bremsstrahlung. If the laser intensity is higher than 10^{22} W/cm^2, strong bremsstrahlung might occur. Accompanying this, the "radiation reaction force (or damping force)" works on the charged particle. Therefore, it is necessary to study the radiation reaction effects in the ultra relativistic laser-electron interaction regime. We can consider this effect as a "self-interaction". In the laser electron-interaction at this intensity level, one of the important equations is the equation of motion with the reaction force.

The study of the radiation reaction was started by L. A. Lorentz [5]. The purpose of his study was research of the electron's characteristics via classical physics. The charge of an electron is distributed on the surface of the sphere with the radius of the classical electron's radius. One part of the electron interacts with another part by the Liénard-Wiechert (L-W) electromagnetic field. When the electron moves, the force which is generated by the integral of the L-W field appears. This force is the radiation reaction force, and the equation of the electron's motion becomes

$$\left(m_0 + \alpha\frac{e^2}{4\pi\varepsilon_0 rc^2}\right)\dot{\boldsymbol{v}} = -eE_{\text{laser}} + \frac{e^2}{6\pi\varepsilon_0 c^3}\ddot{\boldsymbol{v}}. \tag{1}$$

This equation is called the Lorentz-Abraham equation. Where, c is the speed of the light, $e^2\ddot{\boldsymbol{v}}/6\pi\varepsilon_0 c^3$ is the radiation reaction force. The term of $\alpha Q^2/4\pi\varepsilon_0 rc^2$ is called the electromagnetic mass depending on the sphere radius r. α is a constant depending on the charge distribution. In the classical theory, an electron is treated as a point particle, which means the limit, $r \to 0 \Rightarrow \alpha e^2/4\pi\varepsilon_0 rc^2 \to \infty$. This is a difficulty in classical electrodynamics similar to renormalization in QED. We can say that the study of the radiation reaction is researching the way on how to describe the effect of the bremsstrhalung without the electromagnetic mass.

P. A. M. Dirac derived the relativistic equation with the radiation reaction in 1938 [6]. The great point of his method was using the advanced potential as the solution

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*) This article is based on the presentation at the 21st International Toki Conference (ITC21).

of Maxwell equation. Normally, the radiation field is described by the retarded potential (L-W field) because of causality. His equation frees us from the infinity of the electromagnetic mass by the advanced field. This is regarded as the classical renormalization. The equation of motion is named the Lorentz-Abraham-Dirac (LAD) equation,

$$m_0 \frac{\mathrm{d}}{\mathrm{d}\tau} w^\mu = -eF^{\mu\nu}_{\mathrm{laser}} w_\nu + m_0\tau_0 \frac{\mathrm{d}^2 w^\mu}{\mathrm{d}\tau^2} + \frac{m_0\tau_0}{c^2}\frac{\mathrm{d}w_\sigma}{\mathrm{d}\tau}\frac{\mathrm{d}w^\sigma}{\mathrm{d}\tau} w^\mu. \tag{2}$$

Where, $\tau_0 = e^2/6\pi\varepsilon_0 m_0 c^3$, m_0 is the rest mass of the electron, w is the 4-velocity, τ is the proper time of the electron and g is the Lorentz metric with the signature of $(+---)$. It seems that this equation is general. However, the LAD equation has another difficulty. The solution of Eq. (2) is

$$\frac{\mathrm{d}}{\mathrm{d}\tau} w^\mu(\tau) = \frac{\mathrm{d}}{\mathrm{d}\tau} w^\mu(0) \times \mathrm{e}^{\frac{\tau}{\tau_0}} - \int_0^\tau \mathrm{d}\tau' \left[\frac{Q}{m_0\tau_0} F^{\mu\nu}_{\mathrm{laser}} w_\nu + \frac{1}{c^2}\frac{\mathrm{d}w_\sigma}{\mathrm{d}\tau}\frac{\mathrm{d}w^\sigma}{\mathrm{d}\tau} w^\mu \right] \mathrm{e}^{\frac{\tau-\tau'}{\tau_0}}. \tag{3}$$

Since τ_0 is $\mathrm{O}(10^{-23})$, the factor $\exp(\tau/\tau_0)$ diverges to infinity very quickly. This infinity is the "run-away" problem. The run-away is caused by the form of Eq. (2). To avoid this problem, some studies exist: (a) such as the approximation of Eq. (2) [7, 8] or (b) another derivation with new assumptions [9–12].

In the ultra-relativistic laser high energy electron interaction, every method leads to the converged equation,

$$m_0 \frac{\mathrm{d}}{\mathrm{d}\tau} w^\mu = -eF^{\mu\nu}_{\mathrm{laser}} w_\nu - \frac{1}{c^2}\frac{\mathrm{d}W}{\mathrm{d}t} w^\mu. \tag{4}$$

Here, the energy rate of bremsstrahlung is described as

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -m_0\tau_0 g_{\rho\sigma}\frac{\mathrm{d}w_\sigma}{\mathrm{d}\tau}\frac{\mathrm{d}w^\sigma}{\mathrm{d}\tau} = m_0 c^2 \tau_0 \frac{\dot{\boldsymbol{\beta}}^2 - \left(\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}}\right)^2}{\left(1-\boldsymbol{\beta}^2\right)^3}. \tag{5}$$

Then, the solution is

$$w^\mu(\tau) = w^\mu(0) \times \mathrm{e}^{-\int_0^\tau \mathrm{d}\tau' \frac{1}{m_0c^2}\frac{\mathrm{d}W}{\mathrm{d}t}(\tau')} + \int_0^\tau \mathrm{d}\tau' \frac{Q}{m_0} F^{\mu\nu}_{\mathrm{laser}} w_\nu(\tau') \times \mathrm{e}^{-\int_{\tau'}^\tau \mathrm{d}\tau'' \frac{1}{m_0c^2}\frac{\mathrm{d}W}{\mathrm{d}t}(\tau'')}, \tag{6}$$

when significant bremsstrahlung is caused by the electron's motion, the energy-momentum of the electron decreases rapidly as an exponential since $\mathrm{d}W/\mathrm{d}t \geq 0$. This behavior is completely different from the case without the radiation reaction [9, 10]. This is a big reason why we need to consider the radiation reaction in the ultra-relativistic laser electron interactions.

Here, we consider that Dirac's model should be applied to the quantum theory. If the radiation reaction is treated in quantum theory, it is considered as high-order Compton scattering by quantum electrodynamics (QED). But, QED requires us to prepare the initial and final state in the Feynman diagram [13]. Quantum field theory (or QED) can describe the particle's annihilation and creation. But at intensities under electron-positron pair creation (10^{24} W/cm^2), we don't need to consider the annihilation-creation of electrons. In this case, the basic equation is the Dirac equation. If we can describe the radiation reaction in relativistic quantum dynamics (not QED), we can calculate easily without considering many patterns of diagrams (interactions). We suggest the Dirac equation with radiation reaction, in quantum dynamics in this paper.

2. Radiation Field with Advanced Potential

First, we show Dirac's method in the classical theory [6]. The unique point in his method is using the advanced potential. From Maxwell's equations in the Lorentz gauge, the wave equation of the 4-potential is derived.

$$\partial_\lambda \partial^\lambda A^\nu = \mu_0 j^\nu \tag{7}$$

This equation is solved by the method of Green's functions. The Green's function of Eq. (7) satisfies

$$\partial_\lambda \partial^\lambda G(x, x') = \delta^4(x - x'). \tag{8}$$

x and x' are the events in the Minkowski spacetime, $x = (ct, \boldsymbol{x})$. Equation (8) has two solutions,

$$G^{\mp}(x, x') = \frac{1}{4\pi|\boldsymbol{x}-\boldsymbol{x}'|}\delta\left(x^0 - x'^0 \mp |\boldsymbol{x}-\boldsymbol{x}'|\right). \tag{9}$$

Therefore, the solutions of Eq. (7) are

$$A^{\pm\nu}(x) = \int_{\mathrm{R}^4} \mathrm{d}^4x' \mu_0 j^\nu(x') G^{\pm}(x, x') = \int_{\mathrm{R}^3} \mathrm{d}^3\boldsymbol{x}' \frac{\mu_0 j^\nu\left(x^0 \pm |\boldsymbol{x}-\boldsymbol{x}'|, \boldsymbol{x}'\right)}{4\pi|\boldsymbol{x}-\boldsymbol{x}'|}. \tag{10}$$

Each of $A^\pm$ are satisfied with Eq. (7). $A_{\mathrm{ret}} = A^-$ is called the retarded potential and $A_{\mathrm{adv}} = A^+$ is called the advanced potential. Normally, the retarded potential is taken as the solution of Eq. (7) due to causality. We often use the formula of the bremsstrahlung, Eq. (5) which can be derived from the retarded potential. However, Dirac saw that the radiation field can be expressed as

$$A_{\mathrm{radiation}}{}^\nu(x) = \frac{A_{\mathrm{ret}}{}^\nu(x) - A_{\mathrm{adv}}{}^\nu(x)}{2} = \int_{\mathrm{R}^3} \mathrm{d}^3\boldsymbol{x}' \frac{\mu_0}{4\pi|\boldsymbol{x}-\boldsymbol{x}'|}\frac{1}{2}\begin{bmatrix} j^\nu\left(x^0 - |\boldsymbol{x}-\boldsymbol{x}'|, \boldsymbol{x}'\right) \\ -j^\nu\left(x^0 + |\boldsymbol{x}-\boldsymbol{x}'|, \boldsymbol{x}'\right)\end{bmatrix}. \tag{11}$$

Moreover, J. Schwinger derived Eq. (5) using the field $A_{\mathrm{radiation}}$ [14]. The retarded potential is divided two parts,

$$A_{\mathrm{ret}} = A_{\mathrm{radiation}} + A_{\mathrm{symmetry}}. \tag{12}$$

The second term, $A_{\mathrm{symmetry}} = (A_{\mathrm{ret}} + A_{\mathrm{adv}})/2$ has a singular point around the electron. The infinity of the electromagnetic mass causes this. Equation (12) can distinguish the radiation and the part of the electromagnetic mass. This radiation field $A_{\mathrm{radiation}}$ is regarded as the retarded field A_{ret} without the singularity of the electromagnetic mass. The quantum electric field is

$$A^{\nu}(x) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3\, 2k^0} \sum_{\lambda=1,2} \left[\begin{array}{l} \varepsilon_{\lambda}{}^{\nu}(\boldsymbol{k})\, a_{\lambda}(\boldsymbol{k})\, \mathrm{e}^{-ik_{\mu}x^{\mu}} \\ +\varepsilon_{\lambda}{}^{\nu *}(\boldsymbol{k})\, a_{\lambda}{}^{\dagger}(\boldsymbol{k})\, \mathrm{e}^{ik_{\mu}x^{\mu}} \end{array} \right], \tag{13}$$

(Here, $\varepsilon_{\lambda=1,2}{}^{\nu}(\boldsymbol{k})$ is the polarization of a photon) and corresponds to the classical field of the laser and the bremsstrahlung is

$$A_{\mathrm{laser}}(x) \leftrightarrow \varepsilon_{\lambda}{}^{\nu}(\boldsymbol{k})\, a_{\lambda}(\boldsymbol{k})\, \mathrm{e}^{-ik_{\mu}x^{\mu}}, \tag{14}$$

$$A_{\mathrm{radiation}}(x) \leftrightarrow \varepsilon_{\lambda}{}^{\nu *}(\boldsymbol{k})\, a_{\lambda}{}^{\dagger}(\boldsymbol{k})\, \mathrm{e}^{ik_{\mu}x^{\mu}}. \tag{15}$$

In the ultra-relativistic laser electron (or plasma) interaction, a certain charge particle interacts with multi-photons. This means we should consider the multi electron-photon scattering processes in the framework of "non-linear QED". The meaning of the word "non-linear" is a requirement in which the Feynman diagram becomes high-order. But, the diagram method is reasonable for elementary processes which occur in a very short time. The dynamics can be represented by combinations of the scattering elements, however, it becomes complicated to calculate. If we consider the dynamics with the annihilation and the creation of particles, we should consider quantum field theory. But the range in which we apply the theory is limited to only an one electron system. Therefore, only covariant quantum dynamics is required. Our basic equation is Dirac's equation because we only need the electron's (Fermion's) equation [15].

$$\left[i\hbar\gamma^{\mu}\left(\partial_{\mu} - \frac{ie}{\hbar}A_{\mu}\right) - mc\,\mathrm{I} \right]\psi = 0 \tag{16}$$

where, $\psi = {}^{t}[\psi_1, \psi_2, \psi_3, \psi_4]$ is the wave function with 4 elements (the Dirac spinor), $\gamma^{0,1,2,3}$ is called the Dirac matrix with 4×4 components and I is the 4×4 unit matrix. In the case which considers the effect of the bremsstrahlung, Maxwell's equations have to be solved, coupled with Eq. (16). The formula in classical dynamics is the equation which contains the energy and momentum change by the radiation as the solution of Maxwell's equations. In the external field which comes from the laser without the radiation reaction, the equation of motion is

$$m_0 \frac{\mathrm{d}}{\mathrm{d}\tau} w^{\mu} = -eF^{\mu\nu}_{\mathrm{laser}} w_{\nu}. \tag{17}$$

Here, F_{laser} is the electromagnetic field tensor, defined by

$$F^{\mu\nu}_{\mathrm{laser}} = F^{\mu\nu}(A_{\mathrm{laser}}) = \partial^{\mu}A_{\mathrm{laser}}{}^{\nu} - \partial^{\nu}A_{\mathrm{laser}}{}^{\mu}. \tag{18}$$

Following the suggestion of Dirac, the radiation reaction is inserted in Eq. (17) by the potential replacement in which

$$A_{\mathrm{laser}} \mapsto A_{\mathrm{laser}} - A_{\mathrm{radiaton}}. \tag{19}$$

And then, from the linearity of the tensor,

$$\begin{aligned} &-e\left[\partial^{\mu}(-A_{\mathrm{radiation}}{}^{\nu}) - \partial^{\nu}(-A_{\mathrm{radiation}}{}^{\mu})\right] w_{\nu} \\ &= m_0\tau_0 \frac{\mathrm{d}^2 w^{\mu}}{\mathrm{d}\tau^2} + \frac{m_0\tau_0}{c^2} g_{\rho\sigma} \frac{\mathrm{d}w^{\rho}}{\mathrm{d}\tau}\frac{\mathrm{d}w^{\sigma}}{\mathrm{d}\tau} w^{\mu}, \end{aligned} \tag{20}$$

the radiation reaction force is derived.

3. Quantum Description

The formula of the radiation reaction in the classical regime is derived as Eq. (20) with the replacement of Eq. (19). In this section, we suggest a formula of quantum dynamics with the radiation damping. In the classical theory, the equation of the electron's motion is

$$m_0 \frac{\mathrm{d}}{\mathrm{d}\tau} w^{\mu} = -eF^{\mu\nu}(A_{\mathrm{laser}} - A_{\mathrm{radiatoin}})\, w_{\nu} \tag{21}$$

with the notation of Eq. (18). In analogy with this, the equation in quantum dynamics is described as

$$\left[i\hbar\gamma^{\mu}\partial_{\mu} + eg_{\mu\nu}\gamma^{\mu}(A_{\mathrm{laser}}{}^{\nu} - A_{\mathrm{radiatoin}}{}^{\nu}) - mc\,\mathrm{I} \right]\psi = 0. \tag{22}$$

The radiation field is defined by Eq. (11), this equation is derived from Maxwell's equations, Eq. (7). The source term in Eq. (7) is represented by the quantum current,

$$j^{\nu} = -ec\bar{\psi}\gamma^{\nu}\psi. \tag{23}$$

Where, $\bar{\psi}$ is defined as $\bar{\psi} = \psi^{\dagger}\gamma^0 = [\psi_1, \psi_2, \psi_3, \psi_4]\,\gamma^0$, the dual-like spinor of ψ. Therefore, when Eq. (11) is calculated, it requires us to get the information from the far past to the far future. But this is difficult because of causality. We need to explore a way to calculate Eq. (11). Hence, neglecting the time dependence, we express the integrand in Eq. (11) by the Fourier series.

$$\begin{aligned} &j^{\nu}\left(x^0 - |\boldsymbol{x} - \boldsymbol{x}'|, \boldsymbol{x}'\right) - j^{\nu}\left(x^0 + |\boldsymbol{x} - \boldsymbol{x}'|, \boldsymbol{x}'\right) \\ &= \int_{-\infty}^{\infty} \frac{\mathrm{d}k_0}{2\pi} \hat{j}^{\nu}(k_0, \boldsymbol{x}')\, \mathrm{e}^{ik_0x^0}(-i)\sin k_0 |\boldsymbol{x} - \boldsymbol{x}'| \\ &= \int_{-\infty}^{\infty} \frac{\mathrm{d}k_0}{2\pi i} \hat{j}^{\nu}(k_0, \boldsymbol{x}')\, \mathrm{e}^{ik_0x^0} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)!}\left(k_0|\boldsymbol{x} - \boldsymbol{x}'|\right)^{2n-1} \end{aligned} \tag{24}$$

The Taylor series expansion of $\sin k_0 |\boldsymbol{x} - \boldsymbol{x}'|$ can be used in all $|\boldsymbol{x} - \boldsymbol{x}'|$, since the radius of convergence is infinite. It is substituted into Eq. (11),

$$\begin{aligned} &A_{\mathrm{radiation}}{}^{\nu}(x) \\ &= -\frac{\mu_0}{4\pi}\int_{\mathrm{R}^3} \mathrm{d}^3\boldsymbol{x}' \sum_{n=1}^{\infty} \frac{|\boldsymbol{x} - \boldsymbol{x}'|^{2n-2}}{(2n-1)!} \partial_0{}^{2n-1} j^{\nu}\left(x^0, \boldsymbol{x}'\right). \end{aligned} \tag{25}$$

Here, from the formula of the inverse-Fourier transformation,

$$\partial_0{}^{\alpha} j^{\nu}\left(x^0, \boldsymbol{x}'\right) = \int_{-\infty}^{\infty} \frac{\mathrm{d}k_0}{2\pi}(ik_0)^{\alpha}\, \hat{j}^{\nu}(k_0, \boldsymbol{x}')\, \mathrm{e}^{ik_0x^0}. \tag{26}$$

In the final description, the information of the physics from the infinite past to the infinite future appears as high order derivatives. The full equation which we searching for is, using Eq. (22), Eq. (23) and Eq. (25), as follows.

$$\begin{aligned} &\left[i\hbar\gamma^{\mu}\partial_{\mu} + eg_{\mu\nu}\gamma^{\mu}A_{\mathrm{laser}}{}^{\nu} - mc\,\mathrm{I} \right]\psi \\ &= \frac{e^2c\mu_0}{4\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}\int_{\mathrm{R}^3}\mathrm{d}^3\boldsymbol{x}'\,|\boldsymbol{x} - \boldsymbol{x}'|^{2n-2}\,\psi \\ &\quad +\partial_0{}^{2n-1}\left[g_{\mu\nu}\gamma^{\mu}\bar{\psi}\gamma^{\nu}\psi\right]\left(x^0, \boldsymbol{x}'\right)\psi \end{aligned} \tag{27}$$

This Eq. (27) is difficult to solve. So we should use the method of perturbation.

4. Method of Perturbation

In this section, we discuss how to calculate Eq. (27). Here, we use the method of the Green's functions. The Green's function of the Dirac equation $S(x, x')$ is defined by,

$$\left[i\hbar\gamma^{\mu}\partial_{\mu} - mc\,\mathrm{I}\right] S(x, x') = \mathrm{I}\delta(x - x'). \tag{28}$$

and the solution of Eq. (22) is

$$\psi(x) = \psi_{\text{free}}(x) - e\int \mathrm{d}^4x' S(x, x') \times g_{\mu\nu}\gamma^{\mu}\left[A_{\text{laser}}{}^{\nu}(x') - A_{\text{radiatoin}}{}^{\nu}(x')\right]\psi(x'). \tag{29}$$

Here, ψ_{free} is satisfied by the free electron equation,

$$\left(i\hbar\gamma^{\mu}\partial_{\mu} - mc\,\mathrm{I}\right)\psi_{\text{free}} = 0. \tag{30}$$

After this, the method is like a QED perturbation. However, there is a significant difference. In QED, every field is (second) quantized. Any number of particles can be treated by the creation-annihilation operators defined on the full Fock space, in QED. This is complete as a quantum system, however, it can only calculate a given process (scattering). In other words, we need to consider many Feynman diagrams (How many photons and patterns of interactions are there? etc...). Our method is free from these discussions. In this paper, the second term in Eq. (29) is named the "interaction term". The scenario of solving it is as follows: (1) The interaction term is described by ψ_{free}. Then the solution of Eq. (22) is noted as $\psi_{(1)}$. (2) Next, the interaction term is described by $\psi_{(1)}$. We repeat these loops.

$$\psi_{(1)}(x) = \psi_{\text{free}}(x) - e\int \mathrm{d}^4x' S(x, x') \times g_{\mu\nu}\gamma^{\mu}\left[A_{\text{laser}}{}^{\nu}(x') - A_{\text{radiatoin}}{}^{\nu}(x')\right]_{\text{free}}\psi_{\text{free}}(x') \tag{31}$$

and after repeated n loops,

$$\psi_{(n)}(x) = \psi_{\text{free}}(x) - e\int \mathrm{d}^4x' S(x, x') \times g_{\mu\nu}\gamma^{\mu}\left[A_{\text{laser}}{}^{\nu}(x') - A_{\text{radiatoin}}{}^{\nu}(x')\right]_{(n-1)}\psi_{(n-1)}(x'). \tag{32}$$

The subscripts next to the square brackets mean that the electromagnetic field depends on the wave function (see Eq. (25)). However, Eq. (25) is still difficult to calculate. Now, we define that

$$A^{(m)}_{\text{radiation}}{}^{\nu}(x) = -\sum_{l=1}^{m}\frac{\mu_0}{4\pi(2l-1)!} \times \int_{\mathrm{R}^3}\mathrm{d}^3\boldsymbol{x}'\left|\boldsymbol{x} - \boldsymbol{x}'\right|^{2l-2}\partial_0{}^{2l-1}j^{\nu}\left(x^0, \boldsymbol{x}'\right). \tag{33}$$

In the classical method, the main effective terms are the lowest few orders. Relativistic-covariance is broken, but it can be calculated. The loop of $\psi_{(n)}$ should be given the calculation precision of the LHS of Eq. (22). Radiation reaction is described in the quantum system without field quantization.

5. Summary

Radiation reaction is becoming more important, since it is predicted by theories in the ultra-relativistic laser electron interaction with the laser intensity of over 10^{22} W/cm^2. The method of Dirac with the advanced potential is very novel, considering the reversed-causality of Maxwell's electromagnetic theory. However, this method has a significant problem. The run-away characteristic has difficulties in mathematics and classical theory. In this paper, Dirac's model has been applied to quantum theory. Normally, the radiation is treated as a quantum field. But, the QED method by Feynman diagrams requires us to give the initial and final states. Moreover, there are many patterns of diagrams (interactions) which we need to consider, since a laser is a gathering of photons. If there isn't annihilation and creation of particles, we need to consider only relativistic quantum dynamics. Therefore, the Dirac equation is the basic equation if the radiation reaction is taken as in (Eq. (22)). The solution of the Dirac equation is the method of perturbation. In a realistic computational calculation, the radiation field of Eq. (33) may be considered by a few loops $m = 3 \sim 5$, since the lower order terms are more effective in classical dynamics.

Acknowledgments

This work is partly supported by auspices of Japanese Ministry of Education, Culture, Sports, Science and Technology (MEXT) project on "Promotion of relativistic nuclear physics with ultra-intense laser" and by JSPS Grant-in-Aid for Scientific Research (C) (22540511).

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