Scaling Law of Non-Inductive Current Drive Steady State Tokamak

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In non-inductive current drive steady state operation of tokamak reactor, it is desirable that the $Q_{\rm cd}$ value determined by the plasma current balance is the same as the Q value determined by the plasma energy balance [Progress in the ITER Physics Basis, Nucl. Fusion **47**, S285 (2007)]. The more quantitative scaling laws of Q and $Q_{\rm cd}$ are derived from the scaling laws of electron density, beta ratio, energy confinement time and current drive efficiency. The reduced scaling laws of Q and $Q_{\rm cd}$ are examined by comparison with the data of the standard scenario of inductive operation and the reference scenario of non-inductive operation of ITER. Sensitivities of Q and $Q_{\rm cd}$ on the plasma parameters are studied and requirement is examined to satisfy $Q_{\rm cd} \approx Q$.

Keywords: steady state tokamak reactor, non-inductive current drive, ITER, current drive efficiency

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1. Introduction

In current drive steady state operation of tokamak reactors, the $Q_{\rm cd}$ value defined by the ratio of the fusion output power divided by the necessary power to drive the plasma current is desirable to be the same as the Q value determined by the energy (pressure) balance [1]. In the case of $Q_{\rm cd} < Q$, the driven plasma current decreases and it is necessary to add the inductive plasma current. In the case of $Q_{\rm cd} > Q$, some part of the additional heating power must be replaced to the same heating power which does not work to drive the plasma current in order to satisfy the current balance. Or it is necessary to keep the plasma current constant and to charge the magnetic flux of the inductive coil in the case of $Q_{\rm cd} > Q$, to prepare to drive the inductive plasma current in the case of $Q_{\rm cd} < Q$. In any case the condition of $Q_{\rm cd} \approx Q$ is desireable for non-inductive current drive steady state tokamak reactor. In Sec. 2 and 3, the more quantitative scaling laws of Q and $Q_{\rm cd}$ are derived by use of scaling laws of electron density, beta ratio, energy confinement time, and current drive efficiency respectively. They are examined by comparison with the data of standard scenario of inductive operation and reference scenario of non-inductive operation of ITER. In Sec. 4, the sensitivity of Q and $Q_{\rm cd}$ on plasma parameters is evaluated and requirement is discussed to satisfy $Q_{\rm cd} \approx Q$.

2. The Q value for Plasma Energy Balance

Although there are many parameters to specify a tokamak device, there are also many relations and constrains between them. If the plasma radius a, the toroidal field $B_{\rm t}$ and the aspect ratio A are specified, other parameters of tokamak are determined by use of scaling laws of electron density, beta, energy confinement time and burning condition, when the safety factor $q_{\rm I}$, the elongation ratio $\kappa_{\rm s}$, the triangularity δ of plasma cross section, Greenwald fraction $N_{\rm G}$ and the normalized beta $\beta_{\rm N}$ are given. The definition of $q_{\rm I}$ is

$$q_{\rm I} \equiv \frac{Ka}{R}\frac{B_{\rm t}}{B_{\rm p}} = \frac{5K^2 aB_{\rm t}}{AI_{\rm p}},$$

where $2\pi Ka$ is the circumference of plasma boundary and K is given approximately by $K^2 \approx (1+\kappa_{\rm s})/2$ and the average poloidal magnetic field is $B_{\rm p} = \mu_0 I_{\rm p}/2\pi Ka$. Therefore the plasma current $I_{\rm p}$ is given by

$$I_{\rm p} = \frac{5K^2 aB_{\rm t}}{Aq_{\rm I}}.$$

The units of $I_{\rm p}$, $B_{\rm t}$, a are MA, T, m respectively.

The safety factor q_{95} at 95% magnetic flux surface is given approximately by [2]

$$q_{95} \approx q_{\rm I} f_\delta f_{\rm A},$$
$$f_\delta = \frac{1+\kappa_{\rm s}^2(1+2\delta^2-1.2\delta^3)}{1+\kappa_{\rm s}^2}, \quad f_{\rm A} = \frac{1.17-0.65/A}{(1-1/A^2)^2}.$$

The volume average electron density $\langle n\rangle_{20}$ in unit of $10^{20}{\rm m}^{-3}$ is

$$\langle n\rangle_{20} = N_{\rm G}\frac{I_{\rm p}({\rm MA})}{\pi a^2}. \qquad (1)$$

The beta ratio of thermal plasma is

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$$\beta_{\rm th} \equiv \frac{\langle p \rangle}{B_{\rm t}^2/2\mu_0} = \frac{0.0403}{B_{\rm t}^2}(\langle n_{20}T_{\rm e\,keV}\rangle + (f_{\rm DT} + f_{\rm He} + f_z)\langle n_{20}T_{\rm i\,keV}\rangle) = 0.0403(\gamma_T + f_{\rm DT} + f_{\rm He} + f_z)\frac{\langle n_{20}T_{\rm i\,keV}\rangle}{B_{\rm t}^2}, \quad (2)$$

where units of n_{20} and $T_{\rm keV}$ are $10^{20}{\rm m}^{-3}$ and keV respectively. γ_T in (2) is the ratio of the electron temperature to the ion temperature under the assumption that the profiles of $T_{\rm i}$ and $T_{\rm e}$ are the same:

$$\gamma_T \equiv \langle nT_{\rm e}\rangle/\langle nT_{\rm i}\rangle \approx \langle T_{\rm e}\rangle/\langle T_{\rm i}\rangle.$$

The sum $\beta_{\rm total}$ of $\beta_{\rm th}$ (thermal plasma) and $\beta_{\rm fast}$ of fast ion and α particle components is given by

$$\beta_{\rm total} = 0.01\beta_{\rm N}\frac{I_{\rm p}({\rm MA})}{aB_{\rm t}},$$

and

$$\beta_{\rm th} = f_{\rm th}\beta_{\rm total}, \quad f_{\rm th} \equiv \frac{\beta_{\rm th}}{\beta_{\rm total}}.$$

$\langle X\rangle$ is the volume average of X and $f_{\rm DT}$, $f_{\rm He}$, f_z are ratios of ion densities of D+T, He and impurities with atomic number z to electron density respectively.

Thermal energy of plasma $W_{\rm th}$ is

$$W_{\rm th}({\rm MJ}) = \frac{3}{2}\beta_{\rm th}\frac{B_{\rm t}^2}{2\mu_0}V = 0.597\beta_{\rm th}B_{\rm t}^2V, \quad (3)$$

where $W_{\rm th}$ is in unit of MJ and plasma volume V is in unit of ${\rm m}^{-3}$. In divertor configuration, $V = 2\pi^2a^2R\kappa_{\rm s}$ is used to define the elongation ratio $\kappa_{\rm s}$ in this paper.

We utilize the thermal energy confinement scaling of IPB98y2 [3]

$$\tau_{\rm E}=0.144H_{y2}I^{0.93}B_{\rm t}^{0.15}M^{0.19}n_{20}^{0.41}a^{1.97}A^{1.39}\kappa_{\rm s}^{0.78}P_{\rm h}^{-0.69} =0.781H_{y2}q_{\rm I}^{-1.34}M^{0.19}\kappa_{\rm s}^{0.78}(K^2)^{1.34}N_{\rm G}^{0.41}A^{0.05}B_{\rm t}^{1.49}a^{2.49}P_{\rm h}^{-0.69}, \quad (4)$$

where $M(= 2.5)$ is average ion mass unit of D and T. $P_{\rm h}$ is the necessary heating power to compensate the transport loss. The sum of the transport loss $P_{\rm h}$ and the radiation loss $P_{\rm rad}$ is the total heating power. H_{y2} is confinement enhance factor.

The fusion output power $P_{\rm fus}$ is

$$P_{\rm fus} = \frac{Q_{\rm fus}}{4}\langle n_{\rm DT}^2\langle\sigma v\rangle_v\rangle V,$$

where $Q_{\rm fus} = 17.58\,{\rm MeV}$. The fusion rate $\langle\sigma v\rangle_v$ is a function of ion temperature $T_{\rm i}$. Since the fusion rate σv near $T_{\rm i}$ =10 keV is approximately given by

$$\langle\sigma v\rangle_v \approx 1.1\times10^{-24}T_{\rm keV}^2({\rm m}^3/{\rm s}),$$

the following $\Theta(\langle T_{\rm i}\rangle)$ is introduced:

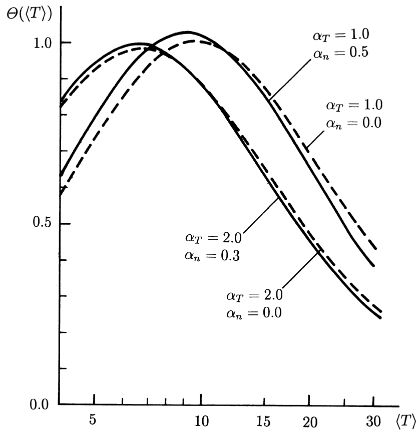

Fig. 1 Θ is the function of the volume average ion temperature $\langle T_{\rm i}\rangle$(keV) in cases with the profiles of parameters ($\alpha_T = 1.0, \alpha_n = 0.0$), ($\alpha_T = 2.0, \alpha_n = 0.0$), ($\alpha_T = 1.0, \alpha_n = 0.5$) and ($\alpha_T = 2.0, \alpha_n = 0.3$).

$$\Theta(\langle T_{\rm keV}\rangle) \equiv \frac{\langle n_{\rm DT}^2\langle\sigma v\rangle_v\rangle}{1.1\times10^{-24}\langle n_{\rm DT}^2T_{\rm keV}^2\rangle} = \frac{\langle n_{\rm DT}^2\langle\sigma v\rangle_v\rangle}{1.1\times10^{-24}\langle n_{\rm DT}T_{\rm keV}\rangle^2}\frac{\langle n_{\rm DT}T_{\rm i}\rangle^2}{\langle n_{\rm DT}^2T_{\rm i}^2\rangle}. \quad (5)$$

Then the fusion reaction rate $\langle n_{\rm DT}^2\langle\sigma v\rangle_v\rangle$ is expressed by

$$\langle n_{\rm DT}^2\langle\sigma v\rangle_v\rangle = 1.1\times10^{-24}\langle n_{\rm DT}T_{\rm keV}\rangle^2\Theta(\langle T_{\rm i}\rangle)f_{\rm prof}, \quad (6)$$

where $f_{\rm prof}$ is the profile parameter defined by

$$f_{\rm prof} = \langle n_{\rm DT}^2T_{\rm i}^2\rangle/\langle n_{\rm DT}T_{\rm i}\rangle^2.$$

Θ is a function of volume average ion temperature $\langle T_{\rm i}\rangle$ in keV and depends on the profiles of density and temperature, and has a peak of around 1 near $\langle T_{\rm i}\rangle \approx 8 \sim 10\,{\rm keV}$. The curves of Θ versus $\langle T_{\rm i}\rangle$ is shown in Fig. 1 [4] in cases of

$$n(\rho) = \langle n\rangle(1-\rho^2)^{\alpha_n}(1+\alpha_n),$$
$$T_{\rm i}(\rho) = \langle T_{\rm i}\rangle(1-\rho^2)^{\alpha_T}(1+\alpha_T),$$

where $\rho^2 = x^2/a^2 + y^2/(\kappa_{\rm s}a)^2$. Then the function Θ is given by

$$\Theta(\langle T_{\rm i}\rangle) = \frac{1+2\alpha_n+2\alpha_T}{1.1\times10^{-24}(1+\alpha_T)^2\langle T_{\rm i}\rangle^2} \times\int_0^1(1-\rho^2)^{2\alpha_n}\langle\sigma v\rangle_v 2\rho{\rm d}\rho,$$

where the fitting functon of $\langle\sigma v\rangle_v$ is [5]

$$\langle\sigma v\rangle_v = \frac{3.7\times10^{-18}}{h(T_{\rm i})}T_{\rm i}^{-2/3}\exp(-20T_{\rm i}^{-1/3})\ {\rm m}^3/{\rm s},$$
$$h(T_{\rm i}) = \frac{T_{\rm i}}{37} + \frac{5.45}{3+T_{\rm i}(1+(T_{\rm i}/37.5)^{2.8})}.$$

In this case, we have $f_{\rm prof} = (\alpha_n+\alpha_T+1)^2/(2\alpha_n+2\alpha_T+1)$. The profile parameter is $f_{\rm prof} = 4/3$ in the case of flat density ($\alpha_n = 0$) and parabolic temperature ($\alpha_T = 1$) profiles

Table 1 (a) Specified design parameters in the case of inductive operation of ITER.

a	B_t	A	q_I	κ_s	N_G	β_N	f_{th}	H_{y2}	$f_{prof}\Theta$	f_{rad}	f_α	γ_T
2.0	5.3	3.1	2.22	1.7	0.85	1.8	0.95	1.063	1.35	0.27	0.95	1.1

$f_{DT} = 0.82$, $f_{He} = 0.04$, $f_{Be} = 0.02$ are specified. $f_{prof}\Theta = 1.35$ and $H_{y2} = 1.063$ are specified in order to be $Q \approx 10$ (Refer to (10)). $q_I = 2.22$ is specified to be $I_p = 15.0$. The triangularity δ is 0.33. $\alpha_n = 0.1$, $\alpha_T = 1.0$.

(b) Reduced parameters.

Q	R	I_p	τ_E	n_{20}	$\langle T_i\rangle$	$\langle T_e\rangle$	W_{th}	P_{fus}	P_{ext}	P_{rad}	β_{total}	q_{95}
9.8	6.2	15.0	3.75	1.01	8.01	8.81	338	424	42	33.2	0.025	3.0

$P_n, P_\alpha, P_{ext}, P_{rad}$ in unit of MW, W_{th} in unit of MJ. I_p in unit of MA. n in unit of $10^{20}\ \mathrm{m}^{-3}$, T in unit of keV.

and $f_{prof} = 9/5$ in the case of more peaked profile of temperature ($\alpha_n = 0, \alpha_T = 2$).

The fusion output power P_{fus} is reduced to

$$\begin{aligned} P_{fus} &= 4.77\frac{f_{DT}^2}{(\gamma_T + f_{DT} + f_{He} + f_I)^2} f_{prof}\Theta(\langle T_i\rangle)\beta_{th}^2 B_t^4 V \\ &= 1.19 f_{dil} f_{prof}\Theta(\langle T_i\rangle)\beta_{th}^2 B_t^4 (2\pi^2\kappa_s A a^3)(\mathrm{MW}) \\ &= 2.35\times 10^{-3} f_{dil} f_{prof}\Theta(\langle T_i\rangle)\kappa_s f_{th}^2 \beta_N^2 I_p^2 B_t^2 A a, \end{aligned} \quad (7)$$

where $I_p = (5K^2/q_I)(aB_t/A)$ and f_{dil} is the dilution parameter of DT fuel due to He and impurities ions: that is,

$$\begin{aligned} f_{dil} &\equiv \left(2\frac{f_{DT}}{(\gamma_T + f_{DT} + f_{He} + f_z)}\right)^2 \\ &= \left(\frac{2}{\gamma_T + 1}\right)^2 \left(\frac{(1 - 2f_{He} - z f_z)}{1 - [f_{He} + (z-1)f_z]/(\gamma_T + 1)}\right)^2. \end{aligned} \quad (8)$$

The α particle fusion output power P_α is

$$P_\alpha = \frac{P_{fus}}{5}.$$

When the absorbed external heating power is denoted by P_{ext} and the heating efficiency of α heating is f_α, the total heating power is $f_\alpha P_\alpha + P_{ext}$. When the fraction of radiation loss power to the total heating power is f_{rad}, the heating power P_h to compensate transport loss is given by

$$P_h = (1 - f_{rad})(f_\alpha P_\alpha + P_{ext}).$$

When Q ratio is defined by the ratio of total fusion output power $P_{fus} = P_n + P_\alpha = 5P_\alpha$ (P_n is neutron output power) to absorbed external heating power P_{ext}, Q is

$$Q = \frac{P_{fus}}{P_{ext}}.$$

Then P_h is reduced to

$$P_h = (1 - f_{rad})\left(f_\alpha + \frac{5}{Q}\right)P_\alpha.$$

Therefore the equation of power balance is

$$\frac{W_{th}}{\tau_E} = P_h = (1 - f_{rad})\left(f_\alpha + \frac{5}{Q}\right)P_\alpha. \quad (9)$$

From (3) (4) (7) and (9), we have

$$\begin{aligned} &\frac{B_t^{0.73} a^{0.42}}{A^{0.26}}\left(f_\alpha + \frac{5}{Q}\right)^{0.31} \\ &= 2.99\left[\frac{1}{(1 - f_{rad}) f_{dil}(f_{prof}\Theta)}\right]^{0.31} \\ &\times \frac{q_I^{0.96}(f_{th}\beta_N)^{0.38}}{H_{y2} M^{0.19} N_G^{0.41} K^{1.92}\kappa_s^{0.09}}. \end{aligned} \quad (10)$$

and

$$\begin{aligned} &\frac{1}{Q} + \frac{f_\alpha}{5} \\ &= \frac{6.83}{(1 - f_{rad}) f_{dil}(f_{prof}\Theta)} \\ &\times \left[\frac{q_I^{0.96}(f_{th}\beta_N)^{0.38}}{H_{y2} M^{0.19} N_G^{0.41}(K^2)^{0.96}\kappa_s^{0.09}}\left(\frac{A^{0.26}}{a^{0.42} B^{0.73}}\right)\right]^{3.226}. \end{aligned} \quad (11)$$

When parameters a, B_t, A are specified, then Q value and other parameters can be evaluated and are shown in Table 1 in the case of inductive operation of ITER. The result of this simple analysis is relatively consistent with ITER design parameters [6] given by Table 2 (inductive operation). $\langle T_i\rangle$ in Table 1 is calculated by use of (1), (2) and (8) as follows;

$$\langle T_i\rangle = \frac{\langle nT_i\rangle}{\langle n\rangle}\frac{\langle T_i\rangle\langle n\rangle}{\langle nT_i\rangle} = \frac{\langle nT_i\rangle}{\langle n\rangle}\frac{1}{f_{prof}^{(2)}}, \quad (12)$$

where $\langle n\rangle$ and $\langle nT\rangle$ are given by (1) and (2) respectively and $f_{prof}^{(2)} \equiv \langle nT\rangle/\langle n\rangle\langle T\rangle \approx (1 + \alpha_n)(1 + \alpha_T)/(1 + \alpha_n + \alpha_T)$.

3. The Q_{cd} Value for Plasma Current Balance

In the case of non-inductive steady state tokamak, we need the necessary power for current drive. The plasma

Table 2 Parameters of ITER [1, 6].

	inductive operation		non-inductive operation
I_p(MA)	15	I_p	9
B_t(T)	5.3	B_t	(5.17)
R/a(m)	6.2/2.0	R/a	(6.35/1.84)
A	3.1	A	3.45
$\kappa_{s\,95}/\delta_{95}$	1.7/0.33	$\kappa_{s\,95}/\delta_{95}$	(1.84/0.41)
$\langle n_e\rangle(10^{20}\mathrm{m}^{-3})$	1.01	$n_e(0)(10^{20}\mathrm{m}^{-3})$	0.6
N_G	0.85	N_G/n_G	~ 0.62/0.85
$\langle T_i\rangle/\langle T_e\rangle$(keV)	8.0/8.8	$T_e(0)/T_i(0)$	37/34
$W_{thermal}/W_{fast}$(MJ)	325/25		
τ_E^{tr}(s)	3.7		
$H_{y2}=\tau_E^{tr}/\tau_E^{IPB98y2}$	1.0	H_{y2}	1.5~1.7
P_{fus}(MW)	410		
P_{ext}(MW)	41	P_{NB}(MW)	34
P_{rad}(MW)	48	P_{EC}(MW)	20
Z_{eff}	1.65		
β_t(%)	2.5	$\beta_{t,th}$(%)	~ 1.9
β_p	0.67	$\beta_{p,th}$	~ 1.2
β_N	1.8	$\beta_{N,th}$	~ 2
q_{95}	3.0	q_{95}	~ 6
q_I	2.22		
l_i	0.86		
Q	10	Q	~ 5
f_R	0.39		
f_{DT}/f_{He}(%)	82/4.1		
f_{Be}/f_{Ar}(%)	2/0.12		

current I_p is the sum of the bootstrap current I_{bs} and the driven current I_{cd}. Bootstrap current density is given by

$$j(r) \approx -\left(\frac{r}{R}\right)^{1/2}\frac{\partial p}{\partial r}\frac{1}{B_p(r)},$$

in the case of circular cross section. The bootstrap current is

$$I_{bs} = -\int_0^a \left(\frac{r}{R}\right)^{1/2}\frac{\partial p}{\partial r}\frac{1}{B_p(r)}2\pi r\mathrm{d}r$$
$$= -\left(\frac{a}{R}\right)^{1/2}\frac{\langle p\rangle}{B_p(a)}2\pi a\int_0^1 \frac{1}{b_p(\rho)}\frac{\partial(p/\langle p\rangle)}{\partial\rho}\rho^{1.5}\mathrm{d}\rho$$
$$\frac{I_{bs}}{I_p} = \left(\frac{a}{R}\right)^{1/2}\frac{\langle p\rangle}{B_p^2(a)/2\mu_0}\left(-0.5\int_0^1 \frac{1}{b_p(\rho)}\frac{\partial(p/\langle p\rangle)}{\partial\rho}\rho^{1.5}\mathrm{d}\rho\right),$$

where $B_p(r) = \mu_0 I_p/2\pi a$, $b_p(\rho) \equiv B_p(r)/B_p(a)$, $\rho = r/a$. When the average poloidal beta is denoted by β_p, we have

$$\frac{I_{bs}}{I_p} = c_b(a/R)^{0.5}\beta_p,$$

where c_b is

$$c_b = -0.5\int_0^1 \frac{1}{b_p(\rho)}\frac{\partial(p/\langle p\rangle)}{\partial\rho}\rho^{1.5}\mathrm{d}\rho$$
$$= -0.5\int_0^1 \frac{q_I(\rho)}{q_I(1)}\frac{\partial(p/\langle p\rangle)}{\partial\rho}\rho^{0.5}\mathrm{d}\rho.$$

$(q_I(r) = (r/R)(B_t/B_p(r))$.

Since $\beta_t = 0.01\beta_N I_p/(aB_t)$, $B_p/B_t = \mu_0 I_p/(2\pi K a B_t) = 0.2K(I_p/aB_t)$, $B_p/B_t = aK/Rq_I$, β_p is reduced to

$$\beta_p = 0.25K^2\beta_N(aB_t/I_p) = 0.05A\beta_N q_I,$$

and

$$\frac{I_{bs}}{I_p} = C_{bs}A^{0.5}\beta_N q_I, \quad C_{bs} = 0.05c_b. \tag{13}$$

When the driven current and the driving power are denoted by I_{cd} and P_{cd}, the current drive efficiency η_{cd} is defined by

$$I_{cd} = \frac{\eta_{cd}}{\langle n\rangle R}P_{cd}.$$

The current drive efficiencies of LHCD [7, 8], ECCD [9]) and NBCD [10–12] are all proportional to the electron temperature T_e. Then the driven current I_{cd} is

$$I_{cd} = \frac{(\eta_{cd}/\langle T_e\rangle)\langle n\rangle\langle T_e\rangle}{\langle n\rangle^2 R}P_{cd}$$
$$= \frac{(\eta_{cd}/\langle T_e\rangle)\langle n\rangle(\langle T_e\rangle + (f_{DT}+f_{He}+f_z)\langle T_i\rangle)}{\langle n\rangle^2 R(1+(f_{DT}+f_{He}+f_z)\langle T_i\rangle/\langle T_e\rangle)}P_{cd},$$

and

$$I_{cd}(\mathrm{MA}) \approx$$
$$2.48\times10^{-2}\left(\frac{\eta_{cd19}/\langle T_{e\,keV}\rangle}{f_{prof}^{(2)}[1+(f_{DT}+f_{He}+f_z)/\gamma_T]}\right)$$

Table 3 (a) Specified design parameters in a case of non-inductive operation of ITER.

a	B_t	A	q_I	κ_s	N_G	β_N	f_{th}	H_{y2}	$f_{prof}\Theta$	f_{rad}	f_α
1.84	5.17	3.45	3.35	1.84	0.63	2.15	0.95	1.702	1.20	0.3	0.95

$f_{DT} = 0.82$, $f_{He} = 0.04, f_{Be} = 0.02$ and $\delta = 0.41$ are specified. $f_{prof}\Theta(\langle T_i \rangle) = 1.2$ is specified in order to be $Q \approx 5$ (Refer to (10)). $q_I = 3.34$ is specified to be $I_p \approx 9.0$ MA. $\alpha_n = 0.03$, $\alpha_T = 2.0$, $\gamma_T = 1.08$

(b) Reduced parameters.

Q	R	I_p	τ_E	n_{20}	$\langle T_i \rangle$	$\langle T_e \rangle$	W_{th}	P_{fus}	P_{ext}	P_{rad}	β_{total}
5.01	6.35	9.02	3.88	0.534	12.0	13.0	241	228	45.5	26.6	0.020

$P_n, P_\alpha, P_{ext}, P_{rad}$ are in the unit of MW and W_{th} is in the unit of MJ. I_p is in the unit of MA. The power of current drive P_{cd} is assumed to be $P_{cd} = P_{ext}$. The approximate value of $q_{95} \approx q_I f_\delta f_A$ is 4.69, which is different from that of Table 2. $T(0) = (1 + \alpha_T)\langle T \rangle \approx 3\langle T \rangle$. Refer to Table 2 (non-inductive operation).

$$\cdot \frac{f_{th}\beta_N I_p(\mathrm{MA}) B_t}{A a^2 \langle n \rangle_{20}^2} P_{cd}(\mathrm{MW}),$$

where η_{cd19} is in the unit of $10^{19}(\mathrm{A/Wm^2})$ and $\langle T_{e\,keV} \rangle$ is the volume average electron temperature in unit of keV, so that we have

$$\frac{I_{cd}}{I_p} = C_{cd} \frac{\beta_N B_t}{A a^2 \langle n \rangle_{20}^2} P_{cd}(\mathrm{MW}),$$
$$C_{cd} = 2.48 \times 10^{-2} \frac{(\eta_{cd19}/\langle T_{e\,keV} \rangle) f_{th}}{[1 + (f_{DT} + f_{He} + f_z)/\gamma_T] f_{prof}^{(2)}}. \quad (14)$$

Since

$$I_{bs}/I_p + I_{cd}/I_p = 1,$$

is the necessary condition for the steady state operation, the required power of current derive P_{cd} is

$$P_{cd} = \frac{(1 - C_{bs} A^{0.5} \beta_N q_I) a R n_{20}^2}{C_{cd} \beta_N B_t}.$$

Fusion power P_{fus} is given by (7) as follows

$$P_{fus} = C_{fus} \beta_N^2 I_p^2 B_t^2 A a,$$
$$C_{fus} = 2.36 \times 10^{-3} f_{dil} (f_{prof}\Theta) \kappa_s f_{th}^2,$$

so that $Q_{cd} \equiv P_{fus}/P_{cd}$ is given by

$$\frac{1}{Q_{cd}} = \frac{(1 - C_{bs} A^{0.5} \beta_N q_I) n_{20}^2 a R}{C_{fus} (\beta_N B_t)^2 I_p(\mathrm{MA})^2 A a C_{cd} \beta_N B_t}$$
$$= \frac{(1 - C_{bs} A^{0.5} \beta_N q_I) N_G^2}{\pi^2 C_{cd} C_{fus} (\beta_N B_t a)^3}. \quad (15)$$

The increase of $A^{1/2}\beta_N q_I$ is favorable to increase the bootstrap current and the increase of $(\beta_N B_t a)^3/N_G^2$ is favorable to increase Q_{cd} ratio, however increase of $q_I \propto 1/I_p$ (decrease of I_p) and decrease of n_e degrade confinement time and need the larger confinement enhance factor H_{y2}.

ITER reference scenario 4, type II in Ref. [1] of non-inductive steady state operation of ITER is selected to examine. In this non-inductive steady state operation scenario, the bootstrap current and driven current are 4.5 MA and 4.5 MA respectively (refer to Table 2). The parameters of R, a, B_t, κ_s/δ in non-inductive operation are referred from Ref. [13]. $N_G, \beta_{t,th}, \beta_{p,th}, \beta_{N,th}$ in non-inductive operation are estimated values of Greenwald parameter and the thermal component of β's from Ref. [1] respectively. Specified values of parameters are shown in Table 3 (a) and the reduced parameters are given in Table 3 (b). These values are relatively consistent with the parameters of reference scenario 4, type II. Refer to Table 2 (non-inductive operation).

The specified bootstrap current $I_{bs} = 4.5$ MA. This specification requires $C_{bs} = 0.0374$ and then $c_b = 0.748$.

In the full non-inductive current drive experiment in JT60-U ($a/R = 0.24$, $\beta_p = 2.7$, reversed shear), the estimated value of c_b is 0.6 [14]. The result of the simulation $I_{bs} = 4.5$ MA of reference scenario 4, type II is probably due to the profile optimization of plasma pressure and safety factor.

The specified externally driven current is $I_{cd} = 4.5$ MA with $P_{cd} = P_{ext} = 41.4$ MW. The necessary value of C_{cd} is $C_{cd} = 0.329 \times 10^{-2}$ and the necessary current drive efficiency η_{cd} is given by (14) as follows

$$\eta_{cd19} = 0.133 \frac{f_{prof}^{(2)} [1 + (f_{DT} + f_{He} + f_z)/\gamma_T]}{f_{th}} \langle T_{e\,keV} \rangle$$
$$\approx 0.259 \langle T_{e\,keV} \rangle.$$

(η_{cd19} is in unit of $10^{19}\mathrm{A/(Wm^2)}$).

The experimental current drive efficiency by the neg-

Table 4 Variables x_i and their exponents α_i in the equation (16).

H_{y2}	M	K^2	$q_{\rm I}$	$N_{\rm G}$	$\beta_{\rm N}$	A	a	$B_{\rm t}$	$\Theta(\langle T\rangle)$
3.23	0.613	3.10	-3.10	1.32	-1.23	-0.839	1.35	2.36	1.0

Note that $\Delta(K^2)/K^2 = (k_{\rm s}^2/K^2)(\Delta k_{\rm s}/k_{\rm s})$. The exponent α of $k_{\rm s}$ is 0.29.

Table 5 Variables y_i and their coefficients γ_i in the equation (17).

$q_{\rm I}$	$N_{\rm G}$	$\beta_{\rm N}$	A	a	$B_{\rm t}$	$\Theta(T)$
$I_{\rm bs}/I_{\rm cd}$	-2	$3 + I_{\rm bs}/I_{\rm cd}$	$0.5 I_{\rm bs}/I_{\rm cd}$	3	3	1

ative ion based neutral beam with the beam energy of 360 keV is

$$\eta_{\rm nb19}^{\rm exp} \approx 0.1 T_{\rm e}(0)_{\rm keV} = 0.1 \frac{T_{\rm e}(0)}{\langle T_{\rm e}\rangle}\langle T_{\rm e\,keV}\rangle \sim 0.3\langle T_{\rm e\,keV}\rangle,$$

in the range of $T_{\rm e}(0) = 1 \sim 13$ keV [15].

The experimental current drive efficiency by the electron cyclotron wave is

$$\eta_{\rm ec19}^{\rm exp} \approx 0.03 T_{\rm e}(0)_{\rm keV},$$

in the range of $T_{\rm e}(0) = 6 \sim 21$ keV [16]. The optimization for higher current drive efficiency has not been made in JT-60U experiments. It is reported that the optimized value is $\eta_{\rm ec19}^{\rm code} \approx 2.0$ in the case of $T_{\rm e}(0) = 20$ keV ($\eta_{\rm ec19}^{\rm code} \approx 0.1 T_{\rm e}(0)_{\rm keV}$) according to the code [16,17]. The theoretical prediction of the electron cyclotron current drive efficiency is $\eta_{\rm ec19}^{\rm theo} \sim 0.1 T_{\rm e}(0)_{\rm keV}$ [18].

The result of the simulation $I_{\rm cd} = 4.5$ MA is due to the assumption of predicted theoretical current drive efficiency of EC wave which is not demonstrated by experiments yet.

4. Sensitivity of Q and $Q_{\rm cd}$ on Plasma Parameters

The Q value and $Q_{\rm cd}$ are quite different quantities with each other. $Q_{\rm cd}$ does not depend on the confinement enhance factor H_{y2}, while Q does not depend on $C_{\rm bs}$ and $C_{\rm cd}$. Even if they are the same value initially, they may change differently. Therefore the sensitivities of their variation ΔQ and $\Delta Q_{\rm cd}$ on their variables must be taken into account. Since Q is given by (11) in the form of

$$\frac{1}{Q} + \frac{f_\alpha}{5} = C\Pi_i x_i^{-\alpha i},$$

we have

$$\frac{\Delta Q}{Q} = \sum \alpha_i \left(1 + \frac{f_\alpha}{5}Q\right)\frac{\Delta x_i}{x_i} + \frac{f_\alpha}{5}Q\frac{\Delta f_\alpha}{f_\alpha}, \qquad (16)$$

where x_i and the exponents α_i are given in Table 4. The Q value is sensitive to H_{y2}, $k_{\rm s}$, $q_{\rm I}$ and $B_{\rm t}$ respectively.

Similarly, $\Delta Q_{\rm cd}$ is given by

$$\frac{\Delta Q_{\rm cd}}{Q_{\rm cd}} = \sum_i \gamma_i \frac{\Delta y_i}{y_i}, \qquad (17)$$

where y_i and γ_i are given in Table 5. $Q_{\rm cd}$ is sensitive to $N_{\rm G}$, $\beta_{\rm N}$, $B_{\rm t}$ and a respectively. Note that $\alpha_{q\rm I} = -3.097$, $\alpha_{\rm NG} = +1.326$ and $\alpha_{\beta\rm N} = -1.26$ in the Q value, while $\gamma_{q\rm I} = I_{\rm bs}/I_{\rm cd}$, $\gamma_{NG} = -2$ and $\gamma_{\beta\rm N} = +4$ in the $Q_{\rm CD}$ value. $\Theta(\langle T_{\rm i}\rangle)$ is the function of the volume average ion temperature $\langle T_{\rm i}\rangle$ and $\langle T_{\rm i}\rangle \propto \beta_{\rm N} B_{\rm t} a/N_{\rm G}$ (refer to (12)). Therefore we have

$$\frac{\Delta\Theta}{\Theta} = \left(\frac{\partial\Theta/\partial\langle T_{\rm i}\rangle}{\Theta/\langle T_{\rm i}\rangle}\right)\frac{\Delta\langle T_{\rm i}\rangle}{\langle T_{\rm i}\rangle}$$
$$= \left(\frac{\partial\Theta/\partial\langle T_{\rm i}\rangle}{\Theta/\langle T_{\rm i}\rangle}\right)\left(\frac{\Delta\beta_{\rm N}}{\beta_{\rm N}} + \frac{\Delta a}{a} - \frac{\Delta N_{\rm G}}{N_{\rm G}} + \frac{\Delta B_{\rm t}}{B_{\rm t}}\right).$$

The value of $\beta_{\rm N}$ is near the stability limit, while the values of $q_{\rm I}$ and $N_{\rm G}$ have the margin to the stability limits in the steady state operation scenario. Therefore we choose $q_{\rm I}$ and $N_{\rm G}$ as the control parameters. Let us consider the two different steady state operations with the same parameters except $(q_{\rm I}, N_{\rm G})$ and $(q_{\rm I} + \Delta q_{\rm I}, N_{\rm G} + \Delta N_{\rm G})$. Then we have

$$\frac{\Delta Q}{Q} = \left(1 + \frac{f_\alpha}{5}Q\right) \times\left[-3.10\frac{\Delta q_{\rm I}}{q_{\rm I}} + \left(1.32 - \frac{\partial\Theta/\partial\langle T_{\rm i}\rangle}{\Theta/\langle T_{\rm i}\rangle}\right)\frac{\Delta N_{\rm G}}{N_{\rm G}}\right],$$

$$\frac{\Delta Q_{\rm cd}}{Q_{\rm cd}} = \frac{I_{\rm bs}}{I_{\rm cd}}\frac{\Delta q_{\rm I}}{q_{\rm I}} - \left(2 + \frac{\partial\Theta/\partial T}{\Theta/T}\right)\frac{\Delta N_{\rm G}}{N_{\rm G}}.$$

In the case of steady state operation reference scenario 4, type II in Ref. [5], the ion temperature is high ($T_{\rm i}(0) = 34$ keV) with peaked profile, so that $\partial\Theta/\partial T \approx -0.0483$ is negative (refer to Fig. 1) and the fusion reaction rate decreases as the average ion temperature increases. Then we have $(\partial\Theta/\partial\langle T_{\rm i}\rangle)/(\Theta/\langle T_{\rm i}\rangle) = -0.76$ and $\Delta(\log Q) = -6.05(\Delta\log q_I) + 4.06(\Delta\log N_{\rm G})$, $\Delta(\log Q_{\rm cd}) = \Delta(\log q_I) - 1.24\Delta(\log N_{\rm G})$.

The dependences of Q and $Q_{\rm cd}$ on $N_{\rm G}$ and q_I are shown in Fig. 2.

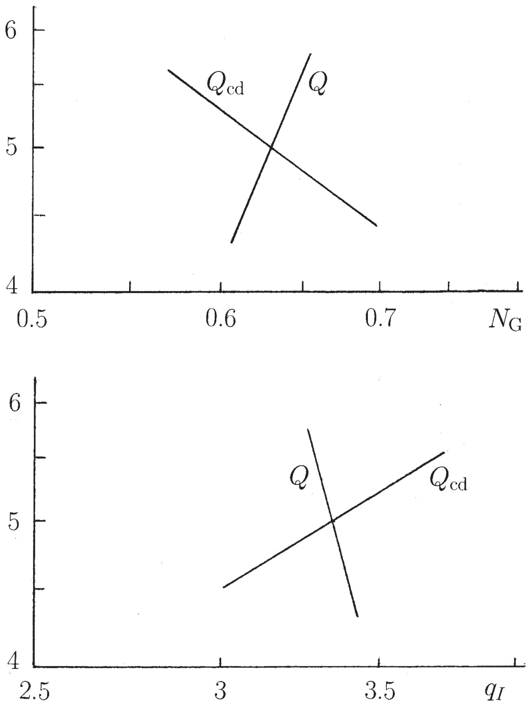

Fig. 2 The dependences of Q and Q_{cd} on N_G are shown in the upper figure and the dependences of Q and Q_{cd} on q_I are shown in the lower figure in the case of the ITER reference scenario 4, type II in Ref. [1]. The vertical axis and the horizontal axis are in log scale.

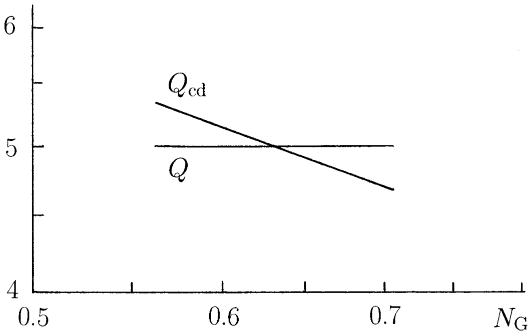

Fig. 3 The dependence of Q_{cd} on N_G under the constraint of $\Delta Q = 0$, that is, $\Delta q_I/q_I = 0.671\Delta N_G/N_G$. The vertical axis and the horizontal axis are in log scale.

Under the constraint of $\Delta Q = 0$, that is, $\Delta q_I/q_I = 0.671\Delta N_G/N_G$, we have $\Delta Q_{cd}/Q_{cd} = -0.569(\Delta N_G/N_G)$ and the dependence of Q_{cd} on N_G is shown in Fig. 3. Reduction of N_G is more effective to increase of Q_{cd} than the effect that the decrease of q_I (increase of I_p) reduces Q_{cd} under the constraint of $\Delta Q = 0$.

5. Conclusion

More quantitative scaling laws of Q and Q_{cd} are derived and examined by comparison with the data of standard scenario of inductive operation and reference scenario of non-inductive operation of ITER. It is cofirmed that the results of scaling laws of Q and Q_{cd} are consistent with the data of both standard scenario of inductive operation and reference scenario of non-inductive operation of ITER.

The dependence of Q and Q_{cd} on plasma parameters are studied and it is found that the control of safety factor q_I and Greenwald fraction N_G is effective to satisfy $Q_{cd} = Q$.

[1] Progress in the ITER Physics Basis, Nucl. Fusion **47**, S285 (2007).
[2] ITER Physics Basis, Nucl. Fusion **39**, 2137 (1999).
[3] ITER Physics Basis, Nucl. Fusion **39**, 2175 (1999).
[4] K. Miyamoto, J. Plasma Fusion Res. **76**, 166 (2000).
K. Miyamoto, *Plasma Physics for Controlled Fusion* (Science and Culture Pub, 2012) http://www.shinanobook.com/genre/book/1481
[5] T. Takizuka and M. Yamagiwa, JAERI-M 87-006 (1987) Japan Atomic Research Institute.
[6] Technical Bases for the ITER-FEAT Outline Design (Draft for TAC Review) 1999; Y. Shimomura *et al.*, 18th IAEA Fusion Energy Conf. (Sorrento) ITER/1, 2000.
[7] N.J. Fisch, Phys. Rev. Lett. **41**, 873 (1978).
[8] C.F.F. Karney *et al.*, Phys. Fluids **22**, 1817 (1979).
[9] N.J. Fisch *et al.*, Phys. Rev. Lett. **45**, 720 (1980).
[10] T. Ohkawa, Nucl. Fusion **10**, 185 (1970).
[11] D.F.H. Start, Plasma Phys. **22**, 303 (1980).
[12] K. Okano, Nucl. Fusion **30**, 423 (1990).
[13] M. Shimada *et al.*, 19th IAEA Fusion Energy Conf. (Lyon) CT-2, 2002.
[14] T. Fujita *et al.*, Phys. Rev. Lett. **87**, 085001 (2001).
[15] K. Oikawa, Nucl. Fusion **41**, 1575 (2001).
[16] T. Suzuki *et al.*, Nucl. Fusion **44**, 699 (2004).
[17] C.C. Petty *et al.*, Nucl. Fusion **42**, 1366 (2002).
[18] R.W. Harvey *et al.*, Nucl. Fusion **37**, 69 (1997).