Scaling Law of Non-Inductive Current Drive Steady State Tokamak

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In non-inductive current drive steady state operation of tokamak reactor, it is desirable that the \( Q_{cd} \) value determined by the plasma current balance is the same as the \( Q \) value determined by the plasma energy balance [Progress in the ITER Physics Basis, Nucl. Fusion 47, S285 (2007)]. The more quantitative scaling laws of \( Q \) and \( Q_{cd} \) are derived from the scaling laws of electron density, beta ratio, energy confinement time and current drive efficiency. The reduced scaling laws of \( Q \) and \( Q_{cd} \) are examined by comparison with the data of the standard scenario of inductive operation and the reference scenario of non-inductive operation of ITER. Sensitivities of \( Q \) and \( Q_{cd} \) on the plasma parameters are studied and requirement is examined to satisfy \( Q_{cd} = Q \).

Keywords: steady state tokamak reactor, non-inductive current drive, ITER, current drive efficiency

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1. Introduction

In current drive steady state operation of tokamak reactors, the \( Q_{cd} \) value defined by the ratio of the fusion output power divided by the necessary power to drive the plasma current is desirable to be the same as the \( Q \) value determined by the energy (pressure) balance [1]. In the case of \( Q_{cd} < Q \), the driven plasma current decreases and it is necessary to add the inductive plasma current. In the case of \( Q_{cd} > Q \), some part of the additional heating power must be replaced to the same heating power which does not work to drive the plasma current in order to satisfy the current balance. Or it is necessary to keep the plasma current constant and to charge the magnetic flux of the inductive coil in the case of \( Q_{cd} > Q \), to prepare to drive the inductive plasma current in the case of \( Q_{cd} < Q \). In any case the condition of \( Q_{cd} \approx Q \) is desirable for non-inductive current drive steady state tokamak reactor. In Sec. 2 and 3, the more quantitative scaling laws of \( Q \) and \( Q_{cd} \) are derived by use of scaling laws of electron density, beta, energy confinement time and current drive efficiency respectively. They are examined by comparison with the data of standard scenario of inductive operation and reference scenario of non-inductive operation of ITER. In Sec. 4, the sensitivity of \( Q \) and \( Q_{cd} \) on plasma parameters is evaluated and requirement is discussed to satisfy \( Q_{cd} \approx Q \).

2. The \( Q \) value for Plasma Energy Balance

Although there are many parameters to specify a tokamak device, there are also many relations and constrains between them. If the plasma radius \( a \), the toroidal field \( B_t \), and the aspect ratio \( A \) are specified, other parameters of tokamak are determined by use of scaling laws of electron density, beta, energy confinement time and burning condition, when the safety factor \( q_t \), the elongation ratio \( \kappa_s \), the triangularity \( \delta \) of plasma cross section, Greenwald fraction \( N_G \) and the normalized beta \( \beta_N \) are given. The definition of \( q_t \) is

\[
q_t = \frac{K a B_t}{\frac{R}{B_p} = 5K^2 a B_t},
\]

where \( 2\pi a \) is the circumference of plasma boundary and \( K \) is given approximately by \( K^2 = (1 + \kappa_s)/2 \) and the average poloidal magnetic field is \( B_p = \mu_0 I_p/2\pi a \). Therefore the plasma current \( I_p \) is given by

\[
I_p = \frac{5K^2 a B_t}{A q_t}.
\]

The units of \( I_p \), \( B_t \), \( a \) are MA, T, m respectively.

The safety factor \( q_s \) at 95% magnetic flux surface is given approximately by [2]

\[
q_s = q_t f_s f_A,
\]

\[
f_s = \frac{1 + \kappa_s^2 (1 + 2 \delta^2 - 1.2 \delta^3)}{1 + \kappa_s^2}, \quad f_A = \frac{1.17 - 0.65/A}{(1 - 1/A^2)^2}.
\]

The volume average electron density \( \langle n \rangle_{20} \) in unit of \( 10^{20} \text{m}^{-3} \) is

\[
\langle n \rangle_{20} = \frac{N_G I_p(\text{MA})}{\pi a^2}.
\]

The beta ratio of thermal plasma is

\[
\beta_T = \frac{1}{2} \left( 1 + \kappa_s^2 (1 + 2 \delta^2 - 1.2 \delta^3) \right).
\]
\[
\beta_{th} = \frac{\langle p \rangle}{\langle n_20 T_{e,keV} \rangle} = \frac{0.6403 B_i^2 / 2\mu_0}{B_i^2 / 2\mu_0 (n_20 T_{e,keV})} + (f_{DT} + f_{He} + f_i (n_20 T_{i,keV}))
\]
\[
= 0.6403 (\gamma_T + f_{DT} + f_{He} + f_i) \frac{(n_20 T_{i,keV})}{B_i^2}, \tag{2}
\]
where units of \(n_20\) and \(T_{keV}\) are \(10^{20} \text{m}^{-3}\) and keV respectively. \(\gamma_T\) in \(2\) is the ratio of the electron temperature to the ion temperature under the assumption that the profiles of \(T_i\) and \(T_e\) are the same:
\[
\gamma_T = \frac{\langle n T_e \rangle / \langle n T_i \rangle}{\langle T_e / T_i \rangle}.
\]
The sum \(\beta_{total}\) of \(\beta_{th}\) (thermal plasma) and \(\beta_{fast}\) of fast ion and \(\alpha\) particle components is given by
\[
\beta_{total} = 0.01 \beta_N \frac{I_p (\text{MA})}{a B_i},
\]
and
\[
\beta_{th} = f_{th} \beta_{total}, \quad f_{th} = \frac{\beta_{th}}{\beta_{total}}.
\]
\(\langle X \rangle\) is the volume average of \(X\) and \(f_{DT}, f_{He}, f_i\) are ratios of ion densities of D+T, He and impurities with atomic number \(Z\) to electron density respectively.

Thermal energy of plasma \(W_{th}\) is
\[
W_{th}(\text{MJ}) = \frac{0.597 \beta_{th} B_i^2}{2\mu_0} V = \frac{3}{2} \beta_{th} B_i^2 V, \tag{3}
\]
where \(W_{th}\) is in unit of MJ and plasma volume \(V\) is in unit of \(\text{m}^3\). In divertor configuration, \(V = 2\pi^2 a^2 R \kappa_s\) is used to define the elongation ratio \(\kappa_s\) in this paper.

We utilize the thermal energy confinement scaling of IPB98y2 [3]
\[
\tau_E = 0.144 H_{y2}^{-0.05} B_i^{-0.15} M^{-0.10} a^{-0.01} A^{1.39} T_{DT}^{-0.78} P_{th}^{-0.69}
\]
\[= 0.781 H_{y2}^{-1.34} M^{-1.19} a^{-0.78} (k_s^{-1.34} N G A)^{-0.05} B_i^{-1.49} a^{-2.49} P_{th}^{-0.69}, \tag{4}
\]
where \(M = 2.5\) is average ion mass unit of D and T. \(P_{th}\) is the necessary heating power to compensate the transport loss. The sum of the transport loss \(P_{t}\) and the radiation loss \(P_{rad}\) is the total heating power. \(H_{y2}\) is confinement enhance factor.

The fusion output power \(P_{fus}\) is
\[
P_{fus} = \frac{Q_{fus}}{4} \langle n_{DT}^2 \sigma v_i \rangle V,
\]
where \(Q_{fus} = 17.58 \text{MeV}\). The fusion rate \(\langle n_{DT}^2 \sigma v_i \rangle\) is a function of ion temperature \(T_i\). Since the fusion rate \(\sigma v_i\) near \(T_i = 10 \text{keV}\) is approximately given by
\[
\langle \sigma v_i \rangle_i \approx 1.1 \times 10^{-24} T_{keV}^2 (\text{m}^3 / \text{s}),
\]
the following \(\Theta(T_i)\) is introduced:

\[
\Theta(T_i) = \frac{\langle n_{DT}^2 \sigma v_i \rangle_i}{(1 + \alpha_T) T_i^{1/3}} \times \int_0^1 (1 - \rho^2)^{2\alpha_T} \langle \sigma v_i \rangle_i 2d\rho, \tag{5}
\]
where the fitting function of \(\langle \sigma v_i \rangle_i\) is [5]
\[
\langle \sigma v_i \rangle_i = \frac{3.7 \times 10^{-18}}{h(T_i)} T_i^{-3/2} \exp(-20 T_i^{-1/3}) \text{m}^3 / \text{s},
\]
\[
h(T_i) = \frac{T_i}{37} + \frac{5.45}{3 + T_i (1 + (T_i/37.5)^{2/3})}.
\]
In this case, we have \(f_{prof} = (\alpha_n + \alpha_T + 1)^2 / (2\alpha_n + 2\alpha_T + 1)\). The profile parameter is \(f_{prof} = 4/3\) in the case of flat density \(\alpha_n = 0\) and parabolic temperature \((\alpha_T = 1)\) profiles.
The total heating power is $Q \approx 10$ (Refer to (10)). $q_l = 2.22$ is specified to be $I_p = 15.0$. The triangularity $\delta$ is 0.33, $\alpha_n = 0.1$, $\alpha_T = 1.0$.

(b) Reduced parameters.

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$R$</th>
<th>$I_p$</th>
<th>$\tau_E$</th>
<th>$n_{20}$</th>
<th>$\langle T_i \rangle$</th>
<th>$\langle T_e \rangle$</th>
<th>$W_{th}$</th>
<th>$P_{fus}$</th>
<th>$P_{ext}$</th>
<th>$P_{rad}$</th>
<th>$\beta_{total}$</th>
<th>$\phi_{55}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.8</td>
<td>6.2</td>
<td>15.0</td>
<td>3.75</td>
<td>1.01</td>
<td>8.01</td>
<td>8.81</td>
<td>338</td>
<td>424</td>
<td>42</td>
<td>33.2</td>
<td>0.025</td>
<td>3.0</td>
</tr>
</tbody>
</table>

$P_{th}$, $P_{fus}$, $P_{ext}$, $P_{rad}$ in unit of MW, $W_{th}$ in unit of MJ. $I_p$ in unit of MA. $n$ in unit of $10^{20}$ m$^{-3}$, $T$ in unit of keV.

Therefore the equation of power balance is

$$W_{th} = P_h = (1 - f_{rad}) \left( f_\alpha + \frac{5}{Q} \right) P_a. \tag{9}$$

From (3), (4), (7) and (9), we have

$$\frac{Q_1^{0.73} \delta_{B}^{0.42}}{A^{0.26}} \left( f_\alpha + \frac{5}{Q} \right)^{0.31} \frac{1}{(1 - f_{rad}) f_{dil}(f_{prof} \Theta)}^{0.31} \times \frac{\delta_{B}^{0.96}(f_{dil} \Phi_B)^{0.38}}{H_{32} \delta_{M}^{0.19} N_5^{24} K_{1.92}^{0.09}}. \tag{10}$$

and

$$\frac{1}{Q} + \frac{f_\alpha}{5} = 6.83$$

When parameters $a$, $B_0$, $A$ are specified, then $Q$ value and other parameters can be evaluated and are shown in Table 1 in the case of inductive operation of ITER. The result of this simple analysis is relatively consistent with ITER design parameters [6] given by Table 2 (inductive operation). $\langle T_i \rangle$ in Table 1 is calculated by use of (1), (2) and (8) as follows;

$$\langle T_i \rangle = \frac{\langle n T_i \rangle}{\langle n \rangle} \langle T_i \rangle / \langle n \rangle = \frac{\langle n T_i \rangle}{\langle n \rangle} f_{\text{prof}}^{(2)}, \tag{12}$$

where $\langle n \rangle$ and $\langle n T \rangle$ are given by (1) and (2) respectively and $f_{\text{prof}}^{(2)} \equiv \langle n T \rangle / \langle n \rangle T = 1 + \alpha_n (1 + \alpha_T)/(1 + \alpha_n + \alpha_T)$.

3. The $Q_{cd}$ Value for Plasma Current Balance

In the case of non-inductive steady state tokamak, we need the necessary power for current drive. The plasma
Table 2 Parameters of ITER [1,6].

<table>
<thead>
<tr>
<th></th>
<th>inductive operation</th>
<th>non-inductive operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_p ) (MA)</td>
<td>15</td>
<td>( I_p ) 9</td>
</tr>
<tr>
<td>( B_t ) (T)</td>
<td>5.3</td>
<td>( B_t ) 5.17</td>
</tr>
<tr>
<td>( R/a ) (m)</td>
<td>6.2/2.0</td>
<td>( R/a ) 6.35/1.84</td>
</tr>
<tr>
<td>( A )</td>
<td>3.1</td>
<td>( A ) 3.45</td>
</tr>
<tr>
<td>( \kappa_{gs}/\delta_{gs} )</td>
<td>1.7/0.33</td>
<td>( \kappa_{gs}/\delta_{gs} ) (1.84/0.41)</td>
</tr>
<tr>
<td>( \langle n_e \rangle (10^{20} \text{m}^{-3}) )</td>
<td>1.01</td>
<td>( n_e (0)(10^{20} \text{m}^{-3}) ) 0.6</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>0.85</td>
<td>( N_G/n_G ) ~0.62/0.85</td>
</tr>
<tr>
<td>( \langle T_i \rangle /\langle T_e \rangle (\text{keV}) )</td>
<td>8.0/8.8</td>
<td>( T_o(0)/T_i(0) ) 37/34</td>
</tr>
<tr>
<td>( W_{\text{thermal}}/W_{\text{fast}}(\text{MJ}) )</td>
<td>325/25</td>
<td></td>
</tr>
<tr>
<td>( r_{th}^2 ) (s)</td>
<td>3.7</td>
<td></td>
</tr>
<tr>
<td>( H_{y2} )</td>
<td>( r_{th}^2 /r_{E} )</td>
<td>1.0 ( H_{y2} ) 1.5~1.7</td>
</tr>
<tr>
<td>( P_{in}(\text{MW}) )</td>
<td>410</td>
<td>( P_{\text{NB}}(\text{MW}) ) 34</td>
</tr>
<tr>
<td>( P_{\text{pol}}(\text{MW}) )</td>
<td>41</td>
<td>( P_{\text{EC}}(\text{MW}) ) 20</td>
</tr>
<tr>
<td>( Z_{\text{eff}} )</td>
<td>1.65</td>
<td></td>
</tr>
<tr>
<td>( \beta_i(%) )</td>
<td>2.5</td>
<td>( \beta_{\text{th}}(%) ) ~1.9</td>
</tr>
<tr>
<td>( \beta_p )</td>
<td>0.67</td>
<td>( \beta_{p,\text{th}} ) ~1.2</td>
</tr>
<tr>
<td>( \beta_{\text{sh}} )</td>
<td>1.8</td>
<td>( \beta_{\text{th}} ) ~2</td>
</tr>
<tr>
<td>( q_{os} )</td>
<td>3.0</td>
<td>( q_{os} ) ~6</td>
</tr>
<tr>
<td>( q_i )</td>
<td>2.22</td>
<td></td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>( Q )</td>
<td>10</td>
<td>( Q ) ~5</td>
</tr>
<tr>
<td>( f_R )</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>( f_{\text{DT}}/f_{\text{He}}(%) )</td>
<td>82/4.1</td>
<td></td>
</tr>
<tr>
<td>( f_{\text{He}}/f_{\text{Ar}}(%) )</td>
<td>2/0.12</td>
<td></td>
</tr>
</tbody>
</table>

Current \( I_p \) is the sum of the bootstrap current \( I_{bs} \) and the driven current \( I_{cd} \). Bootstrap current density is given by

\[
j(r) \approx -\left(\frac{r}{R}\right)^{1/2} \frac{\partial p}{\partial r} \frac{1}{B_p(r)}.
\]

in the case of circular cross section. The bootstrap current is

\[
I_{bs} = -\int_0^\infty \left(\frac{r}{R}\right)^{1/2} \frac{\partial p}{\partial r} \frac{1}{B_p(r)} 2\pi rd r = -\left(\frac{a}{R}\right)^{1/2} \frac{\langle p \rangle}{B_p(a)} \int_0^1 \frac{1}{b_p(\rho)} \frac{\partial (p(\langle p \rangle))}{\partial \rho} \rho^{1.5} d \rho.
\]

where \( B_p(r) = \mu_0 I_p/2\pi a, b_p(\rho) \equiv B_p(r)/B_p(a), r = r/a.\) When the average poloidal beta is denoted by \( \beta_p \), we have

\[
\frac{I_{bs}}{I_p} = c_b(a/R)^{0.5} \beta_p,
\]

where \( c_b \) is

\[
c_b = -0.5 \int_0^1 \frac{1}{b_p(\rho)} \frac{\partial (p(\langle p \rangle))}{\partial \rho} \rho^{1.5} d \rho = -0.5 \int_0^1 \frac{q(\rho)}{q_i(1)} \frac{\partial (p(\langle p \rangle))}{\partial \rho} \rho^{0.5} d \rho.
\]

Since \( \beta_i = 0.01 \beta_N I_p/(aB_t), B_p/B_t = \mu_0 I_p/(2\pi K a B_t) = 0.2K(I_p/aB_t), B_p/B_t = aK/Rq_i, \beta_p \) is reduced to

\[
\beta_p = 0.25K^2 \beta_N (aB_t/I_p) = 0.05 \beta_N q_i,
\]

and

\[
\frac{I_{bs}}{I_p} = C_{bs} a^{0.5} \beta_N q_i, \quad C_{bs} = 0.05 c_b.
\]

When the driven current and the driving power are denoted by \( I_{cd} \) and \( P_{cd} \), the current drive efficiency \( \eta_{cd} \) is defined by

\[
I_{cd} = \eta_{cd} (\langle n \rangle R P_{cd}).
\]

The current drive efficiencies of LHCD [7, 8], ECCD [9] and NBCD [10–12] are all proportional to the electron temperature \( T_e \). Then the driven current \( I_{cd} \) is

\[
I_{cd} = \frac{\eta_{cd}(\langle T_e \rangle) (n)(\langle T_e \rangle)}{(\langle n \rangle)^2 R} P_{cd} = \frac{(\eta_{cd}(\langle T_e \rangle)(n)(\langle T_e \rangle + (f_{\text{DT}} + f_{\text{He}} + f_3(T_i))(T_e))}{(\langle n \rangle)^2 R (1 + (f_{\text{DT}} + f_{\text{He}} + f_3(T_i))(T_e))} P_{cd},
\]

and

\[
I_{cd}(\text{MA}) \approx 2.48 \times 10^{-2} \frac{\eta_{cd}(\langle T_e \rangle)(1 + (f_{\text{DT}} + f_{\text{He}} + f_3)/\gamma T)}{r_{\text{profi}}^2 (1 + (f_{\text{DT}} + f_{\text{He}} + f_3)/\gamma T)}.
\]
Fusion power is the necessary condition for the steady state operation, where $C_{\text{P}}$ is in the unit of $10^{19}$ (A/\text{m}$^2$). The power of current drive $P_{\text{cd}}$ is given by (7) as follows

$$P_{\text{cd}} = \frac{f_{\text{bs}}(T_{\text{he}})(\frac{\alpha R}{c})^{\frac{1}{2}}}{\langle T_{\text{ekeV}} \rangle^{\frac{1}{3}}} \cdot$$

The specified bootstrap current $I_{\text{bs}}$ is 4.5 MA. This specification requires $C_{\text{bs}} = 0.0374$ and then $C_{\text{bs}} = 0.748$.

In the full non-inductive current drive experiment in JT60-U ($a/R = 0.24$, $\beta_{p} = 2.7$, reversed shear), the estimated value of $C_{\text{bs}}$ is 0.6 [14]. The result of the simulation $I_{\text{bs}} = 4.5$ MA of reference scenario 4, type II is probably due to the profile optimization of plasma pressure and safety factor.

The specified externally driven current is $I_{\text{cd}} = 4.5$ MA with $P_{\text{cd}} = P_{\text{ext}} = 4.14$ MW. The necessary value of $C_{\text{cd}}$ is $C_{\text{cd}} = 0.329 \times 10^{-2}$ and the necessary current drive efficiency $\eta_{\text{cd}}$ is given by (14) as follows

$$\eta_{\text{cd}} = 0.133^{\frac{1}{2}} \frac{f_{\text{bs}}^{(2)}}{f_{\text{bs}} + f_{\text{he}} + f_{\beta}} \langle T_{\text{ekeV}} \rangle^{\frac{1}{3}} \approx 0.259\langle T_{\text{ekeV}} \rangle^{\frac{1}{3}}.$$

The experimental current drive efficiency by the neg-
The Q value is sensitive to $H_{q2}$, $k_x$, $q_1$ and $B_t$ respectively. Similarly, $\Delta Q_{cd}$ is given by

$$\frac{\Delta Q_{cd}}{Q_{cd}} = \sum_i \gamma_i \frac{\Delta q_i}{q_i},$$

(17)

where $q_i$ and $\gamma_i$ are given in Table 5. $Q_{cd}$ is sensitive to $N_G$, $\beta_N$, $B_t$ and $a$ respectively. Note that $\alpha_{cd} = -3.097$, $\alpha_{NG} = +1.326$ and $\alpha_{BN} = -1.26$ in the Q value, while $\gamma_{cd} = I_{bs}/I_{cd}$, $\gamma_{NG} = -2$ and $\gamma_{BN} = +4$ in the QCD value. $\Theta(T_i)$ is the function of the volume average ion temperature ($T_i$) and $\langle T_i \rangle \propto b_k B_t a / N_G$ (refer to (12)). Therefore we have

$$\frac{\Delta \Theta}{\Theta} = \left\{ \frac{\partial \Theta / \partial (T_i)}{\langle T_i \rangle} \right\} \left[ \frac{\Delta (T_i)}{\langle T_i \rangle} \right]$$

$$= \left\{ \frac{\partial \Theta / \partial (T_i)}{\langle T_i \rangle} \right\} \left( \frac{\Delta \beta_N}{\beta_N a} + \frac{\Delta a}{a} - \frac{\Delta N_G}{N_G} + \frac{\Delta B_t}{B_t} \right).$$

The value of $\beta_N$ is near the stability limit, while the values of $q_1$ and $N_G$ have the margin to the stability limits in the steady state operation scenario. Therefore we choose $q_1$ and $N_G$ as the control parameters. Let us consider the two different steady state operations with the same parameters except ($q_1$, $N_G$) and ($q_1 + \Delta q_1$, $N_G + \Delta N_G$). Then we have

$$\frac{\Delta Q}{Q} = \left[ 1 + \frac{f_0}{5} \right] \left\{ \frac{\Delta q_1}{q_1} \right\} \times \left\{ -3.10 \frac{\Delta q_1}{q_1} + \left( 1.32 - \frac{\partial \Theta / \partial (T_i)}{\langle T_i \rangle} \right) \frac{\Delta N_G}{N_G} \right\},$$

$$\frac{\Delta Q_{cd}}{Q_{cd}} = \frac{I_{bs} \Delta q_1}{I_{cd} q_1} \left( 2 + \frac{\partial \Theta / \partial (T_i)}{\langle T_i \rangle} \right) \frac{\Delta N_G}{N_G}.$$

In the case of steady state operation reference scenario 4, type II in Ref. [5], the ion temperature is high ($T_i(0) = 34 \text{ keV}$) with peaked profile, so that $\partial \Theta / \partial T < -0.0483$ is negative (refer to Fig. 1) and the fusion reaction rate decreases as the average ion temperature increases. Then we have $\langle \partial \Theta / \partial (T_i) \rangle / \langle (T_i) \rangle = -0.76$ and $\Delta (\log Q) = -6.05 (\log q_1) + 4.06 (\log N_G)$, $\Delta (\log Q_{cd}) = \Delta (\log q_1) - 1.24 (\log N_G)$.

The dependences of $Q$ and $Q_{cd}$ on $N_G$ and $q_1$ are shown in Fig. 2.
Fig. 2 The dependences of $Q$ and $Q_{cd}$ on $N_G$ are shown in the upper figure and the dependences of $Q$ and $Q_{cd}$ on $q_I$ are shown in the lower figure in the case of the ITER reference scenario 4, type II in Ref. [1]. The vertical axis and the horizontal axis are in log scale.

Fig. 3 The dependence of $Q_{cd}$ on $N_G$ under the constraint of $\Delta Q = 0$, that is, $\Delta q_I/q_I = 0.671 \Delta N_G/N_G$. The vertical axis and the horizontal axis are in log scale.

Under the constraint of $\Delta Q = 0$, that is, $\Delta q_I/q_I = 0.671 \Delta N_G/N_G$, we have $\Delta Q_{cd}/Q_{cd} = -0.569(\Delta N_G/N_G)$ and the dependence of $Q_{cd}$ on $N_G$ is shown in Fig. 3. Reduction of $N_G$ is more effective to increase of $Q_{cd}$ than the effect that the decrease of $q_I$ (increase of $I_p$) reduces $Q_{cd}$ under the constraint of $\Delta Q = 0$.

5. Conclusion

More quantitative scaling laws of $Q$ and $Q_{cd}$ are derived and examined by comparison with the data of standard scenario of inductive operation and reference scenario of non-inductive operation of ITER. It is confirmed that the results of scaling laws of $Q$ and $Q_{cd}$ are consistent with the data of both standard scenario of inductive operation and reference scenario of non-inductive operation of ITER.

The dependence of $Q$ and $Q_{cd}$ on plasma parameters are studied and it is found that the control of safety factor $q_I$ and Greenwald fraction $N_G$ is effective to satisfy $Q_{cd} = Q$.