Super-High Magnetic Fields in Spatially Inhomogeneous Plasma

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The new phenomenon of a spontaneous magnetic field in spatially inhomogeneous plasma is found. The criteria for instability are determined, and both the linear and nonlinear stages of the magnetic field growth are considered; it is shown that the magnetic field can reach a considerable magnitude, namely, its pressure can be comparable with the plasma pressure. Especially large magnetic fields can arise in hot plasma with a high electron density, for example, in laser-heated plasma. In steady-state plasma, the magnetic field can be self-sustaining. The considered magnetic fields may play an important role in thermal insulation of the plasma.

Keywords: spatially inhomogeneous plasma, instability, spontaneous magnetic field, self-sustaining magnetic field in steady-state plasma

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The phenomenon of self-excitation of spontaneous magnetic fields (SMFs) in plasma was discussed until the end of 1960s mostly in connection with problems of astrophysics using the "turbulent dynamo" model, which was first formulated by Batchelor [1]. The discussion of this model and the main results of its application can be found in the monograph of Zeldovich et al. [2]. In the late 1960s spontaneous magnetic fields were discovered experimentally in a laser spark [3] and the plasma produced by irradiating a solid surface with a laser pulse [4]. Preliminary estimates of the magnetic field in the laser-produced plasma, which were grossly overestimated ($B \sim 1$ MG at an electron density of $n \sim 10^{21}$ cm^{-3}) gave hope to the possibility of obtaining an effective magnetic thermal insulation of plasma; however, these expectations were not realized. The quantities of the magnetic fields in laser plasmas and the instability mechanism were thoroughly studied in [5–7]; it was found analytically in paper [6] and numerically in paper [7] that at the nonlinear stage, the magnetic field saturates at $\omega\tau_e \sim 1$, i.e., the ratio of the cyclotron frequency of electrons to the frequency of their collisions with ions is of the order of unity. In the same time in compressed targets, where the electron density can reach the values of $\sim 10^{27}$ cm^{-3} or higher, the magnetic fields at $\omega\tau_e \sim 1$ and temperature $T \sim 10$ keV can be of the order of 100 MG and above [8]. The kinetic theory for spontaneous magnetic field was considered in [9].

Below we discuss the appearance of a spontaneous magnetic field on the basis of the mechanism, which is associated with the instability of the thermal diffusion currents flowing perpendicular to the magnetic field and temperature gradient. We will show that in the nonlinear regime, the magnetic field due to this mechanism can reach such values that its pressure becomes comparable with the plasma pressure; in this case, the ion $\omega\tau_I \gg 1$ is reached, resulting in an effective magnetic thermal insulation of plasma. The rate of the increase in the magnetic field due to the considered instability can be much slower than due to non-co linearity of the density and temperature gradients (in the laser plasma [3–9]), but under conditions of a steady-state plasma the SMFs emerging fields reach higher values.

Consider the emergence of a magnetic field for a simple case of a plane plasma layer. Note that the mechanism of instability considered is possible only in the plasma area where plasma current can be closed. It is assumed that the density and temperature gradients of the plasma are parallel to the x axis, their gradients in the symmetry plane $x = 0$ being equal to zero; in turn, electric current of thermal diffusion and the magnetic field are parallel to the y and z axes, respectively. In the hydrodynamic approximation, at $\omega\tau_e \ll 1$, the magnetic field is related to the electric current density $\boldsymbol{j}$ by the equation (below we retain the notation from [10])

$$\frac{\partial B}{\partial t} = -\frac{c}{en}\nabla n\times\nabla T - \frac{c}{e}(\nabla\times\boldsymbol{R}_T)/n - c\left(\nabla\times\frac{\boldsymbol{j}_\perp}{\sigma_\perp}\right) \quad (1)$$

$$\boldsymbol{R}_T = -0.8n\omega\tau_e\boldsymbol{h}\times\nabla T \quad \text{where } \omega\tau_e \ll 1 \quad (2)$$

Here, $\boldsymbol{h}$ is unit vector in the direction of the magnetic field $\boldsymbol{B}$ and ∇T is the temperature gradient. The electric current density $\boldsymbol{j}$ is perpendicular to the temperature gradient and the magnetic field direction. We have omitted terms associated with the magnetic viscosity and hydrodynamic motions in the plasma, which is unimportant for further consideration. In the most common case of an inhomogeneous plasma, where the gradients of the plasma density and temperature are parallel (e.g., in installations for magnetic plasma confinement), the above mechanism of spon-

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taneous magnetic field excitation does not possibly work, but another mechanism, which we will discuss below, is possible. The rate of an increase in the magnetic field in this case may be substantially lower than in the case of non-co linearity, but the magnetic field saturates at a much higher level. We assume that the plasma is in the state when the plasma pressure is independent of the coordinates. After simple transformations, Eq. (1) together with expression (2) for the "thermal force" [10] can be represented in the form (quadratic fluctuations $\delta T \cdot B$ are neglected, as usual):

$$\frac{\partial B}{\partial t} = \psi(x)B + \varphi(x)\frac{\partial B}{\partial x}, \tag{3}$$

Equation (3) is the same for both y- and z-component of the magnetic field, $B = B_y, B_z$; $\psi(x)$ and $\varphi(x)$ are the functions of the electron temperature and gradients of the plasma density and temperature

$$\psi(x) = \frac{0.8}{m}[\nabla_x \tau_e \cdot \nabla_x T_e + \tau_e \cdot \nabla_x^2 T_e] \tag{4}$$

$$\varphi(x) = \frac{0.8}{m}\tau_e \cdot \nabla_x T_e \tag{5}$$

As we see, Eq. (3) is a linear partial differential equation where the coefficients at B and its derivative are the functions of the x-coordinate. It follows from Eq. (3) that the instability is convective in nature. To solve this equation, the semi classical approximation is inapplicable, because the dimensions of the inhomogeneties of the magnetic field and plasma are comparable. Therefore, we will use the following obvious way. We will represent the magnetic field variation over time in the form

$$B(t, x) = B(0, x)\exp \gamma t, \tag{6}$$

where the "increment" γ is a function of the coordinates, $\gamma = \gamma(x)$. After substituting expression (6) in Eq. (3), we obtain for the instability increment

$$\gamma(x) = \psi(x) + \varphi(x)\frac{1}{B(0, x)}\frac{\partial B}{\partial x}(0, x), \tag{7}$$

where $B(0, x)$ is the initial condition, $B(0, x) = B(t, x)$ at $t = 0$. As seen from Eq. (7), the rate of the increase in the magnetic field depends on the coordinates, which is a result of a "drift" of the magnetic field. Note that the magnetic field, depending on the geometry of the problem, may change the sign at point $x = 0$ passing through the yz plane.

It follows from Eq. (6) that the initial magnetic field will increase (or decrease) with time, depending on the sign of $\psi(x)$ and $\varphi(x)$, i.e., on the signs of the gradients τ_e and T_e. In this case, the first term in the right-hand side of Eq. (4) under conditions of a neomagnetic confinement [11], when the sum of total plasma pressure and magnetic field is preserved, is always positive; at the same time, the sign of the second term depends on the sign of the heat source $Q(x, T)$ and the behavior of the temperature dependence of thermal conductivity. In Coulomb collisions, $\gamma > 0$ in the case of the heat source $Q(x, T) < 0$. Thus, in a high-temperature plasma, the instability develops primarily in the temperature region where the bulk energy loss exceeds the heat release (for example, exceeds the energy effect of nuclear fusion reactions). It follows from Eq. (3), the magnetic field "drifts" from this region to the region of higher temperatures. Assuming that $Q < 0$ (bremsstrahlung losses dominate), we give approximate expressions to estimate the rate of the increase in the magnetic field. For the increment γ, we have

$$\gamma \sim v_e^2 \tau_e / a^2 \tag{8}$$

where a is the characteristic size of the plasma and v_e-electron thermal velocity.

Let us present the estimates with respect to the laser plasma. Assuming the plasma temperature to be ~10 keV, the plasma density to be $n \sim 10^{21}\,\mathrm{cm}^{-3}$ we obtain $\gamma \sim 10^9\,\mathrm{s}^{-1}$ for $a \sim 1$ cm. This means that the magnetic field rapidly increases and the electronic $\omega\tau_e \sim 1$ is achieved in a fairly short time. Then, when $\omega\tau_e > 1$ the exponential instability is replaced by a linear increase with time

$$\mathrm{d}B^2/\mathrm{d}t \sim \mathrm{mc}^2/\mathrm{e}^2(qv_e/a^2)nT \sim 10^{20}\,\mathrm{G}^2\mathrm{s}^{-1} \tag{9}$$

for the plasma parameters mentioned above and $a \sim 10^{-2}$ cm where q is the cross section for Coulomb scattering. This means that to obtain a field of the order of ~1 MG we need a fairly short time ($t \sim 10^{-8}$ s). It follows from the formula (9) the rate in the increasing magnetic field depends on plasma pressure.

After the magnetic field reaches its maximum value, this field will be self-sustaining. Assuming that the plasma parameters do not vary with time, we consider this phenomenon by the example of toroidal plasma geometry. We assume that the external magnetic fields are absent. The spontaneous magnetic field is sustained by thermal diffusion currents flowing perpendicular to the gradient of the temperature and magnetic field. To solve the problem, we use the corresponding equation for thermal diffusion currents from [10] (Eq. (6.30), p. 240)

$$\nabla \times \left(\frac{3}{2}\frac{c}{e}\frac{1}{\omega\tau}\boldsymbol{h} \times \nabla T - c\frac{\boldsymbol{j}}{\sigma_\perp}\right) = 0, \tag{10}$$

where $\boldsymbol{j}$ is the current density, which includes the toroidal j_T and poloidal j_P components, and $\sigma_\perp$ is the transverse conductivity of the plasma. It is worth noting

Note. The first term in the right-hand side of Eq. (1) describes the excitation of a spontaneous magnetic field due to non-co linearity of the plasma density and temperature gradients [2–8]; in this case, the increase in the magnetic field for the estimates can be expressed as $\mathrm{d}B/\mathrm{d}t \approx 6 \cdot 10^{19} Tf/a^2$ G/s, where f is the sine of the angle between the directions of the density and temperature gradients, a is the spatial scale of inhomogeneities (in cm), and T is the plasma temperature (in CGS units). As seen from the above formula, the spontaneous magnetic field increases at a high rate, and during the laser pulse ~1-10 ns reaches its maximum. In compressed spherical targets, where the electron density reaches a value of $n \sim 10^{27}\,\mathrm{cm}^{-3}$ and higher, the magnetic field at $\omega\tau_e \sim 1$ may exceed the value of $B > 100$ MG [8].

that the results of calculations by the Chapman – Enskog method [10], the Grad method [12, 13] and in the kinetic consideration within the framework of the Lorentz approximation [14] of thermal diffusion coefficients (in our case, the value of the coefficient 3/2 and the sign in Eq. (10)), which are important for self-sustaining the magnetic field, completely coincide. After integrating Eq. (10) in r and taking into account Maxwell's equation

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{j}, \tag{11}$$

we obtain for toroidal B_T and poloidal B_P components of magnetic field

$$\frac{3}{2}\frac{c}{e}\frac{1}{\omega\tau}\frac{\mathrm{d}T}{\mathrm{d}r} + \frac{c}{4\pi\sigma}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rB_\mathrm{P}) = 0, \tag{12}$$

where $\omega = eB_\mathrm{P}/mc$

$$\frac{3}{2}\frac{c}{e}\frac{1}{\omega\tau}\frac{\mathrm{d}T}{\mathrm{d}r} + \frac{c}{4\pi\sigma}\frac{\mathrm{d}}{\mathrm{d}r}B_\mathrm{T} = 0, \tag{13}$$

where $\omega = eB_\mathrm{T}/mc$
Further, assuming the plasma equilibrium [15]

$$\nabla p + \frac{1}{4\pi r}B_\mathrm{p}\frac{\mathrm{d}}{\mathrm{d}r}rB_\mathrm{p} + \frac{1}{4\pi}B_\mathrm{T}\frac{\mathrm{d}}{\mathrm{d}r}B_\mathrm{T} = 0 \tag{14}$$

from Eq. (12), (13) and (14) we find

$$p = p_0(T/T_0)^k, \tag{15}$$

where $p = n_\mathrm{e}(T_\mathrm{e} + T_\mathrm{i})$ is the total plasma pressure, p_0 is its value at $r = 0$ and n_e is the density of electrons (ions). In the plasma layer adjacent to the plasma edge (and at a low toroidicity, $a \ll R$, where a and R are small and large radii of the torus), we can neglect the curvature of magnetic field lines; in this case, for the poloidal magnetic field component we obtain the relation

$$B_\mathrm{P}^2/8\pi \approx p_0[1 - (T/T_0)^k] \tag{16}$$

For the toroidal component we obtain a similar equation, which however remains valid for the whole cross section of the plasma. The power k in Eq. (15) – (16) depends on the ratio between the electron and ion temperatures. When $T_\mathrm{e} = T_\mathrm{i}$, we have $k = 3/2$. Thus, in this case when the temperature decreases, the plasma pressure drops and the magnetic field increases, reaching the values such that the magnetic field pressure at the plasma edge $T \to 0$ becomes comparable with the plasma pressure at $T = T_0$. For example, in laser plasma at density of electrons $n \sim 10^{21}\,\mathrm{cm}^{-3}$ and temperature $T \sim 10\,\mathrm{keV}$ we have the estimate $B \sim 30\,\mathrm{MG}$. At the same time in the compressed plasma when plasma density can reach values of about $10^{26}\,\mathrm{cm}^{-3}$ we could have a field of $\sim 10^6$ T (!).

At temperatures close to the maximum value, the magnetic field is weak and the plasma pressure exceeds the pressure of magnetic field, $\beta = 8\pi p/B^2 \gg 1$. At low temperatures $T \sim$ 10-100 eV, the confinement of plasma by the magnetic field is disturbed and the plasma equilibrium is reduced to the fact that the total pressure of the plasma and neutral gas becomes equal to the pressure of the plasma in the center [11, 16]. In this case, the density of the "cold" plasma (and neutral gas) towards its periphery rapidly increases with decreasing temperature. Thus, at a temperature $T \sim 1000\,\mathrm{K}$, the density of neutral particles near the surface will reach the value of $N \sim 10^{19}\,\mathrm{cm}^{-3}$, if the plasma density in the center is $\sim 10^{14}\,\mathrm{cm}^{-3}$ at $T \sim 10\,\mathrm{keV}$. In the deuterium plasma, the temperature is higher and, therefore, the protective layer of neutral particles will be denser. In addition, the density of neutral particles will increase with increasing plasma density in the center. Thus, when the plasma density on the axis is $\sim 10^{15}\,\mathrm{cm}^{-3}$, we have $N \sim 10^{20}\,\mathrm{cm}^{-3}$, which corresponds to a pressure of about 10 bar. For a sufficiently large area occupied by the cold plasma and neutral gas with a high density of particles, fast charged particles cannot reach the walls without a complete loss of energy.

Consider a poloidal magnetic field in the vicinity of $r \sim 0$, where the field is minimal and vanishes at $r = 0$. Given that $B \to 0$ at $r \to 0$, the ratio B/r can be replaced by the derivative $\mathrm{d}B/\mathrm{d}r$ and the equation of plasma equilibrium can 8π be transformed; as a result, we find the local relation between the magnetic field and plasma temperature

$$B_\mathrm{P}^2/8\pi \approx \frac{1}{2}p_0[1 - (T/T_0)^k] \tag{17}$$

Thus, the relation for the magnetic field strength with the temperature near the major axis of the torus is defined by a similar relation (16), but the magnetic field strength in this case is twice lower. The relation between the plasma pressure and temperature is similar to the previously obtained Eq. (15). The self-sustaining magnetic field increases towards the plasma periphery; therefore, the minimum B takes place in the center of the plasma column.

We have considered the emergence of the instability of spatially inhomogeneous plasma with respect to the excitation of the magnetic field. The rate of the increase in the field is sufficient for the magnetic field to reach, in a fairly short time, such values when its pressure becomes comparable with the plasma pressure. In the steady-state or quasi-stationary plasma, self-sustaining of the magnetic field is possible when the magnetic field strength and its spatial dependence are determined by the dependence of plasma parameters on the spatial coordinates.

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