The Stability of Flute Modes in the GAMMA10 A-divertor*)

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The linear growth rates of flute instability are investigated in the GAMMA10 A-divertor magnetic geometry. It is found that the minimum-B in the remaining anchor cell can stabilize the flute mode even in the GAMMA10 A-divertor which contains an axisymmetric divertor mirror cell, although the flute modes are not stabilized in case of weak magnetic well.

Keywords: flute mode, interchange mode, mirror, MHD, simulation

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1. Introduction

The GAMMA10 tandem mirror is planning to replace one anchor cell with an axisymmetric divertor mirror cell, which is called "the GAMMA10 A-divertor" at present shown in Fig. 1. The main purpose of installing a divertor mirror cell is to perform the simulation experiments of a divertor of a big torus such as LHD and ITER.

We are here interested in the stability of the flute mode in the GAMMA10 A-divertor. The min.B mirror and the divertor mirror in the GAMMA10 A-divertor have a different flute mode stability mechanism with each other. That is, there are the good magnetic field line curvatures in the anchor cell, but the bad curvatures in the divertor mirror in the core region. The long thin approximation of magnetic field lines can be applied to the anchor mirror, but not to the divertor mirror.

It is known that the flute mode is stabilized mainly by the plasma compressibility in a divertor mirror [1, 2], while it is stabilized by the good magnetic field line curvature in the min.B anchor mirror cell [3]. The stability boundary of the flute modes was found to depend on the radial profiles of mass density and temperature in an axisymmetric divertor mirror [1, 2].

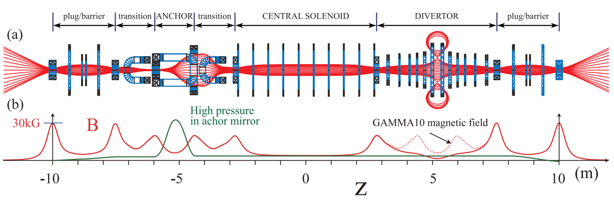

Fig. 1 GAMMA10 A-divertor. (a) is the magnetic field lines with coils and (b) is the axial magnetic field and pressure profiles.

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The GAMMA10 A-divertor requires the three-dimensional treatment for the flute mode linear analysis. There are the basic equations for the flute mode fluctuations which are applicable to the axisymmetric magnetic field including divertor mirror cell [4, 5]. In the paper we carry out the stability analysis by extending the equations to the GAMMA10 A-divertor.

2. Basic Equation

The flute mode stability criterion in the open systems such as a tandem mirror is given by [3],

$$\int \frac{[\hat{p}_\perp(\chi) + \hat{p}_\parallel(\chi)]\kappa_\psi}{B^2}\, \mathrm{d}\chi \geq 0\,. \tag{1}$$

Here the curvature of a magnetic field line is represented by the covariant components $\boldsymbol{\kappa}\,(\equiv \hat{e}_\parallel \cdot \nabla \hat{e}_\parallel) = \kappa_\psi \nabla\psi + \kappa_\theta \nabla\theta$, where the coordinates (ψ, θ, χ) satisfy $\boldsymbol{B} = \nabla\psi \times \nabla\theta = \nabla\chi$. The components, parallel and perpendicular to the magnetic field $\boldsymbol{B}$, of plasma pressure $p_{\parallel,\perp}$ are described by a separation of variables $p_{\parallel,\perp}(\psi,\chi) = \hat{p}_{\parallel,\perp}(\chi)\nu(\psi)$. Note that Eq. (1) can be applied to the non-axisymmetric mirror such as GAMMA10.

In the case of isotropic pressure the stability criterion (1) is written as

$$\frac{\partial U}{\partial \psi} = -2 \int \frac{\kappa_\psi}{B^2}\, \mathrm{d}\chi \leq 0\,. \tag{2}$$

Here U is the specific volume of a magnetic field line defined by

$$U = \int \frac{1}{B^2}\, \mathrm{d}\chi, \tag{3}$$

and the relation $\nabla_\perp B = B\boldsymbol{\kappa}$, which holds in the vacuum magnetic field, was used to obtain Eq. (2).

Equation (2) indicates that $\partial U/\partial\psi$ represents the effect of a magnetic field line curvature. That is, if $\partial U/\partial\psi \leq 0$ then there is a magnetic well having the stabilizing effects on the flute modes. Namely, the specific volume of a magnetic field line includes the effect of the magnetic field line curvature.

For the purpose that the equations derived on the assumption of axisymmetric magnetic field are applied to the non-axisymmetric (but effectively axisymmetrized) GAMMA10 A-divertor, the specific volume U is redefined as

$$U = \int \frac{\hat{p}_\perp(\chi) + \hat{p}_\parallel(\chi)}{B^2}\, \mathrm{d}\chi\, . \tag{4}$$

Equation (4) assures that the condition of magnetic well $\partial U/\partial\psi \leq 0$ gives the same stability criterion as Eq. (1).

The basic equations (the reduced MHD equations) for flute mode fluctuations are written as [4, 5]

$$\frac{\partial \hat{w}}{\partial t} + [\![\Phi, \hat{w}]\!] - [\![\hat{\rho}, \langle\frac{v_\alpha^2}{2}\rangle]\!] + \frac{1}{U^\gamma}\frac{\partial U}{\partial\psi}\frac{\partial\hat{\rho}\hat{T}}{\partial\theta} = \{DT\}, \tag{5}$$

$$\frac{\partial}{\partial\psi}\left(\hat{\rho}\langle r^2\rangle\frac{\partial}{\partial\psi}\Phi\right) + \frac{\partial}{\partial\theta}\left(\hat{\rho}\left\langle\frac{1}{r^2B^2} + \lambda^2B^2\right\rangle\frac{\partial}{\partial\theta}\Phi\right) = \hat{w}, \tag{6}$$

$$\frac{\partial\hat{\rho}}{\partial t} + [\![\Phi, \hat{\rho}]\!] = \{DT\}, \tag{7}$$

$$\frac{\partial\hat{T}}{\partial t} + [\![\Phi, \hat{T}]\!] = \{DT\}\, . \tag{8}$$

The symbol $\{DT\}$s in the right hand side in Eqs. (5), (7), (8) represent the diffusion terms resulting from the classical viscosity, resistivity and conductivity. The quantity $\hat{T}$ is defined as $\hat{T} \equiv (T_i + T_e)U^{2/3}/M_i$ where T_i (T_e) is the ion (electron) temperature and M_i is ion mass, $\hat{\rho}$ is defined as $\hat{\rho} \equiv \rho U$ where ρ is mass density, $\hat{w}$ is defined as $\hat{w} = wU$ where w is the plasma vorticity described in Eq. (6), Φ is the electrostatic potential, v_α is the plasma adiabatic velocity, γ is specific heat index. The symbol $\langle A\rangle$ means the average quantity of A along a magnetic field line, i.e., $\langle A\rangle \equiv (1/U)\int(A/B^2)\mathrm{d}\chi$ and $\lambda = U(\partial/\partial\psi)(\int_0^\chi[1/UB^2]\mathrm{d}\chi) + (\nabla\psi\cdot\nabla\chi)/(r^2B^4)$. The notation $[\![\ ]\!]$ defined by the equation,

$$[\![A, B]\!] \equiv \frac{\partial A}{2x\partial x}\frac{\partial B}{\partial\varphi} - \frac{\partial A}{\partial\varphi}\frac{\partial B}{2x\partial x}, \tag{9}$$

is known as the Poisson bracket and this term represents the convective $\boldsymbol{E}\times\boldsymbol{B}$ flow term in Eqs. (5)–(8).

Although Eqs. (5)–(8) are derived on the assumption of the axisymmetric magnetic field, we apply these equations to the effectively axisymmetrized GAMMA10 A-divertor by changing the specific volume of a magnetic field line to Eq. (4)

3. Non-local Linear Analysis

Henceforth all variables are normalized as $D = \hat{\rho}/\hat{\rho}_M$, $T = \hat{T}/\hat{T}_M$, $w = \hat{w}\psi_b/\epsilon\rho_M U_M b c_{sM}$, $\phi = \Phi b/\epsilon c_{sM}\psi_b$. Here subscript $_M$ means the quantity at the midplane of a divertor mirror cell, ψ_b is the coordinate at the separatrix (x-point), c_s is sound speed, $b = \sqrt{\psi_b/B_M}$, $x = \sqrt{\psi/\psi_b}$, $\varphi = \theta$, ϵ is the small expansion parameter which are used to obtain Eqs. (5)–(8), where we assume $\epsilon^2 = 10^{-2}$ in the paper.

The variables are divided into the equilibrium quantities and the perturbed quantities as

$$\begin{aligned}
D(x,\varphi) &= D_E(x) + \epsilon^2\textstyle\sum_{m\neq0} D_{f(m)}(x)\exp\{\mathrm{i}m\varphi - \mathrm{i}\omega\tau\},\\
T(x,\varphi) &= T_E(x) + \epsilon^2\textstyle\sum_{m\neq0} T_{f(m)}(x)\exp\{\mathrm{i}m\varphi - \mathrm{i}\omega\tau\},\\
w(x,\varphi) &= w_0(x) + \textstyle\sum_{m\neq0} w_{f(m)}(x)\exp\{\mathrm{i}m\varphi - \mathrm{i}\omega\tau\},\\
\phi(x,\varphi) &= \phi_0(x) + \textstyle\sum_{m\neq0} \phi_{(m)}(x)\exp\{\mathrm{i}m\varphi\},
\end{aligned} \tag{10}$$

where $\tau = \epsilon t c_{sM}/b$ is the normalized time and m is the azimuthal mode number.

The non-local linear dispersion equation is obtained by linearizing Eqs. (5)–(8) with neglect of the terms $\{DT\}$,

$$\begin{aligned}
&\frac{\partial}{\partial x}\left(\frac{D_E f_1}{x}\frac{\partial\phi_{(m)}}{\partial x}\right) - \frac{\partial}{\partial x}\left(\frac{mf_1}{2x^2}\phi_{(m)}\frac{\partial\phi_0}{\partial x}\frac{\partial D_E}{\partial x}\Big/\left(\omega - \frac{m}{2x}\frac{\partial\phi_0}{\partial x}\right)\right)\\
&\quad - \left(4xD_E(f_3+f_4)m^2 + \frac{4xw_{f(m)}}{\phi_{(m)}}\right)\phi_{(m)} = 0,
\end{aligned} \tag{11}$$

where $\phi_0(x)$ is determined by

$$\frac{\partial}{\partial x}\left(\frac{D_E f_1}{x}\frac{\partial\phi_0}{\partial x}\right) = 4xw_0, \tag{12}$$

and the last term in left-hand side of Eq. (11) is written as

$$\begin{aligned}
\frac{4xw_{f(m)}}{\phi_{(m)}} = &-\frac{m^2}{2x}\frac{\partial(V^2)_{(0)}}{\partial x}\frac{\partial D_E}{\partial x}\Big/\left(\omega - \frac{m}{2x}\frac{\partial\phi_0}{\partial x}\right)^2\\
&-2m\frac{\partial w_0}{\partial x}\Big/\left(\omega - \frac{m}{2x}\frac{\partial\phi_0}{\partial x}\right)\\
&-\frac{3m^2}{5\epsilon^2 x}\frac{1}{u^{5/3}}\frac{\partial u}{\partial x}\frac{\partial(D_E T_E)}{\partial x}\Big/\left(\omega - \frac{m}{2x}\frac{\partial\phi_0}{\partial x}\right)^2.
\end{aligned} \tag{13}$$

Here $(V^2)_0 = \frac{1}{2\pi\epsilon^2c_{sM}^2}\int\langle v_\alpha^2\rangle\mathrm{d}\varphi$, $f_1 = \frac{\langle r^2\rangle}{b^2}$, $f_3 = \langle\frac{B_M^2b^2}{B^2r^2}\rangle$, $f_4 = \langle\lambda^2B^2\rangle B_m^2b^2$. The specific volume is normalized to be $u(x) \equiv U(x)/U(0)$.

Because Eq. (11) is the second order differential equation with the eigen-value ω, Eq. (11) can be solved on the boundary condition $\phi_{(m)}(x=0)=0$ and $\phi_{(m)}(x=1)=0$ with help of a shooting method numerically, where the condition $\phi_{(m)}(x=1)=0$ comes from the effect of the electric short circuit along azimuthal magnetic null line.

4. Specific Volume *U*

The axial pressure profile is assumed to be

$$\hat{p}(\chi) \equiv \hat{p}_\perp(\chi) + \hat{p}_\parallel(\chi) = \max\left(p_A\frac{(B_m^2 - B^2)}{(B_m^2 - B_c^2)},\ 1\right), \tag{14}$$

where p_A is the pressure at the anchor midplane; B_c is the magnetic field at the midplane on axis in anchor cell and $B_m = 1.7B_c$, $B = B(\chi)$ is the magnetic field on axis, and the pressure $\hat{p}$ in the other region is set to be unity, the example of which is plotted in Fig. 2.

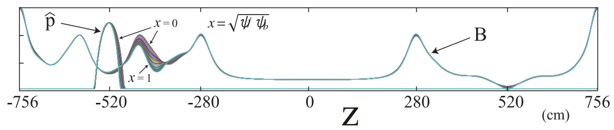

Fig. 2 Axial pressure $\hat{p}$ and magnetic field B profiles at various radii.

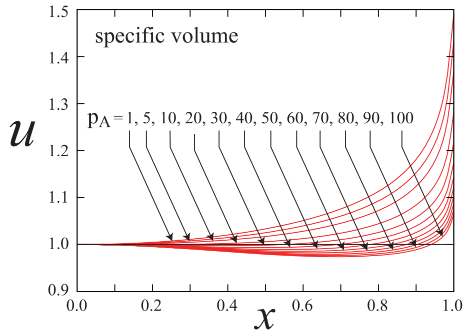

Fig. 3 Radial profiles of normalized specific volume $u(x)$.

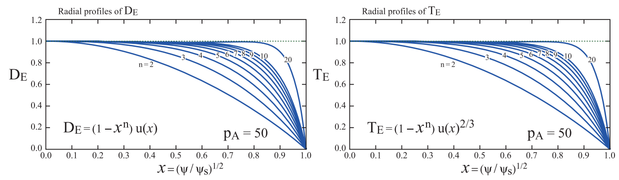

Fig. 4 Radial profiles of $D_E(x)$ and $T_E(x)$ for various parameters of n in Eq. (18).

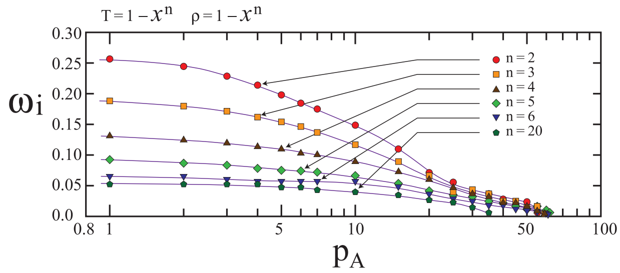

Fig. 5 Linear growth rate ω_i of $m = 1$ flute mode.

The specific volume $u(x)$ defined by Eq. (4) is plotted for various p_A in Fig. 3, where $u(x)$ is the same as Eq. (3) in the case of $p_A = 1$. There is the magnetic well in the core region $x \lesssim 1/2$ for $p_A \gtrsim 30$. Although $u(x)$ becomes infinitely large at the separatrix, the coordinate x is cut off just before the separatrix for the numerical problem in Fig. 3.

Now the flute modes stability criterion is mentioned in the following briefly [6]. The plasma internal energy in the unit magnetic flux tube is $Q_p = pU/(\gamma - 1)$, where uniform pressure p is assumed. The exchange of two neighboring magnetic flux tube with the same unit magnetic flux gives the change of the internal energy,

$$\delta Q_p = \delta p \delta U + \gamma p \frac{(\delta U)^2}{U} . \tag{15}$$

In the long thin device such as GAMMA10, where $|\delta p/p| \gg |\delta U/U|$ is satisfied, the stability criterion is given by

$$\delta Q_p \simeq \delta p \delta U \geq 0 . \tag{16}$$

The stability criterion is $\delta U \leq 0$ because $\delta p < 0$ in the real experimental device, which gives the same stability criterion as Eq. (1).

On the other hand, there is the region $U \to \infty$ in the neighborhood of x-point of a divertor mirror cell, where $|\delta p/p| \simeq |\delta U/U|$ is expected. The stability criterion in this case is written as

$$\delta Q_p = \frac{\delta U}{U^\gamma} \delta(pU^\gamma) \geq 0 . \tag{17}$$

The radial profile of the normalized specific volume $u(x)$ in Fig. 3 has a minimum point at $x \simeq 0.55$ for the case of $p_A = 50$, while there is a minimum at $x \simeq 0.50$ for $p_A = 40$. In order to satisfy $\delta Q_p \geq 0$ everywhere in the case of $p_A = 50$, $\partial(pu^\gamma)/\partial x \leq 0$ in the good magnetic field line curvature region of $x < 0.5$, while $\partial(pu^\gamma)/\partial x \geq 0$ in the bad curvature region of $x > 0.5$.

5. Numerical Results of $m = 1$ Flute Mode

One problem is to make clear how strong the min.B anchor cell can stabilize the flute mode for the experimentally expected pressure radial profile in the GAMMA10 A-divertor. The radial profiles of $D_E(x)$ and $T_E(x)$ are assumed to be

$$D_E(x) = (1 - x^n)\, u(x) , \quad T_E(x) = (1 - x^n)\, u(x)^{2/3}, \tag{18}$$

the profiles of which are plotted in Fig. 4 for the case of $p_A = 50$.

The radial profiles of $D_E(x)$ and $T_E(x)$ are the monotonically decreasing functions of x in Fig. 4 except for $n = 20$ in Eq. (18). In the case of $n = 20$ both $D_E(x)$ and $T_E(x)$ have a local minimum at $x \sim 0.5$.

Figure 5 plots the growth rate ω_i of $m = 1$ flute instability as a function of p_A, which was obtained by solving Eq. (11) with $D_E(x)$ and $T_E(x)$ in Eq. (18). It is found that the growth rate ω_i becomes smaller as p_A is larger; the $m = 1$ flute mode is stable in the range of $p_A \gtrsim 60$. The radial profiles of $D_E(x)$ and $T_E(x)$ in Fig. 4 are the unstable profile to the flute modes in the divertor mirror only. Therefore, the min.B anchor mirror stabilizes the flute modes in the GAMMA10 A-divertor for $p_A \gtrsim 60$. As mentioned in the previous section, however, the specific volume $u(x)$ shown in Fig. 3 indicates that magnetic well (min.B) region exists for $p_A \gtrsim 30$. That is, a shallow magnetic well can not stabilize the flute instability in the system containing an axisymmetric divertor mirror cell.

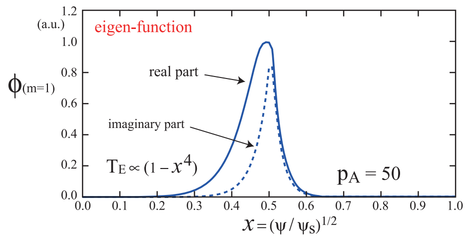

Fig. 6 Eigen-function of $m = 1$ flute mode.

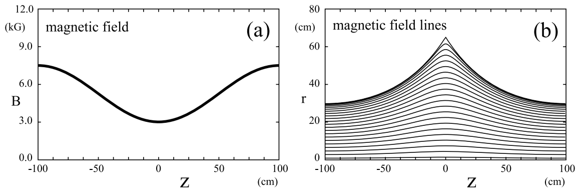

Fig. 7 A modeled single divertor mirror [4].

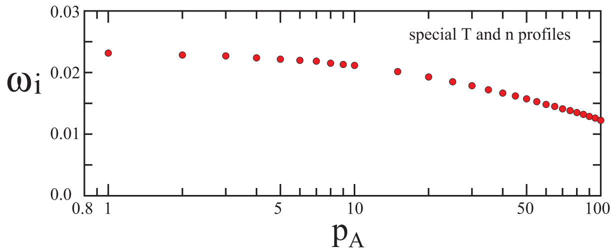

Fig. 8 Linear growth rate ω_i of $m = 1$ flute mode.

The growth rate is lower as n is bigger in Fig. 5. Note that $D_E(x)T_E(x) \propto pU^{\gamma}$. The radial profile with bigger n approaches $D_E(x) \simeq 1$ and $T_E(x) \simeq 1$, that is $pU^{\gamma} \simeq const$, which is a marginally stable profile to the flute mode as shown in Eq. (17). Thus the growth rate becomes lower with the radial profile of a bigger n.

Figure 6 plots the eigen-function of the $m = 1$ flute instability in the case of $n = 4$ and $p_A = 50$, which is localized around $x \sim 0.5$ with a peak just outside the magnetic well ($\partial u/\partial x < 0$).

If $pu^{\gamma} = const$ everywhere, the stability condition of flute modes is satisfied and the system is a marginally stable state. However, the classical transport is very large around x-point and so $pu^{\gamma} = const$ breaks around x-point. A question is that whether the magnetic well can stabilize the flute mode in such the pressure radial profile. We have performed the numerical simulation in the modeled single divertor mirror shown in Fig. 7, which is found in Ref. 4. In the linear growing phase of the flute instability we observed the radial profiles in the simulation [Eq. (28) in Ref. 4].

Figure 8 plots the linear growth rate ω_i of the $m = 1$ flute instability, where the radial profiles of $w_0(x)$, $D_E(x)$, $T_E(x)$, observed in the simulation [Eq. (28) in Ref. 4], were used. The remarkable point is that the flute instability is not stabilized by the magnetic well at all. The reason that the growth rate decreases with p_A in Fig. 8 is that the gradient

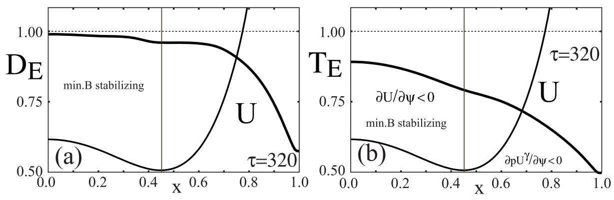

Fig. 9 Radial profiles of $D_E(x)$ and $T_E(x)$ observed in the simulation [7].

of specific volume at $x \gtrsim 0.7$ decreases with p_A in Fig. 3.

It is found in Figs. 5 and 8 that the stabilizing effects of min.B anchor cell on the flute modes depend strongly on the radial profile of D_E and T_E. Experimentally, plasma is sustained by gas puffing and externally injected micro-wave, that is the radial profile control is not so easy. Recently we obtained the results shown in Fig. 9 in the GAMMA10 A-divertor magnetic geometry, where the computer simulation was performed by solving Eqs. (5)–(8) with the initial conditions of $D_E(x) = 1$, $T_E(x) = 1$ [7].

Figure 9 is the radial profiles of D_E and T_E in the quasi-steady state, where plasma continues to be lost radially by the classical transport. The radial profiles in $x \gtrsim 0.45$ is unstable to the flute mode in the divertor, while those in $x \lesssim 0.45$ are stable in the min.B. That is, the flute modes are stabilized by the min.B anchor clearly in the GAMMA10 A-divertor. We, therefore, expect that the min.B anchor can stabilize the flute modes in the GAMMA10 A-divertor even with a magnetic divertor.

6. Summary and Discussion

We calculate the linear growth rate of the $m = 1$ flute mode for various radial profile of mass density and temperature in the GAMMA10 A-divertor. The GAMMA10 A-divertor contains a min.B anchor mirror cell and an axisymmetric divertor mirror cell. The flute modes are stabilized by a good magnetic field line curvature in an anchor mirror cell, while these are stabilized by mainly the plasma compressibility in a divertor mirror cell where the magnetic field line curvature is bad.

It is found that there is a radial profile of mass density and temperature where $pU^{\gamma} = const$ in the magnetic well, in the profile of which the min.B mirror can not stabilize the flute modes at all.

As long as the pressure radial profile is a monotonically decreasing function in the min.B region, the min.B mirror can stabilize the flute modes. It is also found that the shallow magnetic well, however, does not stabilize the flute modes even in the monotonically decreasing pressure radial profile.

The previous works [4, 7] have defined the specific volume as

$$U = \int \frac{\hat{p}_{\perp}(B) + \hat{p}_{\parallel}(B)}{B^2} d\chi, \qquad (19)$$

instead of Eq. (4). That is, the plasma pressures are represented by $p_{\perp,\parallel}(\psi, B) = \hat{p}_{\perp,\parallel}(B)\nu(\psi)$ in the MHD analysis [3]. The reason that B was used one of axes in the MHD analysis is to take into account the plasma current along a magnetic field line [8]. However, Eq. (19) gives $\partial U/\partial\psi = \int [\hat{p}_\perp(B) + \hat{p}_\parallel(B)]\kappa_\psi \mathrm{d}\chi/B^2 + \int \left(\partial[\hat{p}_\perp(B) + \hat{p}_\parallel(B)]/\partial\psi\right) \mathrm{d}\chi/B^2$, which includes the unnecessary term for the stability criterion. Equation (19) is valid in the paraxial approximation. Therefore the coordinates (ψ, θ, χ) are used in the present paper to take into account the flute stability criterion in Eq. (1) exactly in the non-paraxial divertor mirror. The results in the present paper revealed that the linear phase of the flute instability was almost the same as those in the previous works [4, 7].

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