Ignition Analysis for D Plasma with Non-Maxwellian ^{3}He Minority in Fusion Reactors*)

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Possible fusion reactivity enhancement due to ^{3}He minority ICRF heating in D-^{3}He toroidal plasma is demonstrated in present numerical simulations. On this purpose the particle code based on test-particle approach is developed. This code solves guiding center equations for ^{3}He ions in toroidal magnetic field including Coulomb collisions of these ions with the background deuterons and electrons. A simple Monte Carlo model for ICRF heating is implemented in this code as well. The transformation of ^{3}He distribution function from Maxwellian to non-Maxwellian due to heating plays the key role for reactivity enhancement. The formation of significant energetic tail gives rise to the reactivity enhancement. This is an important issue for the performance of fusion reactors with minority heating of ICRF.

Keywords: fusion plasma, test-particle, ^{3}He minority, ICRF heating, reactivity enhancement

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1. Introduction

One of the possible techniques to decrease neutron load on plasma facing components and superconducting coils in fusion reactors is to use fuel cycle based on D-^{3}He reaction as alternative to D-T. Taking into account that the thermal reactivity of D-^{3}He is much lower than that of D-T, new approach such as ICRF catalyzed fusion should be developed. The main idea of this technique is to modify reagent distribution function in order to achieve favorable reaction rate for nuclear fusion energy production. The effect of transformation from the Maxwellian to non-Maxwellian plasma is essential for reactor aspects studies both in tokamaks and heliotrons.

To provide ignition analysis for D-^{3}He plasma with ^{3}He minority the set of particle and power balance equations should be solved [1]. The main point is that these equations include the term, which is the volume averaged reaction rate. This term gives the fusion reaction intensity and is proportional to the product of densities of fusion reagents and the averaged reactivity. The reactivity itself depends on the distribution functions of fusion reagents.

In this paper we demonstrate the possibility to increase the averaged reactivity by modification of distribution function of ^{3}He minority due to ^{3}He selective ICRF heating. This study is done by means of numerical code, based on test-particle approach. A simple model for ICRF heating is included in code as well.

The structure of this paper is the following: firstly we introduce the particle code, and secondary we present the calculation results of ^{3}He minority distribution function under RF heating. Thirdly, we show the data from reactivity enhancement calculations, and then we summarize our study and describe the directions for the further research.

2. Particle Code

To calculate the distribution function of ^{3}He minority in D plasma we developed a numerical code based on the test-particle approach [2]. This code solves the guiding center equation of a general vector form given by

$$\boldsymbol{v}_{\mathrm{g}} = v_{\parallel}\frac{\boldsymbol{B}}{B} + \frac{c}{B^2}\boldsymbol{E}\times\boldsymbol{B} + \frac{m_a c(2v_{\parallel}^2 + v_{\perp}^2)}{2\,Z_a e\, B^3}\boldsymbol{B}\times\nabla B + \frac{m_a c\, v_{\parallel}^2}{Z_a e B^4}\,(\boldsymbol{B}\times\mathrm{rot}\,\boldsymbol{B})\times\boldsymbol{B}. \tag{1}$$

We solve Eq. (1) by means of the fourth-order Runge-Kutta integrating scheme for the test-particle of mass m_a and charge $Z_a e$ in toroidal magnetic field $\boldsymbol{B}$, taking into account the effect of the electric field $\boldsymbol{E}$. It is considered that the adiabatic invariant of motion $\mu = v_{\perp}^2/B = \mathrm{const}$ together with the total energy $W = m_a(v_{\perp}^2 + v_{\parallel}^2)/2 + Z_a e\,\Phi = \mathrm{const}$ are conserved in the absence of collisions and RF heating.

To simulate the Coulomb collisions of test-particle with the other species the discretized collision operator based on binomial distribution is used [3]. The idea of this operator is that after each integration time step, the test-particle gets a collision kick in pitch-angle $\lambda \equiv v_{\parallel}/v$ and kinetic energy $K \equiv m_a v^2/2$. In case of ^{3}He ions colliding

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with the background deuterons and electrons the operator reads for pitch-angle scattering as

$$\lambda_n = (1 - \nu_d \tau)\lambda_0 \pm \sqrt{\nu_d\left(1-\lambda_0^2\right)\tau}, \tag{2}$$

and for kinetic energy slowing down and scattering

$$K_n = K_o - 2\,\nu_K\left[K_o - T\frac{x\,\psi'(x)}{\psi(x)}\right]\tau \pm 2\sqrt{K_o\nu_K T\tau} \tag{3}$$

where ν_d is the deflection frequency, $\nu_K = \nu_S - x\nu_{\|}$ is the combination of slowing down and parallel velocity diffusion frequencies. The Maxwell integral $\psi(x)$ is a function of the square of ratio of test-particle velocity to the background species thermal velocity. The subscripts 'n' and 'o' stand for the new and old values respectively, and τ is the integration time step [4].

The ICRF heating of minorities is modeled by modifying the perpendicular velocity of ^{3}He when it passes the resonant layer $2\pi F_{\mathrm{RF}} = n\omega_c$ by the value

$$\Delta v_\perp = \frac{Z_a e}{2m_a} I\,|E_+|\,J_{n-1}(k_\perp\rho_L)\cos(\phi_r) + \frac{Z_a e}{8m_a^2 v_\perp}(I\,|E_+|\,J_{n-1}(k_\perp\rho_L))^2\sin^2\phi_r, \tag{4}$$

where E_+ and ϕ_r are the left-circularly polarized component of RF wave electric field and random phase respectively [5]. The argument of Bessel function J_{n-1} is the product of test-particle Larmor radius ρ_L and wave field parameter $k_\perp$. At the same time $k_{\|} = 0$ for our further treatment. The time that a particle needs to pass the resonant layer is given in terms of time derivatives of cyclotron frequency ω_c and harmonic number n as $I = \min\left(\sqrt{2\pi/n\dot{\omega}_c}, 2\pi(n\ddot{\omega}_c/2)^{-1/3}Ai(0)\right)$, where $Ai(0)$ is Airy function.

A simple electric field distribution is assumed throughout this study $E_+ = E_{+0}\tanh((1 - r/a_{pl})/l)\cos\vartheta$ with a plasma minor radius a_{pl}, poloidal angle variable ϑ and wave field structure parameter l.

For our simulations we employ a simple toroidal magnetic field model with magnetic field components presented in the toroidal coordinates as

$$\boldsymbol{B} = B_0\frac{1}{h}\left(\boldsymbol{e}_\vartheta\,\varepsilon_t\frac{1}{q} + \boldsymbol{e}_\varphi\right), \tag{5}$$

where $h = 1 - \varepsilon_t\cos\vartheta$, $\varepsilon_t = r/R_0$ and q is the safety factor.

3. ^{3}He Minority Distribution Function under RF Heating

In the present analysis, two cases of ^{3}He ion fractions were traced during 1 second in fusion plasma with the following parameters given in Table 1.

In the first fraction, 24000 ions were heated on the main harmonic with the RF frequency 50 MHz at the magnetic field of 5.3 T [6]. This is the condition planned for the ICRF heating at ITER. We also employ $k_\perp = 62.8\,\mathrm{m}^{-1}$ and $l = 0.2$ in the present calculations. In this case, the resonance layer $R_{\mathrm{res}} = 6.69$ m appears in greater value of the periodic wave electric field and the time averaged acceleration of the particles is higher than in the other case with 15000 ions under the RF frequency 51 MHz having the resonance layer at $R_{\mathrm{res}} = 6.55$ m.

Table 1 Parameters of fusion plasma.

Major radius R_0	6.2 m
Minor radius of plasma a_{pl}	2 m
Deuterium density n_{D}	$5{\times}10^{19}\ \mathrm{m}^{-3}$
Electron density n_{e}	1.2 n_{D}
Plasma temperature T_{pl}	15 keV
Toroidal magnetic field on axis B_0	5.3 T
RF electric field amplitude E_{+0}	12 kV/m

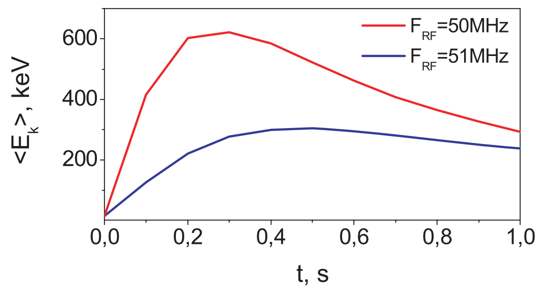

Fig. 1 The averaged kinetic energy for two ^{3}He minority fractions under different heating frequencies versus the tracing time.

Numerical method used to integrate guiding center equation (1) has an error term $O(\tau^5)$ that could be estimated following common algorithm [7]. In present calculations we are able to evaluate this term on a single integration step. Let us note the fact that the value of integration time step varies adaptively to the plasma parameters and the characteristic scale length of magnetic field inhomogenity. Hence, for our test-particle with the kinetic energy varying from 1 keV up to 600 keV we obtain $2.87\times10^{-12} \le O(\tau^5) \le 5.96\times10^{-10}$ for radial coordinate, $5.39\times10^{-9} \le O(\tau^5) \le 1.71\times10^{-6}$ for the poloidal and $6.43\times10^{-9} \le O(\tau^5) \le 2.05\times10^{-6}$ for the toroidal angles.

In Fig. 1 the averaged ion kinetic energy for both particle fractions is displayed versus the tracing time. As one can see for both cases the energy is growing up to 600 keV for 50 MHz heating whereas up to 300 keV for 51 MHz. The mean absorbed power by a single particle from the test-fraction is 6.7×10^{-13} W and 2.9×10^{-13} W respectively.

The kinetic energy decreases after reaching the maximum by the energy transfer to the background deuterons and electrons, which are assumed Maxwellian with a constant temperature 15 keV. The other reason of the energy decay is the escaping of energetic particles from the con-

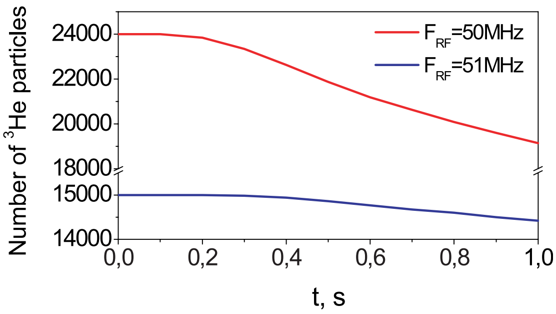

Fig. 2 The number of ^{3}He ions in the confinement volume versus the tracing time.

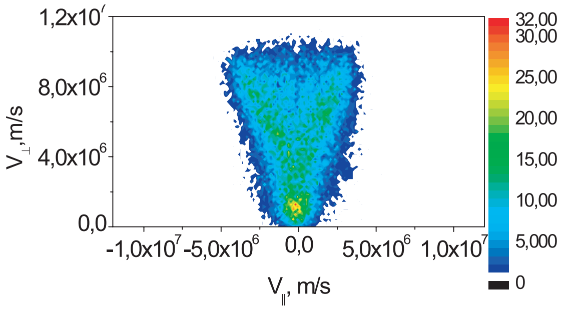

Fig. 3 Distribution function of ^{3}He ions in $(v_\parallel, v_\perp)$ velocity space under F_{RF} = 50 MHz heating at the time slice t = 0.3 s.

finement volume. In Fig. 2 the number of ^{3}He ions in the plasma is plotted versus the tracing time. We should note that there is no source term of ^{3}He ions in the present calculations.

As it was pointed out in the previous section, the energy from the RF heating is deposited in the perpendicular velocity of the test-particles. This effect is included in the model by means of perpendicular velocity alteration given by (4). Hence the distribution function of ^{3}He is modified to anisotropic shape with an elongated tail in the $v_\perp$ direction. The further elongation is prevented by collisions of test-species with background plasma. The energy from the test-particles transfers to the background species and spreads in the pitch-space. This is called the gyro-relaxation effect, which is included in our model by Eqs. (2) and (3). As an example, in Fig. 3 the distribution function of ^{3}He ions in $(v_\parallel, v_\perp)$ velocity space under F_{RF} = 50 MHz heating at t = 0.3 s is demonstrated. The size and the shape of the energetic tail depend on the heating efficiency, particle losses and energy transfer from the energetic fraction to the background plasma.

The distribution functions of ^{3}He ions under the F_{RF} = 50 MHz heating versus the absolute velocity value for several time slices starting from 0.1 s up to 1 s are displayed in Fig. 4.

At each time slice there is a certain energy, which is gained from RF wave and stored in ^{3}He fraction. With the green curves on Fig. 4, the Maxwellian distributions of

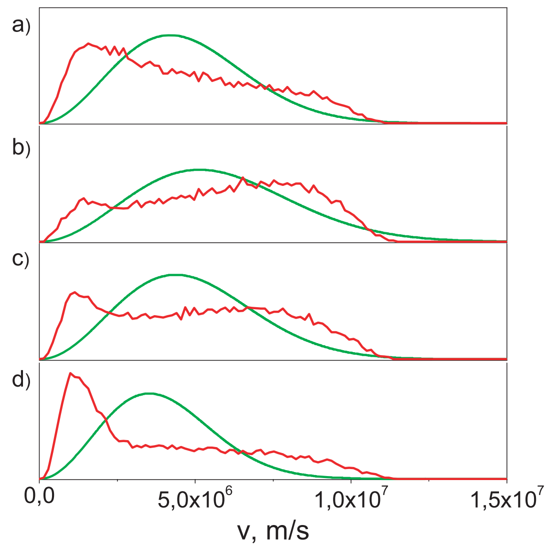

Fig. 4 Distribution functions of ^{3}He ions under F_{RF} = 50 MHz heating (red curves) in time slices 0.1, 0.3, 0.6 and 1 seconds on graphs a), b), c) and d) respectively. Maxwellian plasma distribution functions (green curves) with the same energy stored as in cases of heating are also plotted. Vertical axes are in arbitrary units.

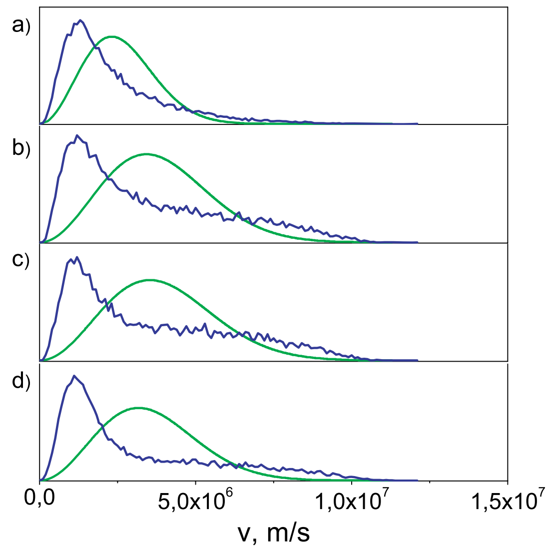

Fig. 5 Similar plots as in Fig. 4 but with F_{RF} = 51 MHz.

^{3}He ions are given for the same stored energy values as in the non-Maxwellian case. The similar plot is presented in Fig. 5 with blue curves for ^{3}He ions under the F_{RF} = 51 MHz heating. The energetic ion tail is still observed, but as it was mentioned above due to less effective heating the number of particles accelerated up to high energies is smaller compared to the previous case.

Collisions with the background plasma particles (deuterons and electrons) truncate the tail as well. Despite the losses of energetic particles, displayed in Fig. 2, up to 1s of tracing we still observe a number of particles that form energetic tails in both heating scenarios.

It should be noted that RF electric field amplitude E_{+0} in both cases is the same. It is caused by assumption that equal RF antenna feeding power is used, and same conditions for wave propagation in plasma are considered. The same absorbed power could be achieved in second case due to increasing of the RF electric field amplitude E_{+0}. But such increasing is limited by technical feasibility of RF antenna feeding system and RF antenna itself. The value for E_{+0} is chosen for the reasonable maximum power of ICRF antennas [8] which are used and designed in fusion devices.

4. Enhancement of Fusion Reactivity

The fusion reactivity is calculated in general as a six-fold integral

$$\langle \sigma v \rangle_{\mathrm{D}\,^3\mathrm{He}} = \iint f_{\mathrm{D}}(\boldsymbol{v}_{\mathrm{D}}) f_{^3\mathrm{He}}(\boldsymbol{v}_{^3\mathrm{He}})\, \sigma(v)\, v \,\mathrm{d}\boldsymbol{v}_{\mathrm{D}}\, \mathrm{d}\boldsymbol{v}_{^3\mathrm{He}}, \tag{6}$$

where $v = |\boldsymbol{v}_{\mathrm{D}} - \boldsymbol{v}_{^3\mathrm{He}}|$ is the relative particle velocity, f_{D} and $f_{^3\mathrm{He}}$ are the distribution functions for deuterium and ^{3}He respectively and $\sigma(v)$ is the fusion cross-section [9]. By means of this expression we are able to calculate the reactivity for modified distribution functions displayed in Figs. 4 and 5.

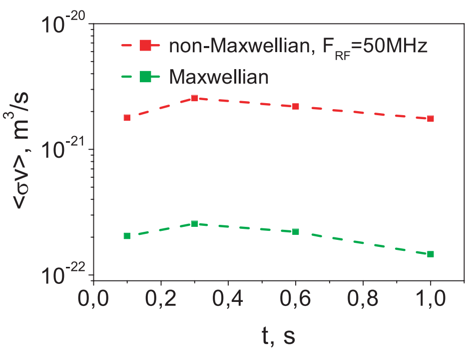

Fig. 6 Reactivity rates ^{3}He (d, p) α for non-Maxwellian ^{3}He distribution function (red) under F_{RF} = 50 MHz heating and corresponding Maxwellian distribution (green) with the same amount of stored energy versus the tracing time.

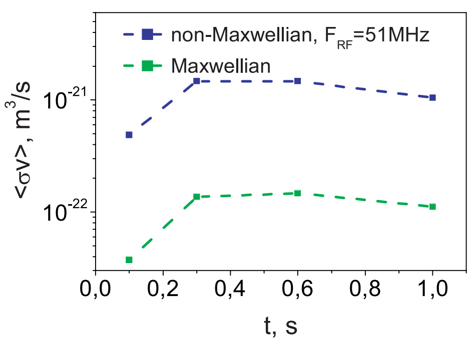

Fig. 7 Similar plot as in Fig. 6 but with F_{RF} = 51 MHz.

On this purpose we include in expression (6) the non-Maxwellian ^{3}He distribution functions $f_{^3\mathrm{He}}$ for both heating scenarios. At the same time deuterium distribution function f_{D} is assumed to stay in the Maxwellian with the constant temperature 15 keV. The reactivity values plotted in Fig. 6 (red) are related to the distribution functions at four time slices displayed in Fig. 4 (red curves) for the F_{RF} = 50 MHz heating scenario. The reactivity rates for F_{RF}= 51 MHz scenario are displayed in Fig. 7 (in blue) with each point on this figure related to the distribution function shape shown (in blue) in Fig. 5.

Then we assume that the ^{3}He fraction could have Maxwellian distribution function but with the stored energy equal to that gained from heating. These reactivities are pointed with green color in Figs. 6 and 7.

The reactivity depends on the relative velocity of the interacting particles. When we increase the energetic tail for one reacting specie (^{3}He) we increase the relative velocity and hence the reactivity by itself. In our case we observe an increment by a factor of 10.

5. Conclusions

The ICRF heating of ^{3}He minority in D-^{3}He toroidal plasma is studied by means of numerical simulations. On this purpose the particle code based on test-particle approach is developed. This code solves the guiding center equations for ^{3}He test particles taking into account Coulomb collisions by means of a discretized collision operator. The detailed analysis of the RF field parameter effect on the reactivity enhancement should be done after accurate account of RF wave penetration and propagation processes in plasma. These issues are complicated by geometry effects of real plasma column and real RF antenna. Besides that self-consistent analytical and numerical models of ICRF heating are under way now. That is why a simple Monte Carlo model for ICRF heating is used in present calculations.

Two heating scenarios with 50 MHz and 51 MHz antenna frequency are examined at the magnetic field of 5.3 T and ITER-like plasma. It is observed that the effective heating of ^{3}He on the main harmonic with 50 MHz frequency is followed by formation of non-Maxwellian distribution function with a significant energetic tail. The same situation but with a less energetic tail appears under 51 MHz heating.

The non-Maxwellian shape of the ^{3}He distribution function plays the key role for reactivity enhancement. It is calculated that the formation of the energetic tail gives rise to the reactivity increase by the factor of 10 in comparison with thermal reactivity for both heating scenarios in the considered fusion plasma.

The increase of reactivity rate is an important issue for the performance of fusion reactors, which needs further detailed studies.

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