Effect of Magnetic-Field Gradient on Positron Acceleration along the Magnetic Field in an Oblique Shock Wave*)

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Ultrarelativistic positron acceleration along the magnetic field due to shock waves in an electron-positron-ion plasma is studied with use of one-dimension (one space coordinate and three velocities), fully kinetic, fully electromagnetic, particle simulations. First, ultrarelativistic acceleration to $\gamma \sim 10^4$ in a uniform external magnetic field $\boldsymbol{B}_0$ is demonstrated with a simulation with the shock speed $v_{\rm sh}$ close to $c\cos\theta$, where c is the speed of light and θ is the angle between the external magnetic field and the wave normal. Then, the effect of non-uniformity of $\boldsymbol{B}_0$ is investigated; comparisons are made of two different cases: 1) the strength of the external magnetic field increases as a shock wave propagates (∇B_0 is parallel to the wave normal), and 2) it decreases (∇B_0 is unti-parallel to the wave normal). It is found that positron acceleration in the latter tends to be stronger than in the former.

Keywords: ultrarelativistic positron acceleration, electron-positron-ion plasma, magnetosonic shock wave, parallel electric field, particle simulation

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1. Introduction

It has been shown with theory and electromagnetic particle simulations [1, 2] that a shock wave propagating obliquely to an external magnetic field $\boldsymbol{B}_0$ in an electron-positron-ion plasma can accelerate positrons to ultrarelativistic energies along the magnetic field with the electric field parallel to the magnetic field; in the simulation of Ref. [2], positron energies reached $\gamma \sim 2000$, where γ is the Lorentz factor.

The time rate of change of γ of an accelerated positron is given as

$$\frac{1}{\Omega_{\rm p}}\frac{{\rm d}\gamma}{{\rm d}t} = \frac{c\cos\theta}{v_{\rm sh}}\frac{(\boldsymbol{E}\cdot\boldsymbol{B})}{(\boldsymbol{B}\cdot\boldsymbol{B}_0)}, \tag{1}$$

where $\Omega_{\rm p}$ is the nonrelativistic positron gyrofrequency, c is the speed of light, $v_{\rm sh}$ is the propagation speed of the shock wave, θ is the angle between $\boldsymbol{B}_0$ and the wave normal, $\boldsymbol{E}$ is the electric field, and $\boldsymbol{B}$ is the total magnetic field. As can be seen from Eq. (1), the energy increase rate is proportional to the parallel electric field,

$$E_\parallel = \frac{(\boldsymbol{E}\cdot\boldsymbol{B})}{B}. \tag{2}$$

Since the strength of the parallel electric field $E_\parallel$ and thus its integral along the magnetic field, $F = -\int E_\parallel {\rm d}s$, are approximately proportional to $\boldsymbol{B}_0^2$ [3, 4], the acceleration is strong when the magnetic field is intense; the parallel pseudo potential F in a shock wave is given as

$$n_{\rm e0}eF \sim \left(\rho v_{\rm A}^2 + \Gamma_{\rm e}p_{\rm e0}\right)\frac{n_{\rm i0}}{n_{\rm e0}}\epsilon, \tag{3}$$

where $n_{\rm e0}$ and $n_{\rm i0}$ are, respectively, the equilibrium electron and ion densities, ρ is the mass density, $v_{\rm A}$ is the Alfvén speed, $\Gamma_{\rm e}$ is the specific heat ratio of electrons, $p_{\rm e0}$ is the electron pressure, and ϵ is the wave amplitude. This equation is valid when the wave amplitude is large [$\epsilon \sim O(1)$]. (For small-amplitude pulses ($\epsilon \ll 1$), F takes different forms [3, 4].)

The acceleration is particularly enhanced when the shock propagation speed $v_{\rm sh}$ is close to $c\cos\theta$,

$$v_{\rm sh} \sim c\cos\theta. \tag{4}$$

In this situation, relativistic particles with their parallel velocity $v_\parallel$ [$= (\boldsymbol{v}\cdot\boldsymbol{B})/B$] close to the speed of light can move with the shock wave for long periods of time.

Provided that the external magnetic field is not uniform, then even if the condition (4) is met at some point, it will break down as the shock wave goes away from that point.

This paper investigates the effect of magnetic-field gradient on the positron acceleration due to shock waves, with use of one-dimension (one space coordinate and three velocities), fully kinetic, fully electromagnetic, particle simulations [5, 6]. First, with a simulation with a uniform external magnetic field we will demonstrate positron acceleration to $\gamma \sim 10^4$. Then, we show that positron acceleration tends to be stronger in decreasing external magnetic field than in increasing one.

2. Simulation Model

Shock waves are supposed to propagate in the positive x direction ($\partial/\partial y = \partial/\partial z = 0$) in an external magnetic field $\boldsymbol{B}_0 = [B_{x0}, 0, B_{z0}(x)]$, where B_{x0} is constant, which is due to $\nabla\cdot\boldsymbol{B} = 0$, while B_{z0} depends on x. We compare

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three cases: 1) constant B_{z0}, 2) increasing magnetic field ($\partial B_{z0}/\partial x > 0$), in which v_A and thus v_{sh} rise whereas $\cos\theta$ goes down as a shock wave propagates, and 3) decreasing magnetic field ($\partial B_{z0}/\partial x < 0$).

The strength of the external magnetic field is taken to be $|\Omega_e|/\omega_{pe} = 12$ for the constant B_{z0} case, where Ω_e (< 0) is the nonrelativistic electron gyrofrequency and ω_{pe} is the plasma frequency. The ion-to-electron mass ratio is $m_i/m_e = 1836$; the ion thermal velocity is $v_{Ti}/(\omega_{pe}\Delta_g) = (T_i/m_i)^{1/2}/(\omega_{pe}\Delta_g) = 0.012$, where T_i is the ion temperature and Δ_g is the grid spacing, while the electron and positron thermal velocities are $v_{Te}/(\omega_{pe}\Delta_g) = v_{Tp}/(\omega_{pe}\Delta_g) = 0.50$; the speed of light is $c/(\omega_{pe}\Delta_g) = 10$. The positron-to-electron density ratio is taken to be $n_{p0}/n_{e0} = 0.02$; hence, the Alfvén speed is

$$v_A = \left(\frac{B_0^2}{4\pi(\Sigma_j n_{j0} m_j)}\right)^{1/2} = 2.8(\omega_{pe}\Delta_g), \qquad (5)$$

where the subscript j refers to electrons (j = e), positrons (j = p), or ions (j = i), and m_j and n_{j0} represent the mass and equilibrium density of particle species j, respectively. The total system length is $L = 65,536\Delta_g$ with the total number of electrons $N_e \simeq 2.5 \times 10^6$ with $N_e = N_i + N_p$. (For more details of the method of shock simulations, see Refs. [1, 2].)

3. Simulation Results

Figure 1 shows positron phase spaces and the profiles of B_z near shock fronts. Three different cases are presented.

In the uniform $\boldsymbol{B}_0$ case depicted in the top panel, the propagation angle is taken to be $\theta = 43°$, and the shock propagation speed is observed to be $v_{sh} = 2.6 v_A$; the condition (4) is therefore met [$v_{sh}/(c\cos\theta) = 1.01$]. Positron energies reach $\gamma \sim 10^4$ by the end of the simulation run, $\omega_{pe}t = 7000$. Since the acceleration has not been saturated, γ will continue to rise if we perform a longer simulation run.

We also note that electrons are accelerated to $\gamma \sim 7000$ in this simulation, with a different mechanism from that of positrons. Some electrons are trapped near the shock transition region and gain a great amount of energy from the strong electric field formed in the shock wave. Since this phenomenon has been described in detail in Refs. [7, 8] we will not discuss this phenomenon in this paper.

The second panel shows the increasing B_{z0} case, in which the gradient of B_{z0} is given as

$$\frac{B_{z0}(x) - B_{z0}(x_0)}{B_{z0}(x_0)} = \frac{x - x_0}{a}, \qquad (6)$$

with the scale length $a = 1.5 \times 10^4 c/\omega_{pe}$. The angle θ becomes 43° at the position x_0, which is taken to be $3600c/\omega_{pe}$. Although the maximum γ is $\sim$ 8000, only a small fraction of positrons have γ greater than 4000. Since

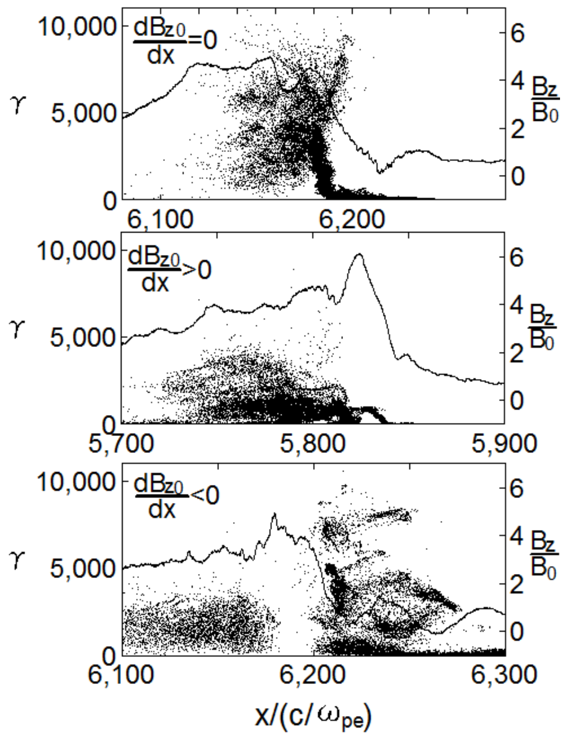

Fig. 1 Snapshots of positron phase spaces (x, γ) near shock fronts. The solid lines show the profiles of B_z. The top panel presents the case of uniform external magnetic field, while the second and the bottom panels display, respectively, increasing and decreasing B_{z0} cases. In the top panel, which is plotted at $\omega_{pe}t = 7000$, the highest energy reaches $\gamma \sim 10^4$, and the acceleration has not been saturated by the end of the simulation run. In the second panel plotted at $\omega_{pe}t = 6500$, the number of positrons with $\gamma > 4000$ are quite small. Particles that enter the shock wave eventually move to the downstream region. In the bottom panel at $\omega_{pe}t = 7000$, we find many high energy positrons with $\gamma > 5000$ near the shock front, behind which there is a region of low positron density.

the difference $(v_{sh} - c\cos\theta)$ keeps increasing as the shock wave propagates, all the positrons eventually move to the downstream region after entering the shock wave.

In the decreasing B_{z0} case (bottom panel), the gradient of B_{z0} is negative; however, the magnitude of a and other parameters are taken to be the same as those in the increasing B_{z0} case. Here, the acceleration is strongest when the shock front is near the point x_0 and has been finished by $\omega_{pe}t = 7000$, with the highest energy close to $\gamma \sim 10^4$. After the encounter with the shock wave, positrons move to the downstream region if $v_{sh} > c\cos\theta$ and go away ahead of the shock front if $v_{sh} < c\cos\theta$. Therefore, positrons that enter the shock wave in the early phase in which $v_{sh} > c\cos\theta$ move to the downstream region, while the positrons encountering the shock wave after v_{sh} has exceeded $c\cos\theta$ are reflected forward from the shock front to the upstream region. As shown in the bottom panel, therefore, there appears a region where the positron density is quite low.

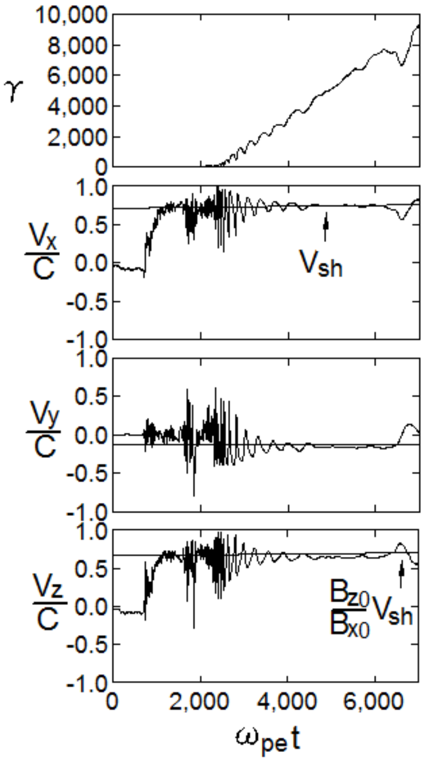

Fig. 2 Time variations of γ and $\boldsymbol{v}$ of a positron accelerated by an oblique shock wave in a uniform external magnetic field. The horizontal lines represent the theoretical values (7)-(9). The value of v_x is close to $v_{\rm sh}$, and the particle velocity is nearly parallel to the magnetic field with v_y much smaller than v_x and v_z. These are consistent with the theory.

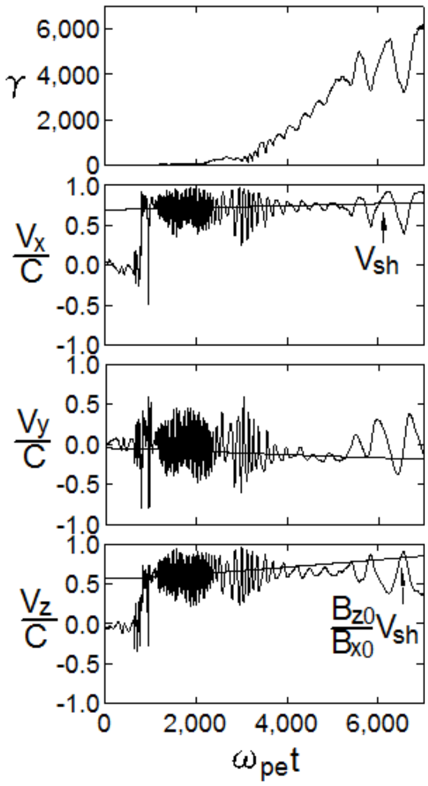

Fig. 3 Time variations of γ and $\boldsymbol{v}$ of a positron accelerated by an oblique shock wave in an increasing B_{z0}. While it is near the shock front, the behavior of γ and $\boldsymbol{v}$ is similar to those in Fig. 2. This particle eventually moves to the downstream region, and its γ and $\boldsymbol{v}$ begin to oscillate at $\omega_{\rm pe}t = 5200$.

Although the magnitudes of the field gradient ($|\partial B_{z0}/\partial x|$) are the same for the second and third panels in Fig. 1, the decreasing B_{z0} case creates higher energy positrons than the increasing B_{z0} case. This arises because in the former case, positrons that encounter the shock wave when the difference $(v_{\rm sh} - c\cos\theta)$ is about to change from positive to negative values tend to penetrate deep into the shock wave and thus spend long periods of time in the shock transition region before going out to the upstream region.

Figure 2 displays the time variations of γ and $\boldsymbol{v}$ of an accelerated positron in a shock wave in a uniform magnetic field. This particle encounters the shock wave at $\omega_{\rm pe}t \simeq 2500$.

After $\omega_{\rm pe}t \simeq 2500$, when the particle is in the shock transition region, its energy keeps rising with nearly a constant increase rate, close to that given by Eq. (1). The observed energy increase rate is $d\gamma/d(\omega_{\rm pe}t) = 1.9$, while the theory (1) gives $d\gamma/d(\omega_{\rm pe}t) = 1.8$ for the present simulation parameters.

The velocity v_x shown in the second panel becomes nearly equal to the shock speed,

$$v_x = v_{\rm sh}, \tag{7}$$

and v_z is given by

$$v_z = \frac{B_{z0}}{B_{x0}} v_x, \tag{8}$$

in the acceleration process. That is, the particle velocity is nearly parallel to the magnetic field. Furthermore, the values of v_y are small compared with v_x and v_z and are close to the theoretical value

$$\frac{v_y}{c} = \frac{B_{x0}B_y\gamma_{\rm sh}^{-2} - E_x B_{z0}(v_{\rm sh}/c)}{B_{z0}B_z(v_{\rm sh}/c) + B_{z0}^2\gamma_{\rm sh}^2(v_{\rm sh}/c)^3}, \tag{9}$$

where $\gamma_{\rm sh}$ is the Lorentz factor corresponding to the shock speed, $\gamma_{\rm sh} = (1 - v_{\rm sh}^2/c^2)^{-1/2}$. These indicate that this particle is accelerated by the mechanism predicted by the theory in Refs. [1, 2].

Figure 3 shows the time variations of γ and $\boldsymbol{v}$ of a positron accelerated by the shock wave in an increasing magnetic field ($\partial B_{z0}/\partial x > 0$). While this particle is in the shock transition region, its energy rises, and its velocity approaches the one given by Eqs. (7)–(9). After it has passed the shock transition region, its γ and velocity components begin to oscillate at $\omega_{\rm pe}t = 5200$ owing to the gyromotion. Since the plasma is moving across the magnetic field behind the shock front, nearly a constant electric field is present there. Particle energies therefore oscillate in association with their gyromotions.

We plot in Fig. 4 the orbit of this particle in the

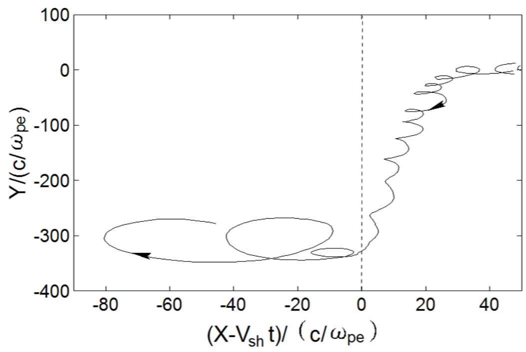

Fig. 4 Orbit in the $(x - v_{\mathrm{sh}}t, y)$ plane of the particle shown in Fig. 3. The vertical dashed line indicates the peak position of B_z. This particle stays in the shock transition region for $\omega_{\mathrm{pe}}t \simeq 2200$, after which goes out to the downstream region.

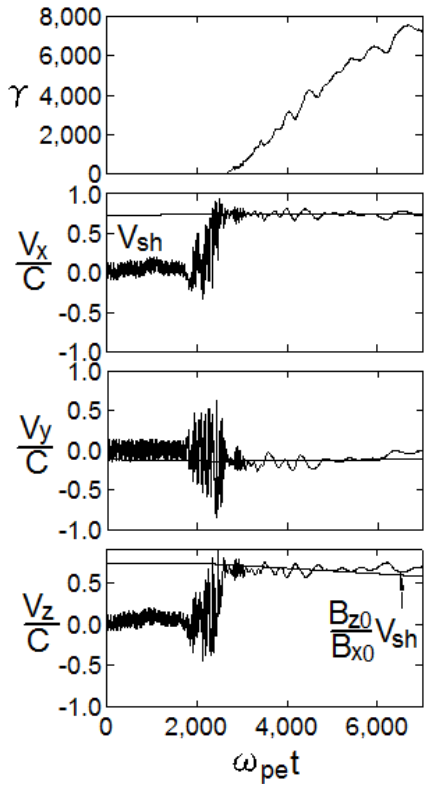

Fig. 5 Time variations of γ and $\boldsymbol{v}$ of a positron accelerated by an oblique shock wave in a decreasing B_{z0}. While it is near the shock front, γ and $\boldsymbol{v}$ exhibit behavior similar to those in Fig. 2. This particle goes out of the shock wave to the upstream region at $\omega_{\mathrm{pe}}t = 6500$.

$(x - v_{\mathrm{sh}}t, y)$ plane. The vertical dashed line represents the position at which B_z takes its maximum value. After entering the shock transition region, this positron stays there for $\omega_{\mathrm{pe}}t \simeq 2200$ and then moves to the downstream region.

Figure 5 shows the case with decreasing magnetic field $(\partial B_{z0}/\partial x < 0)$. This particle is accelerated similarly to that in Fig. 2 when it is near the shock front. It is detrapped (goes away ahead of the shock front) at $\omega_{\mathrm{pe}}t \simeq 6500$, after which its energy does not change much.

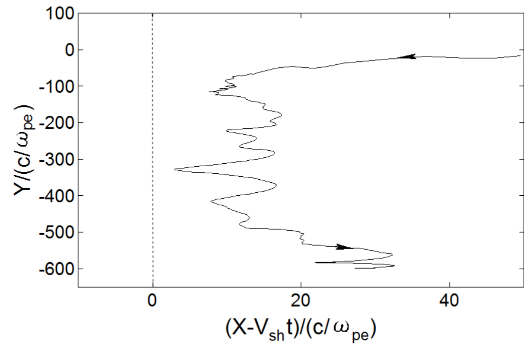

Fig. 6 Orbit in the $(x - v_{\mathrm{sh}}t, y)$ plane of the particle shown in Fig. 5. After entering the shock transition region and staying there for rather a long period of time, this particle goes back to the upstream region.

The time period that this particle is in the shock transition region is $\omega_{\mathrm{pe}}t \simeq 4000$ and is much longer than that of the particle in Fig. 3, $\omega_{\mathrm{pe}}t \simeq 2200$, which leads to the difference in the energies of these particles.

The orbit of this particle in the $(x - v_{\mathrm{sh}}t, y)$ plane is depicted in Fig. 6. This particle penetrates deep into the shock transition region and stays in the shock wave for a period $\omega_{\mathrm{pe}}t \simeq 4000$, after which it is reflected forward to the upstream region.

4. Summary

With use of one-dimension, fully kinetic, fully electromagnetic particle simulations, we have studied positron acceleration along the magnetic field caused by shock waves in an electron-positron-ion plasma, with particular attention to the effect of the non-uniformity of the external magnetic field.

First, we have demonstrated positron acceleration to $\gamma \sim 10^4$ in a uniform external magnetic field. Since the acceleration has not been saturated by the end of the simulation run, positron energies would continue to rise in a longer simulation. We have then compared the cases with increasing and decreasing external magnetic fields and found that the positron acceleration is more enhanced in the latter than in the former.

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