Linear Gyrokinetic Analyses of ITG Modes and Zonal Flows in LHD with High Ion Temperature

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Ion temperature ($T_i$) gradient modes (ITG modes) and zonal flows for high $T_i$ discharges in the Large Helical Device (LHD) are investigated by linear gyrokinetic Vlasov simulation. In recent LHD experiments, high $T_i$ plasmas are generated by neutral beam injection, and spatial profiles of density fluctuations are measured by phase contrast imaging (PCI) [K. Tanaka et al., Plasma Fusion Res. 5, S2053 (2010)]. The observed fluctuations most likely propagate in the direction of the ion diamagnetic rotation in the plasma frame, and their amplitudes increase with the growth of the temperature gradient. The results show the characteristics of ITG turbulence. To investigate the ITG modes and zonal flows in the experiment, linear gyrokinetic simulations were performed in the corresponding equilibria with different $T_i$ profiles by using the GKV-X code [M. Nunami et al., Plasma Fusion Res. 5, 016 (2010)]. The simulation results predict unstable regions for the ITG modes in radial, wavenumber, and phase velocity spaces, in agreement with the PCI measurements. Thus, the fluctuations observed in the experiment are attributed to ITG instability. The responses of the zonal flows show clear contrasts in different field spectra that depend on the $T_i$ profile and agreement with the radial position. In addition to the dependence on the field spectra, the zonal flow residual levels are enhanced by increasing the radial wavenumber as theoretically predicted.

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1. Introduction

Achieving high ion temperature ($T_i$) in confinement plasmas is one of the keys to realize a magnetic fusion reactor, and anomalous transport of the plasmas which can be driven by drift wave plasma turbulence [1] is a critical issue in fusion research. In recent Large Helical Device (LHD) [2] experiments, high-$T_i$ discharges ($T_i \approx 5.6$ keV) have been achieved by high-power neutral beam injection [3]. For better understanding of transport physics in such experiments, quantitative comparisons should be conducted between the experimental observations and computational simulations including zonal flows which play a significant role in regulating turbulent transport in toroidal plasmas [4-6]. In fact, nonlinear gyrokinetic simulations have shown that ion temperature gradient (ITG) turbulent transport in LHD plasma is reduced when the zonal flow generation is enhanced in the inward-shifted LHD configuration [7,8].

Figures 1-(a) and (d) show the radial profiles of the density and the ion and electron temperatures obtained from the LHD high-$T_i$ discharge of shot number 88343 [9,10] in the low-$T_i$ ($r = 1.833$ s) and high-$T_i$ phases ($r = 2.233$ s), respectively. The spatial distributions of the density fluctuations were also measured by two-dimensional phase contrast imaging (2D-PCI) [11]. Figures 1-(b) and (e) show the distributions of the density fluctuations in $k_\rho$-$\rho$ space, and Figs. 1-(c) and (f) show them in $v_{lab}$-$\rho$ space. Here $k_\rho$ is the poloidal wavenumber, $v_{lab}$ is the phase velocity in the laboratory frame, and $\rho$ is the normalized minor radius defined below Eq. (3) in the next section. From the results, the positions in $k_\rho$-$\rho$ space where the fluctuation increases are obtained as

$$v_{peak}^{exp} \sim \begin{cases} 0.8 & \text{for low-$T_i$ phase} \\
0.6 - 0.8 & \text{for high-$T_i$ phase} \end{cases}$$

and

$$(k_\rho \rho_i)_{peak}^{exp} \sim \begin{cases} 0.26 & \text{for low-$T_i$ phase} \\
0.45 & \text{for high-$T_i$ phase} \end{cases}$$

where the ion thermal gyroradius is defined as $\rho_i \equiv v_{th}/\Omega_i$, with the ion gyro frequency $\Omega_i = eB/m_i c$ and the ion thermal speed $v_{th} = \sqrt{T_i/m_i}$ which is obtained from the $T_i$ profiles in the experiment. For example, $\rho_i \sim 1.0 \times 10^{-3}$ m at $\rho \sim 0.9$ in the low-$T_i$ phase, and $\rho_i \sim 1.8 \times 10^{-3}$ m at $\rho \sim 0.7$ in the high-$T_i$ phase. The phase velocities of the fluctuations in the $E \times B$ rotating frame (plasma frame), $v_{\phi ph}^{exp} = v_{lab} - v_{E \times B}$, are given by

$$v_{\phi ph}^{exp} \sim \begin{cases} -1.7 \times 10^3 \text{ m/s} & \text{at } \rho = 0.98 \text{ for low-$T_i$ phase} \\
-3.4 \times 10^3 \text{ m/s} & \text{at } \rho = 0.80 \text{ for high-$T_i$ phase} \end{cases}$$

where the observed velocities in the laboratory frame are $v_{lab} = -1.1 \times 10^3$ m/s (low-$T_i$) and $v_{lab} = -3.9 \times 10^3$ m/s.
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Fig. 1 Radial profiles of electron density, ion temperature, and electron temperature ((a) and (d)), the density fluctuations in wavenumber space ((b) and (e)), and phase velocity space ((c) and (f)), for low-$T_i$ ((a), (b), and (c)) and high-$T_i$ ((d), (e), and (f)) phases. Light blue curves in (c) and (f) represent the $E \times B$ poloidal rotation velocities $v_{E \times B}$ in the laboratory frame. Here, “i-dia” and “e-dia” indicate the directions of ion diamagnetic and electron diamagnetic rotation in the laboratory frame, respectively. The plots are cited from Ref. [10].

In this study, to investigate the characteristics of the experimentally observed density fluctuations, we perform linear gyrokinetic simulations in the equilibrium magnetic field corresponding to the LHD discharge by using the GKV-X code [12] and compare the simulation results with the experimental observations. This paper is organized as follows. In Sec. 2, we briefly describe GKV-X and the basic equations employed in the calculation. In Sec. 3, we show the linear simulation results for the ITG modes and the zonal flow responses in the LHD experiment with different $T_i$ profiles and compare the results for the ITG modes obtained from the simulations and the fluctuation measurements. Finally, we present our conclusions in Sec. 4.

2. GKV-X Code

GKV-X is a gyrokinetic Vlasov flux-tube code that can handle the three-dimensional magnetic field corresponding to experiments in non-axisymmetric systems such as the LHD. GKV-X incorporates full geometrical information on the non-axisymmetric confinement field, as well as the Fourier components of the field obtained from the MHD equilibrium code VMEC [13] through coordinate transformation into Boozer coordinates [14], $\{\rho, \theta_B, \zeta_B\}$. The magnetic field strength is represented in the coordinate system as

$$B = \sum_{n=0}^{n_{max}} B_{mn}(\rho) \cos n\zeta_B + \sum_{m=1}^{m_{max}} \sum_{n=-n_{max}}^{n_{max}} B_{mn}(\rho) \cos (m\theta_B - n\zeta_B).$$

(3)

where the flux labeling index (or normalized minor radius) is defined by $\rho \equiv \sqrt{\psi/\psi_a}$, with the toroidal magnetic flux $\Psi = B_{av} r^2/2$ at the minor radius $r$ and $\psi_a$ at the last closed surface ($r = a$). Here, $B_{ax}$ is the field strength at the magnetic axis, $B_{mn}(\rho)$ is the Fourier component of the magnetic field with the poloidal and toroidal mode numbers ($m, n$), and the maximum mode numbers for poloidal and toroidal directions used in the VMEC calculation are denoted by $m_{max}$ and $n_{max}$, respectively. From the information of the flux surface shapes obtained using the VMEC code, the metric tensor can be obtained. In addition, the Jacobian on
the flux surface is also obtained from the covariant components of the magnetic field, $B_0, B_z$ in Boozer coordinates,

$$\sqrt{\theta_0} = \left(\nabla \phi \times \nabla \theta_0 \cdot \nabla \theta_0\right)^{-1} = \frac{\psi'}{B_0}\left(B_z + q^{-1}(\rho)B_0\right),$$

where $q$ is the safety factor, and the prime symbol represents the derivative with respect to the flux label $\rho$, i.e., $\lambda' = dA/d\rho$. In the code, we used the local flux-tube model [15] with the field-aligned coordinates $\{x, y, z\} = \{r - r_0, (r_0/q_0)\phi - \zeta_0 \}, \{\theta_0\}$ and the safety factor $q_0$ at the minor radius $r = r_0$ defined by the toroidal magnetic flux $\Psi(r_0) = B_0 r_0^2/2$. Then, we regard the coordinate $z = \zeta_0$ as a coordinate along the field line labeled by $\alpha = \zeta_0 - q_0\theta_0$ constant.

GKV-X solves the electrostatic gyrokinetic function for the perturbed ion gyrocenter distribution function $\delta f$ [16, 17],

$$\left(\frac{\partial}{\partial t} + v_1 b \cdot \nabla + \frac{c_B}{B} \nabla \Phi \cdot \nabla - \frac{\mu}{m_i} b \cdot \nabla B \frac{\partial}{\partial \psi} + v_0 \frac{\partial}{\partial \psi}\right) \delta f = (e - v_0 - v_1 b) \cdot \frac{e^2 v_0 \Phi}{T_i} f_M + C(\delta f),$$

(5)

where the velocity-space coordinates $v_0$ and $\mu = m_i v_0^2/2B$ represent the parallel velocity and magnetic moment, respectively. The Maxwellian distribution with temperature $T_i$ is denoted by $f_M$, the collision term is written as $C(\delta f)$, and $b = B/B$ is the unit vector parallel to the magnetic field. The magnetic and diamagnetic drift velocities are defined by $v_0 = (c_B/eB) b \times (\mu v B + m_i v^2 b \cdot \nabla b)$ and $v_1 = (c_B/eB) b \times [\nabla \ln n + (m_i v^2/2T_i - 3/2)\nabla \ln T_i]$, respectively. In the wavenumber space $(k_x, k_y)$, the gyro-phase-averaged electrostatic potential at the gyrocenter position, $\Phi$, is related to the electrostatic potential at the particle position, $\psi$, as $\Phi_{k_x, k_y} = J_0(k_x v_x/\Omega_i) \psi_{k_x, k_y}$. The zeroth-order Bessel function $J_0(k_x v_x/\Omega_i)$ represents the finite gyroradius effect. The electrostatic potential $\psi_{k_x, k_y}$ is calculated from the quasi-neutrality condition,

$$\int d^3 v J_0(\delta \psi_{k_x, k_y} - n_0 \frac{e \psi_{k_x, k_y}}{T_i} [1 - \Gamma_0(b_k)]) = n_{e,k_x, k_y},$$

(6)

where $\delta \psi_{k_x, k_y}$ is the Fourier component of $\delta f$, $n_0$ is the average electron density, $\Gamma_0(b_k) = I_0(b_k) \exp(-b_k)$ with $b_k = (k_x v_x/\Omega_i)^2$, and $J_0$ is the zeroth-order modified Bessel function. The electron density perturbation $n_{e,k_x, k_y}$ is assumed to be given in terms of the electron temperature $T_e$ and the average density $n_0$ by

$$n_{e,k_x, k_y} = \left\{ \begin{array}{ll} e [\Phi_{k_x, k_y} - \langle \Phi_{k_x, k_y} \rangle] / T_e & \text{if } k_y = 0 \\ e \psi_{k_x, k_y} / T_e & \text{if } k_y \neq 0 \end{array} \right.$$

(7)

Also, $\langle \cdots \rangle$ denotes the flux surface average.

3. Linear Simulations

To investigate ITG modes and zonal flows in LHD discharges, we perform linear simulations using the linearized version of GKV-X. We use the code to solve the Fourier transformed expression of Eq. (5) in the linear, collisionless and zero-beta case,

$$\left(\frac{\partial}{\partial t} + v_1 b \cdot \nabla - \frac{\mu}{m_i} b \cdot \nabla B \frac{\partial}{\partial \psi} + i\omega_B\right) \delta f_{k_x, k_y} = F_M(-v_1 b \cdot \nabla - i\omega_B + i\omega_{T_i}) J_0(k_x v_x/\Omega_i) \frac{e \psi_{k_x, k_y}}{T_i},$$

(8)

where $\omega_B$ is the magnetic drift frequency, and $\omega_{T_i}$ is the diamagnetic drift frequency with the following form:

$$\omega_{T_i} = -\frac{c T_i}{e} \frac{\psi}{n_i} \frac{e}{k_B} \left[ \frac{1}{L_n^2} + \frac{1}{L_{T_i}} \left( \frac{m_i v^2}{2 T_i} - \frac{3}{2} \right) \right].$$

(9)

Here, we assume that the equilibrium radial electric field is zero. Therefore, the frequencies observed in the following simulations are regarded as those measured in the plasma frame, i.e., $\omega_{Bim} = \omega_B - \omega_{ExB}$, if there exists an equilibrium electric field. In Eq. (9), $L_{T_i}$ is the background gradient scale length for the density defined by $L_{n_i} = \ln n_i/d\ln r$, and $L_{T_i}$ is the gradient scale length of the ion temperature defined by $L_{T_i}^{-1} = -d\ln T_i/dr$. For other terms and factors in the gyrokinetic equation (8), i.e., the magnetic drift frequency, mirror force term, perpendicular wavenumber, and parallel derivative term, GKV-X employs exact forms including geometrical effects as written in Eqs. (24)-(28) in Ref. [12].

3.1 Equilibria in LHD discharges

For simulations using GKV-X, we should prepare equilibrium configurations corresponding to the experiment. In the LHD discharge of shot number 88343, the profiles of the electron density ($n_e$), ion temperature ($T_i$), and electron temperature ($T_e$) are obtained in the low-$T_i$ phase ($r = 1.833 s$) and the high-$T_i$ phase ($r = 2.233 s$) [10], as shown in Figs. 1-(a) and (d). Using the profiles as input to the VMEC calculation, we obtain the equilibrium field configurations corresponding to the discharge in each $T_i$ phase. Hereafter, we assume $n_i/n_e = 1$ and $T_i/T_e = 1$. The radial profiles of the temperature gradient normalized by the major radius $R_0$, i.e., $R_0/L_T$, and the normalized density gradient $R_0/L_n$ are determined from the sixth-order fitting functions of $\rho$ for the experimental profiles of $T_i$ and $n_i$, as shown in Figs. 2-(a) and (b), respectively. For the obtained equilibria, we plot the radial profiles of the safety factor in Fig. 2-(c) and the profiles along the field line of the magnetic field strength, magnetic drift frequency, and mirror force term in Figs. 3-(a), (b), and (c), respectively, where the safety factor $q = 1.7$ at $\rho = 0.61$ in the low-$T_i$ phase and at $\rho = 0.65$ in the high-$T_i$ phase. The field strength shown in Fig. 3-(a) for the low-$T_i$ phase takes almost the same minimal value at the bottom of each helical ripple, which implies that particles trapped deep in helical ripples have very low radial drift velocities. Comparing the field strength profiles at $q = 1.7$ in Fig. 3-(a), the ripple-trapped particles are considered to have larger radial drift velocities in the high-$T_i$ phase than in the low-$T_i$ phase.
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fig. 2 radial profiles of (a) normalized ion temperature gradient \( R_0/L_{Ti} \), (b) normalized density gradient \( R_0/L_n \), and (c) safety factor \( q \) in the low-\( Ti \) (blue) and high-\( Ti \) (red) phases.

fig. 3 profiles along the field line for (a) normalized magnetic field strength \( B/B_0 \), (b) magnetic drift frequency \( \omega_{Drift} \) normalized by \( \nu_i R_0/k_B \), and (c) mirror force term normalized by \( (\mu B_0/m_i)R_0^{-1} \). All profiles are evaluated with \( k_y = 0 \) and \( q = 1.7 \) at \( \rho = 0.65 \) in the high-\( Ti \) phase (red) and \( \rho = 0.61 \) in the low-\( Ti \) phase (blue). In (b), we use \( \mu/(m_i\nu_i) \) = 0.71 and \( \nu_i/\nu_i = 1.3 \).

fig. 4 growth rates \( \gamma \) (top) and real frequencies \( \omega_k \) (bottom) of the linear ITG modes as functions of the normalized poloidal wavenumber \( k_y \rho_i \) in the low-\( Ti \) (blue) and high-\( Ti \) (red) phases. Open circles and open triangles show the results in the low-\( Ti \) phase at \( \rho = 0.75 \) and \( \rho = 0.83 \), respectively. Solid circles and solid triangles correspond to the results in the high-\( Ti \) phase at \( \rho = 0.65 \) and \( \rho = 0.83 \), respectively.

3.2 profiles of ITG growth rates

Figure 4 exhibits the linear growth rates and real frequencies of the ITG modes as functions of the normalized poloidal wavenumber \( k_y \rho_i \) in the low-\( Ti \) and high-\( Ti \) phases. The plots are evaluated with \( k_y = 0 \) at several radial points. The magnitudes of the growth rates and the real frequencies in the high-\( Ti \) phase are higher than those in the low-\( Ti \) phase, and the growth rates have maximum values for each radial position. For example, at \( \rho = 0.65 \) in the high-\( Ti \) phase, the maximum growth rate is \( \gamma_{max} = 0.25 (\nu_i/R_0) \) for \( k_y \rho_i = 0.35 \). Therefore, we can obtain the radial profiles of the maximum growth rates as shown in Fig. 5. The unstable regions of the ITG modes are localized at the outer radial region, which agrees with the linear eigenvalue analyses [19, 20] with the modeled temperature profiles in other LHD discharges. From the plots, we can find the peaks of \( \gamma_{max} \) which exist at

\[
\rho_{peak}^{-1} \sim \begin{cases} 0.83, & \text{for low-}\ Ti \text{ phase} \\ 0.65, & \text{for high-}\ Ti \text{ phase} \end{cases}
\]

(10)
in radial and wavenumber spaces, respectively. When the results are compared with the fluctuation measurements
in the LHD discharge, the regions of the density fluctuation peaks shown in Eq. (1) are located around the positions where the ITG modes are most unstable in radial and wavenumber spaces. From the real frequencies $\omega_r$ in Fig. 4, the phase velocities at $\rho = \rho_{\text{peak}}^{\text{sim}}$ in the plasma frame, $v_{\text{ph}}^{\text{sim}} = \omega_r/k_y$, can also be roughly estimated as

$$v_{\text{ph}}^{\text{sim}} \sim \begin{cases} 
-2.0 \times 10^5 \text{ m/s} & \text{for low-} T_i\text{ phase} \\
-1.2 \times 10^5 \text{ m/s} & \text{for high-} T_i\text{ phase}
\end{cases} \tag{11}$$

The phase velocities are qualitatively consistent with the experimental observation that the velocity in the high-$T_i$ phase is faster than that in the low-$T_i$ phase, although the absolute values of the velocities are smaller than those in the experiment. Thus, considering the error bars of the experimental data, the density fluctuations observed in the LHD experiments are attributed to the ITG modes.

At $\rho = \rho_{\text{peak}}^{\text{sim}}$, the eigenfunctions of the electrostatic potentials $\phi_k = \phi_i + i\phi_r$ are also obtained as shown in Fig. 6. Both eigenfunctions have ballooning structures, which are typical of the ITG instability. Because helical ripples in the magnetic field have larger amplitudes in the outer radial region, the eigenfunctions at $\rho = 0.83$ in the low-$T_i$ phase are more strongly corrugated than those at $\rho = 0.65$ in the high-$T_i$ phase. In the high-$T_i$ phase, the ITG mode has higher growth rates and more localized structure at the outside of the torus, $-\pi/2 < z < \pi/2$, than the low-$T_i$ phase.

### 3.3 Critical ion temperature gradient

To find the critical ion temperature gradient for the ITG mode in the low- and high-$T_i$ phases, we investigate the dependence of the ITG mode growth rates on the temperature gradient. In Fig. 7, the dependence of $\gamma_{\text{max}}$ on the normalized ion temperature gradient $R_0/L_{Ti}$ is plotted with all parameters except the temperature gradient fixed at the equilibrium values for several radial positions. From the plots, we calculate the linear fitting function for each radial position and obtain the critical values of $R_0/L_{Ti}$ by extrapolating the fitting functions to the point where $\gamma_{\text{max}}$ vanishes. In Fig. 8, we show the critical values for the low- and high-$T_i$ phases as functions of $\rho$. Noting the deviation of the temperature gradient from the critical value in the figure, we find that the $\gamma_{\text{max}}$ value given by Fig. 5 peaks at a radial position where the deviation is largest. The critical values are higher than the experimental values for $\rho \leq 0.6$ in the low-$T_i$ phase and for $\rho \leq 0.2$ in the high-$T_i$ phase, where $\gamma_{\text{max}}$ vanishes in Fig. 5.

In the radial profiles of the critical temperature gradient, the critical values for the high-$T_i$ phase are higher than those for the low-$T_i$ phase. The higher critical tem-
In Sec. 3.1, the magnetic drift frequency in Fig. 9 are evaluated at $\rho/T = 0$ because $R_0/L_{T_i}$ is fixed. We find that the ITG mode is stabilized at $R_0/L_a \gtrsim 14 (\eta_T \lesssim 0.85)$ for the high- $T_i$ phase and at $R_0/L_a \gtrsim 17 (\eta_T \lesssim 0.82)$ for the low- $T_i$ phase. The changes in $\gamma_{\text{max}}$ for the density gradient lengths at $\rho \sim \rho_{\text{peak}}^{\text{sim}}$, that is, $R_0/L_a \sim 9.3 \pm 0.2$ in the low- $T_i$ phase and $R_0/L_a \sim -0.9 \pm 0.2$ in the high- $T_i$ phase, are within a few percent of the central values. Concluding from the $R_0/L_{T_i}$ dependence of $\gamma_{\text{max}}$ in Fig. 7, it can be stated that the effect of errors in the density gradient on the ITG growth rates is negligible, whereas changes in the ion temperature gradients greatly affect the growth rates in the LHD discharge.

3.4 Zonal flow response

Zonal flows are produced by an electrostatic potential perturbation that varies in the radial direction but takes a constant value on a flux surface (hence $k_y = 0$). Therefore, the diamagnetic drift frequency $\omega_{T_i}$ in Eq. (8) does not appear in the gyrokinetic equation for the zonal flow perturbation, excepting from the $f_{\text{peak}}^{\text{sim}}$ phases. In the high- $T_i$ phase, the ITG modes are unstable for a broad range of $R_0/L_a$ with positive and negative signs. In the figure, an increase in $R_0/L_a$ implies an increase in the pressure gradient and a decrease in $\eta_T \equiv L_a/L_{T_i}$ because $R_0/L_{T_i}$ is fixed. We find that the ITG mode is stabilized at $R_0/L_a \gtrsim 14 (\eta_T \lesssim 0.85)$ for the high- $T_i$ phase and at $R_0/L_a \gtrsim 17 (\eta_T \lesssim 0.82)$ for the low- $T_i$ phase. The changes in $\gamma_{\text{max}}$ for the density gradient lengths at $\rho \sim \rho_{\text{peak}}^{\text{sim}}$, that is, $R_0/L_a \sim 9.3 \pm 0.2$ in the low- $T_i$ phase and $R_0/L_a \sim -0.9 \pm 0.2$ in the high- $T_i$ phase, are within a few percent of the central values. Concluding from the $R_0/L_{T_i}$ dependence of $\gamma_{\text{max}}$ in Fig. 7, it can be stated that the effect of errors in the density gradient on the ITG growth rates is negligible, whereas changes in the ion temperature gradients greatly affect the growth rates in the LHD discharge.

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3.4 Zonal flow response

Zonal flows are produced by an electrostatic potential perturbation that varies in the radial direction but takes a constant value on a flux surface (hence $k_y = 0$). Therefore, the diamagnetic drift frequency $\omega_{T_i}$ in Eq. (8) does not appear in the gyrokinetic equation for the zonal flow perturbation, excepting from the $f_{\text{peak}}^{\text{sim}}$ phases. In the high- $T_i$ phase, the ITG modes are unstable for a broad range of $R_0/L_a$ with positive and negative signs. In the figure, an increase in $R_0/L_a$ implies an increase in the pressure gradient and a decrease in $\eta_T \equiv L_a/L_{T_i}$ because $R_0/L_{T_i}$ is fixed. We find that the ITG mode is stabilized at $R_0/L_a \gtrsim 14 (\eta_T \lesssim 0.85)$ for the high- $T_i$ phase and at $R_0/L_a \gtrsim 17 (\eta_T \lesssim 0.82)$ for the low- $T_i$ phase. The changes in $\gamma_{\text{max}}$ for the density gradient lengths at $\rho \sim \rho_{\text{peak}}^{\text{sim}}$, that is, $R_0/L_a \sim 9.3 \pm 0.2$ in the low- $T_i$ phase and $R_0/L_a \sim -0.9 \pm 0.2$ in the high- $T_i$ phase, are within a few percent of the central values. Concluding from the $R_0/L_{T_i}$ dependence of $\gamma_{\text{max}}$ in Fig. 7, it can be stated that the effect of errors in the density gradient on the ITG growth rates is negligible, whereas changes in the ion temperature gradients greatly affect the growth rates in the LHD discharge.
Radial positions are determined by the radial profiles of the field spectrum. Figure 11 shows that the initial behavior of the residual zonal flow levels are evaluated at $t \sim 23\left( R_0/\nu_i \right)$, as presented in Table 1, for $k_x \rho_i = 0.12$ and $k_x \rho_i = 0.24$. According to an analytical study [21], the early behavior of $K_L(t)$ depends mainly on the Fourier spectrum of the field strength. In the late evolution, the residual zonal flow level is affected by the perpendicular wavenumber as well as the field spectrum. Figure 11 shows that the initial behavior of $K_L$ for $t \leq 1.0\left( R_0/\nu_i \right)$ does not change for different perpendicular wavenumbers, but is influenced by the equilibrium with the different $T_i$ profiles for each safety factor. We also observe that the residual zonal flow levels are enhanced by increasing the radial wavenumber and are also influenced by the helical Fourier components, which depend on the radial position and the $T_i$ profile, as seen in Fig. 3-(a). These simulation results are consistent with the analytical conclusions for the zonal flow responses [21]. Note that the neoclassical transport optimized LHD configuration, the so-called inward-shifted LHD configuration, yields a larger $K_L(t)$ than that in the standard LHD configuration [24]. In our simulations, in fact, the field strength spectrum of the low-$T_i$ phase is similar to that of the inward-shifted LHD configuration used in Refs. [7,24].

Hence $K_L(t)$ is larger in the low-$T_i$ phase than in the high-$T_i$ phase.

### 4. Conclusions

In this study, we investigated ITG modes and zonal flows in LHD discharges by using linear gyrokinetic simulations. We focused on the high-ion-temperature LHD discharge where the density fluctuations are measured in radial, wavenumber, and phase velocity spaces. In the simulations of the low- and high-$T_i$ phases in the discharge, the unstable ITG modes with typical ballooning structures have growth rates that peak at radial positions with the largest deviation of the temperature gradient from the critical...
ical values. The peak positions of the growth rate in radial and wavenumber spaces are close to the regions where density fluctuation peaks are measured experimentally. The critical temperature gradients in the high-$T_i$ phase are higher than those in the low-$T_i$ phase. The former phase has more favorable magnetic curvature and lower density gradients than the latter. In addition, the characteristics of the phase velocities of the ITG modes agree qualitatively with the experimental observations. From the simulation results, therefore, the experimentally observed increases in the fluctuations are attributed to ITG instability. On the other hand, we observed that the Fourier spectra of the field strengths affect the early behavior of the zonal flow responses in different $T_i$ phases, whereas the residual levels are influenced by the radial wavenumber, the radial position, and the field spectra, as predicted theoretically.

ITG turbulence is considered to play a vital role in anomalous ion heat transport in low-$\beta$ plasmas. Nonlinear gyrokinetic ITG turbulence simulations are required to quantitatively evaluate the amplitudes and spectra of the turbulent fluctuations and the resulting anomalous transport coefficients. Although the present analyses are limited to linear simulations of the ITG modes and zonal flows, nonlinear simulations of ITG turbulence under the LHD’s experimental conditions are currently in progress as the next step of this study, and the results will be reported elsewhere.

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